

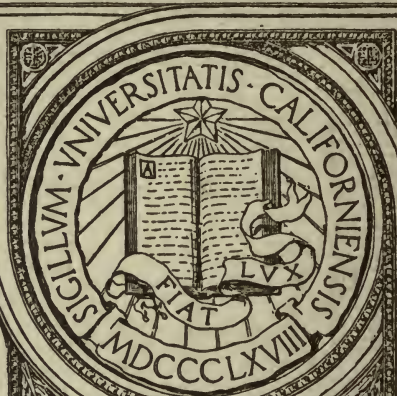
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THE ELEMENTS OF
PLANE AND SPHERICAL
TRIGONOMETRY



THE ELEMENTS OF PLANE AND SPHERICAL TRIGONOMETRY

BY

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AND

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PREFACE.

The subject matter of the chapters of this book devoted to spherical trigonometry has been used in pamphlet form for the last four years in Princeton University, and that of the chapters on plane trigonometry for the last three years.

The aim of the authors has been to present in as brief and clear a manner as possible the essentials of a short course in trigonometry. It has been found that the plane trigonometry may be covered in about thirty recitations, and the spherical trigonometry in somewhat less than this time.

It has been thought advisable to devote some time to drawing the graphs of simple equations in polar coördinates. The reason for this is two-fold. Firstly, because such problems aid in giving the student a clearer idea of the way in which the trigonometric functions vary as the angle is changed; and secondly, because of a very common lack of sufficient knowledge of polar coördinates on the part of students beginning the study of calculus.

The fact that the trigonometric functions are ratios of line segments is emphasized, and their representation by means of lengths of lines is used as little as is conveniently possible.

Certain of the proofs of theorems are shorter than in many text books, and, it is hoped, thereby made more clear; notably the proofs of the formulæ for $\sin(A \pm B)$ and $\cos(A \pm B)$.

The logarithmic tables were taken from Crockett's five place tables, and the proof sheets carefully compared with Albrecht's tables. It is hoped that few errors will be found. The explanation of the tables will be found at the end of the book. It is designed that the matter there contained be given as lessons, and for this reason the student is given examples involving the difficulties usually encountered by the beginner.

The authors wish to thank the members of the mathematical department of Princeton University who have kindly suggested several changes from the form used in the pamphlet editions.

PRINCETON, N. J.,
March 1, 1911.

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ELEMENTS OF PLANE TRIGONOMETRY.

CHAPTER I.

1. Positive and Negative Quantities. It is of fundamental importance for the student of trigonometry to realize at the outset the meaning of negative as well as of positive quantities. That a negative result may have an actual meaning is shown by the following examples: Let us suppose that a man has x dollars and that he owes y dollars. Then the difference $x - y$ denotes the number of dollars he will have after paying his debts. If now this difference happens to be negative, it will indicate the number of dollars he will still owe after he has paid out all of his cash.

As a second example consider the following: A man A is observed to pass a certain inn at exactly noon, walking at the rate of a miles per hour. At two o'clock another man B passes the same point at the rate of b miles per hour. How far will they be beyond the inn when B overtakes A ?

Let the required distance be denoted by x . A will take x/a hours to walk the distance, and B will take x/b hours. Also since A has a two-hour start on B , the latter must walk the x miles in two hours less than A requires. We therefore have the equation

$$\frac{x}{b} = \frac{x}{a} - 2$$

and hence

$$x = \frac{2ab}{b - a}.$$

Now suppose that A walks two miles and B four miles per hour. Then B will overtake A $2 \cdot 2 \cdot 4 / (4 - 2)$ or eight miles beyond the inn. If on the other hand A walks three miles and B two miles per hour, they will be together $2 \cdot 3 \cdot 2 / (2 - 3)$ or -12 miles beyond the inn. The interpretation of this

negative result is evidently that they were together twelve miles before they came to the inn. That this is true is easily verified as follows: Suppose they are together at this place. It will take A $12/3$ or four hours to reach the inn, and B $12/2$ or six hours. Hence, if A gets there at noon, B will arrive at two o'clock as given in the problem.

2. Positive and Negative Lines. The second problem leads us naturally to the idea of positive and negative directions along a line; that is, it brought out the fact that if we assume

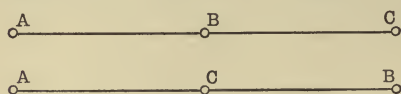


FIG. 1.

one direction of a line to be positive, the opposite direction algebraically is negative. Let A and B be two points of a line.

By the symbol AB we shall mean not only the distance between A and B , but also the direction from A to B . The easiest way in which to think of the symbol AB is to consider it as expressing a motion along the line from A to B . Let us now have a third point C on the line. By the expression $AB + BC$ we mean a motion from A to B and thence to C . The total motion is therefore merely from A to C . This is expressed by the equation $AB + BC = AC$. It is to be carefully observed that this equation is true no matter in what order the points A , B , and C may occur on the line. In particular we have $AB + BA = AA = 0$, or $AB = -BA$.

3. Positive and Negative Angles. Let OX be a fixed line and let the positive direction be from O to X . Also let a second line OP , whose positive direction is from O to P , rotate about O in the plane of the paper. The two lines OX and OP are said to form the angle XOP . Just as the symbol AB denotes not only the distance between A and B , but also the direction from A to B , so the symbol XOP denotes not only the size of the angle formed by the lines OX and OP , but also the direction of rotation from OX into OP . We may consider XOP as denoting a turning of the line OX through a given angle in a given direction. Let us now have another line OQ through O . The symbol $XOP + POQ$ then denotes a rotation of OX into the position of OP and thence into that of OQ .

The total rotation is merely from OX into OQ , and therefore we have the equation $XOP + POQ = XOQ$. In particular $XOP + POX = XOX = 0$, or $XOP = -POX$. In the angle XOP , OX is known as the *initial line* and OP as the *terminal line*.

We may choose either direction of rotation as positive. It is however customary to take as positive the counter-clockwise rotation, that is, a rotation in the direction opposite to that of the hands of a clock. Thus in Fig. 2 XOP , XOQ , and POQ are positive angles, while POX , QOX , and QOP are negative angles.

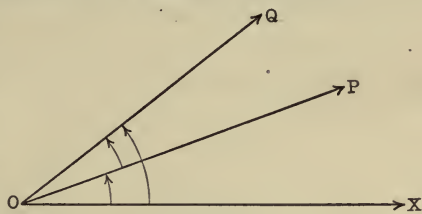


FIG. 2.

4. Measurement of Angles. Two systems of measuring angles are in common use. They are known as the *sexagesimal* system and the *radian* or *circular measure* system.

In the sexagesimal system a right angle is divided into ninety equal parts called degrees. Each degree is in turn divided into sixty equal parts called minutes, and finally each minute is divided into sixty equal parts called seconds. To avoid writing degrees, minutes and seconds, the symbols $^{\circ}$, $'$, $''$ are employed. Thus $69^{\circ} 34' 26''$ is read sixty-nine degrees, thirty-four minutes and twenty-six seconds.

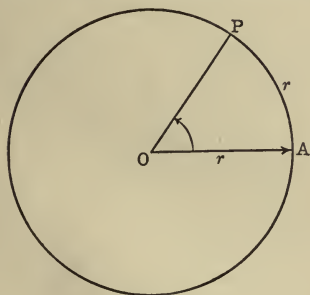


FIG. 3.

In the radian system the unit angle is that angle which at the centre of a circle subtends an arc equal in length to the radius. This unit angle is known as a radian.

In Fig. 3 let AOP be an angle at the centre of a circle of radius r . If this angle is equal to a

radian, we have

$$\begin{aligned} AOP : 360^{\circ} &= \text{arc } AP : \text{circumference} \\ &= r : 2\pi r, \end{aligned}$$

and hence $AOP = \text{one radian} = 360^{\circ}/2\pi = 57.2^{\circ} +$.

If we draw a circle of any radius r with the centre at the vertex of a given angle, this angle will contain as many radians as the arc subtended by it contains r . Hence the radian measure of an angle is the ratio of the length of the arc to the radius. Thus the arc subtended by 360° is $2\pi r$, and therefore the radian measure of 360° is $2\pi r/r$ or 2π . Similarly the radian measure of 90° is $\pi/2$, that of 60° is $\pi/3$, etc.

5. Exercise I.

1. Express each of the following angles in radians: 30° , 150° , 750° , 45° , 260° , -75° .

2. Express each of the following angles in degrees, minutes, and seconds $\frac{2}{3}\pi$, $\frac{1}{5}\pi$, $\frac{2}{7}\pi$, $\frac{3}{2}\pi$, $-\frac{2}{11}\pi$.

3. Find the angle subtended at the centre of a circle of radius 3 inches by an arc of 5 inches.

4. If the radius of the earth is 3,960 miles, find the angle at the centre between radii drawn to two points of the surface 200 miles apart.

6. Coördinates. Let $X'OX$ and $Y'OY$ be two perpendicular lines intersecting at O . These lines divide the plane into four parts called quadrants. The portion of the plane above $X'OX$ and to the right of $Y'OY$ is called the first quadrant. The

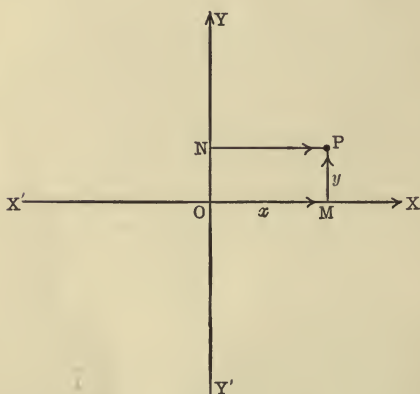


FIG. 4.

second quadrant includes the portion of the plane above $X'OX$ and to the left of $Y'OY$, the third quadrant that below $X'OX$ and to the left of $Y'OY$, and finally the fourth quadrant that below $X'OX$ and to the right of $Y'OY$. By long usage it has become customary to consider the directions OX and OY as positive. In what fol-

lows we shall therefore consider horizontal lines as positive when drawn to the right, and vertical lines as positive when drawn upwards.

Let now P be any point of the plane, and let MP and NP be the perpendicular distances of P from OX and OY respectively. If the point P be given, the distances NP and MP (or the equivalent pair OM and MP) can be measured. Moreover OM and MP cannot have the same pair of values for two different points. This is obviously true if the two points lie in the same quadrant. If on the other hand they lie in different quadrants, one or both of the quantities OM and MP must have different signs in the two quadrants.

For convenience we shall denote the horizontal distance OM by x , and the vertical distance MP by y . The two numbers x and y are called the *coördinates* of the point P . If P be in the first or fourth quadrant OM is measured to the right. Hence x is positive if P be in the first or fourth quadrant. If P be in the second or third quadrant OM is measured to the left, and hence x is negative if P be in either of these quadrants. Similarly y is positive in the first and second quadrants, for in these quadrants MP is measured upwards. In the third and fourth quadrants MP is measured downwards, and therefore in these quadrants y is negative.

Example. On a figure draw the points for which $x=2$, $y=3$; $x=-2$, $y=3$; $x=-2$, $y=-3$; $x=2$, $y=-3$.

To construct the first of these points we must make $OM = x = 2$. Then M is two units to the right of O on the line OX . Also $MP = y = 3$. We must then draw MP perpendicular to OX at M and mark off three units above M on this line.

To construct the second point we must make $OM = x = -2$. Then M is two units to the left of O on the line OX' . The required

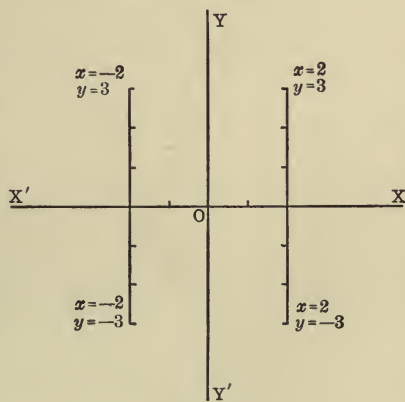


FIG. 5.

point is then three units above M . The other two points may be similarly constructed. They will be found to lie in the

third and fourth quadrants respectively. The four points evidently form the vertices of a rectangle whose centre is at O .

Example. Construct the points for which

(a) $x=7$, $y=2$; (b) $x=-3$, $y=5$;

(c) $x=-3$, $y=-3$; (d) $x=6$, $y=-5$.

7. Trigonometric Functions. In general one quantity is said to be a function of another quantity if the value of the former depends only upon that of the latter. As a simple example suppose that a man is walking at the rate of four miles per hour, and that he walks for t hours. If x denote the distance he walks, we evidently have the relation $x=4t$. The value of x depends only upon that of t . Hence we say that x is a function of t .

Let us now as in page 4 have two perpendicular lines $X'OX$ and $Y'OY$. With OX as initial line draw the angle XOP . This angle is said to be in the first, second, third, or fourth quadrant according as the terminal line OP falls in one or other

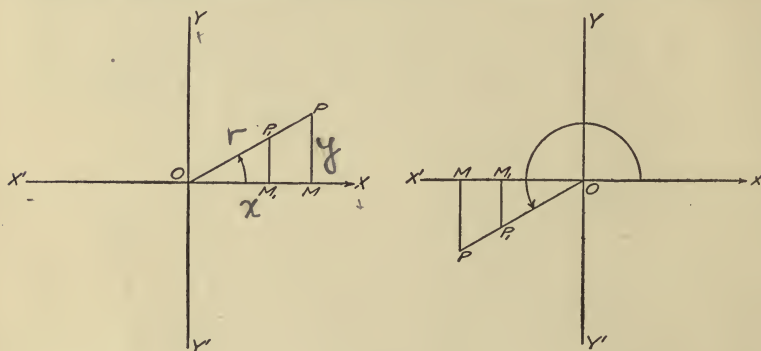


FIG. 6.

of these quadrants. In any one of these cases take a point P on the terminal line of the angle, and let fall the perpendicular MP from P upon OX . Also let P_1 be any other point on OP , and let the corresponding perpendicular be M_1P_1 . Any one of the triangles OMP , OM_1P_1 , etc., is called a reference triangle for the angle XOP .

The two right triangles OMP and OM_1P_1 are similar, and hence the ratio of any two sides of the triangle OMP is equal

to the ratio of the corresponding sides of the triangle OM_1P_1 . Hence the values of these ratios depend not upon the position of the point P upon OP , but only upon the size of the angle XOP . They are then functions of this angle. Let the angle XOP be denoted by θ . Also let r denote the length of OP , and x and y the values of OM and MP respectively. From the three quantities x, y, r we can form ratios all of which we have just seen are functions of the angle θ . We define these functions as follows:

$$\text{sine of } \theta = \frac{MP}{OP} \quad \text{or} \quad \sin \theta = \frac{y}{r},$$

$$\text{cosine of } \theta = \frac{OM}{OP} \quad \text{or} \quad \cos \theta = \frac{x}{r},$$

$$\text{tangent of } \theta = \frac{MP}{OM} \quad \text{or} \quad \tan \theta = \frac{y}{x},$$

$$\text{cotangent of } \theta = \frac{OM}{MP} \quad \text{or} \quad \cot \theta = \frac{x}{y},$$

$$\text{secant of } \theta = \frac{OP}{OM} \quad \text{or} \quad \sec \theta = \frac{r}{x},$$

$$\text{cosecant of } \theta = \frac{OP}{MP} \quad \text{or} \quad \csc \theta = \frac{r}{y},$$

$$\text{versed sine of } \theta = 1 - \frac{OM}{OP} \quad \text{or} \quad \text{vers } \theta = 1 - \frac{x}{r}.$$

8. The Signs of the Trigonometric Functions. Since the positive direction of the terminal line of θ is from O to P , the value of r is in all cases positive. The signs of the trigonometric functions will then depend only upon the signs of x and y .

The Sine. The sine will be positive whenever y is positive; that is, $\sin \theta$ is a positive number if θ be an angle in the first or second quadrant. In the third and fourth quadrants y is negative and therefore $\sin \theta$ is a negative number if θ be an angle in the third or fourth quadrant.

The Cosine. The cosine of θ will be positive or negative according as x is positive or negative. Hence $\cos \theta$ is positive in the first and fourth quadrants, and negative in the second and third.

The Tangent. The tangent of θ will be positive when and only

when x and y are of the same sign, for then only can their ratio be positive. Hence $\tan \theta$ is positive in the first and third quadrants, and negative in the second and fourth.

Let the student similarly consider the cases of the cotangent secant, and cosecant. We may arrange the results of this section in the form of a table as follows:

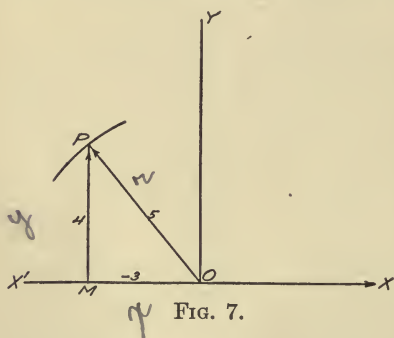
	I	II	III	IV
sine, cosecant.	+	+	-	-
cosine, secant.	+	-	-	+
tangent, cotangent.	+	-	+	-

9. Given One Function of an Angle to Find all the Other Functions of This Angle. Consider the following problem. Given that θ is an angle in the second quadrant and that $\cos \theta = -\frac{3}{5}$, find all the other functions of θ .

We have

$$\cos \theta = \frac{x}{r} = -\frac{3}{5}.$$

In constructing the reference triangle for θ we may take for OP any convenient length. Hence we may put $r = 5$, and so the conditions of our problem give $x = -3$. We shall now



try to construct an angle θ in the second quadrant such that $x = -3$ and $r = 5$. On $X'OX$ lay off $OM = x = -3$. Then M is the point three units to the left of O . At M erect a perpendicular to $X'OX$. With O as centre draw the arc of a circle of radius 5 cutting the perpendicular at P . The angle

$\theta = XOP$ satisfies the conditions of the problem. The length of MP may be calculated from the relation $MP^2 = OP^2 - OM^2$. This gives $MP^2 = 25 - 9 = 16$, and therefore $MP = \pm 4$. But MP is drawn in the positive direction and therefore $MP = 4$.

Hence we have

$$\sin \theta = \frac{y}{r} = \frac{4}{5}, \quad \csc \theta = \frac{r}{y} = \frac{5}{4}, \quad \tan \theta = \frac{y}{x} = -\frac{4}{3},$$

$$\cot \theta = \frac{x}{y} = -\frac{3}{4}, \quad \sec \theta = \frac{r}{x} = -\frac{5}{3}.$$

In an exactly similar way any problem of this type may be solved.

10. Exercise II.

1. Find all the functions of θ when given that

(a) θ is an angle in the third quadrant and $\tan \theta = \frac{3}{4}$.

(b) θ is an angle in the second quadrant and $\sec \theta = -\frac{1}{2}$.

(c) θ is an angle in the fourth quadrant and $\sin \theta = -\frac{2}{3}$.

(d) $\cos \theta = -\frac{3}{7}$ and $\sin \theta$ is negative.

(e) $\sin \theta = \frac{1}{2}$ and $\sec \theta$ is negative

(f) $\text{vers } \theta = \frac{3}{2}$ and $\tan \theta$ is positive.

(g) $\tan \theta = -3$ and $\cos \theta$ is positive.

(h) $\csc \theta = \frac{1}{5}$ and $\sec \theta$ is negative.

(i) $\sin \theta = -\frac{1}{2}$ and $\text{vers } \theta$ is greater than 1.

(j) $\text{vers } \theta = \frac{3}{4}$ and $\csc \theta$ is negative.

2. If θ is an angle in the fourth quadrant and $\tan \theta = -\frac{4}{3}$, find the value of $(\sin \theta + 4 \cos \theta)(\text{vers } \theta - 2 \tan \theta)^2$.

3. If θ is an angle in the second quadrant and $\sin \theta = \frac{3}{5}$, find the value of $\sqrt{\tan \theta + \text{vers } \theta}(\cos \theta - 2 \sec \theta)$.

4. State which of the following problems are possible, and in those cases which have solutions find all the functions of θ :

(a) $\cos \theta = -\frac{3}{5}$, $\tan \theta = \frac{4}{3}$,

(b) $\sec \theta = \frac{3}{2}$, $\tan \theta = \frac{-\sqrt{5}}{2}$,

(c) $\sin \theta = -\frac{1}{2}$, $\csc \theta = 2$,

(d) $\csc \theta = \frac{1}{3}$,

(e) $\csc \theta = 3$, $\cos \theta = \frac{2\sqrt{3}}{3}$.

5. Determine the signs of the following quantities: $\sin 150^\circ$, $\tan 310^\circ$, $\csc 50^\circ$, $\sec 123^\circ$, $\sec \frac{2\pi}{3}$, $\cot \frac{5\pi}{4}$.

11. Variation in Size of the Trigonometric Functions. With

the vertex of the angle XOP as centre draw a circle of unit radius. Let this circle cut OX , OP , and OY in A , P and B respectively. At A and B draw tangents to the circle to meet OP produced in T and S respectively.

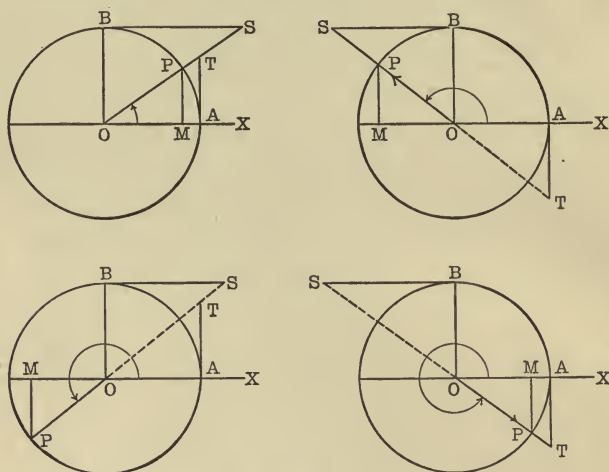


FIG. 8.

We have from the similar triangles OMP , OAT , and SBO

$$\sin \theta = \frac{MP}{OP} = \frac{MP}{1} = MP, \quad \cos \theta = \frac{OM}{OP} = \frac{OM}{1} = OM,$$

$$\tan \theta = \frac{MP}{OM} = \frac{AT}{OA} = \frac{AT}{1} = AT, \quad \cot \theta = \frac{OM}{MP} = \frac{BS}{OB} = \frac{BS}{1} = BS,$$

$$\sec \theta = \frac{OP}{OM} = \frac{OT}{OA} = \frac{OT}{1} = OT, \quad \csc \theta = \frac{OP}{MP} = \frac{OS}{OB} = \frac{OS}{1} = OS.$$

$$\text{vers } \theta = 1 - \cos \theta = 1 - OM = OA - OM = MA.$$

The numerical value of the ratio $\sin \theta$ is equal to the number of units of length of the line MP , where the radius of the circle is taken as this unit; and similarly for the other functions. Observe that the signs of the functions of any angle may be read off from this figure. Such a function is positive or negative according as the line corresponding to it is drawn in the positive or negative direction. Thus if θ be an angle in the second quadrant, AT is drawn downwards. Hence the tangent of an

angle in the second quadrant is negative. These results will be found to agree with those of page 8.

Let us now consider the changes in the values of the trigonometric functions as the angle θ increases from 0° to 360° .

The Sine. As θ approaches 0° the length of the line MP approaches zero. We have therefore $\sin 0^\circ = 0$. As θ increases from 0° to 90° MP continually increases from 0 to OB or 1. Therefore $\sin 90^\circ = 1$, and the sine of any angle in the first quadrant is a *positive number less than unity*. Now as θ increases from 90° to 180° MP decreases from 1 to 0 remaining positive. Hence $\sin 180^\circ = 0$, and the sine of an angle in the second quadrant is a positive number less than unity. Similarly as θ increases from 180° to 270° MP becomes negative and decreases from 0 to -1 . Then $\sin 270^\circ = -1$, and the sine of an angle in the third quadrant is a negative number between 0 and -1 . In a similar manner, as θ increases from 270° to 360° MP remains negative and increases in value from -1 to 0. Hence $\sin 360^\circ = 0$, and the sine of an angle in the fourth quadrant is a negative number between -1 and 0.

The Tangent. When $\theta = 0^\circ$ we have $AT = 0$, and hence $\tan 0^\circ = 0$. As θ increases from 0° to 90° AT increases without limit. For as θ approaches 90° the line OT approaches the position of a parallel to AT . Hence when $\theta = 90^\circ$, $AT = \tan 90^\circ = \infty$. The tangent of an angle in the first quadrant is therefore a positive number, and may have any value between 0 and ∞ . As θ enters the second quadrant AT becomes negative, and as the angle approaches 180° the length of AT approaches 0. Hence the tangent of an angle in the second quadrant is a negative number, and may have any value between $-\infty$ and 0. In the third quadrant the tangent again becomes positive, and increases in value from 0 at 180° to ∞ at 270° . Finally as θ increases from 270° to 360° $\tan \theta$ becomes negative and increases in value from $-\infty$ at 270° to 0 at 360° .

12. Exercise III.

1. What angle between 0° and 360° has the greatest sine? The least sine? The greatest cosine?

2. As θ increases from 0° to 360° explain how the following functions vary: (a) The cosine, (b) the cotangent, (c) the secant, (d) the cosecant, (e) the versed sine.

3. For what values of θ is $\sin \theta = \cos \theta$? $\sin \theta = -\cos \theta$?

4. Given an angle in the first quadrant, show on a figure how to draw an angle in the second quadrant such that the sines of the two angles shall be equal. Could the same problem be given with cosines instead of sines?

5. Given an angle θ in the first quadrant, in what quadrant must ϕ be if $\tan \theta = \tan \phi$? On a figure show how to construct ϕ when given θ .

13. Periodic Functions. A body is said to have a *periodic* motion if it repeats the same cycle of positions over and over after equal intervals of time. Thus the motion of the earth around the sun is a periodic motion since every year it repeats the same relative set of positions. The length of time that elapses before the same position is again reached is called the *period*. In a similar manner a function is said to be periodic if its value for a given value of the variable is always the same as its value when the variable is increased by a certain constant. The amount by which the variable must be increased in order to have the function repeat its original value is known as the period of the function.

In Art. 11 we considered the changes in the values of the trigonometric functions as the angle increased from 0° to 360° . If now the angle increase beyond 360° the terminal line OP will evidently run through the same set of positions that it occupied when the angle was between 0° and 360° . The reference triangles for the two angles θ and $360^\circ + \theta$ will therefore be exactly the same. It follows that all the trigonometric functions of $360^\circ + \theta$ are exactly the same as the corresponding functions of θ . In other words the trigonometric functions are periodic functions of period 360° . As a matter of fact it will be seen later that the tangent and cotangent are periodic functions of period 180° , for it will be shown that $\tan (180^\circ + \theta) = \tan \theta$ and that $\cot (180^\circ + \theta) = \cot \theta$. (Page 36, ex. 7.)

14. Acute Angles; Right Triangles. Let θ be an acute angle, that is an angle in the first quadrant. The six trigonometric functions of θ are positive. The angle θ is one of the acute angles of the triangle OMP . In this triangle OP or r is the *hypotenuse*, OM or x is the side *adjacent* to the angle θ , and MP

or y is the side *opposite* to this angle. We then have for the six trigonometric functions of an *acute* angle:

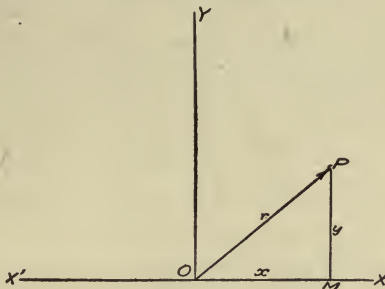


FIG. 9.

$$\sin \theta = \frac{y}{r} = \frac{\text{opposite side}}{\text{hypotenuse}}, \quad \csc \theta = \frac{r}{y} = \frac{\text{hypotenuse}}{\text{opposite side}},$$

$$\cos \theta = \frac{x}{r} = \frac{\text{adjacent side}}{\text{hypotenuse}}, \quad \sec \theta = \frac{r}{x} = \frac{\text{hypotenuse}}{\text{adjacent side}},$$

$$\tan \theta = \frac{y}{x} = \frac{\text{opposite side}}{\text{adjacent side}}, \quad \cot \theta = \frac{x}{y} = \frac{\text{adjacent side}}{\text{opposite side}}.$$

In problems concerning right triangles these relations are very useful. Thus let ABC be a triangle right-angled at C . Also let the lengths of the sides be denoted by a , b and c , where a is the side opposite the angle A , etc. We have

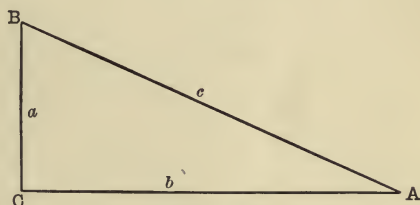


FIG. 10

$$\sin A = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{a}{c},$$

and similarly for the other functions of A .

In the same way we may find the functions of the angle B . For this angle of course b is the opposite side and a the adjacent side.

Hence

$$\tan B = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{b}{a}, \quad \sec B = \frac{\text{hypotenuse}}{\text{adjacent side}} = \frac{c}{a}, \text{ etc.}$$

Observe that

$$\sin A = \cos B = \frac{a}{c}, \quad \tan A = \cot B = \frac{a}{b}, \text{ etc.}$$

Hence since $B = 90^\circ - A$, we have the following relations:

$$\begin{aligned} \sin A &= \cos (90^\circ - A), & \cos A &= \sin (90^\circ - A), \\ \tan A &= \cot (90^\circ - A), & \cot A &= \tan (90^\circ - A), \\ \sec A &= \csc (90^\circ - A), & \csc A &= \sec (90^\circ - A). \end{aligned}$$

Of the two functions sine and cosine either is said to be the *co-function* of the other. Similarly the tangent and cotangent, and the secant and cosecant are called *co-functions*. The six relations above may be expressed as follows: *Any trigonometric function of an angle A is equal to the co-function of the complement of A .* In this section we have proved this rule only when A is an acute angle. The rule however is true for all values of A . [Art. 31.]

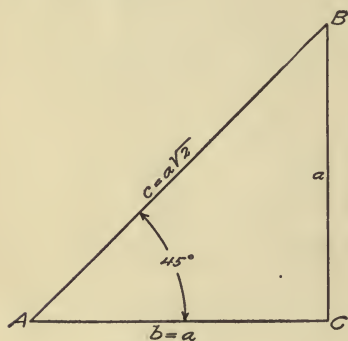


FIG. 11.

15. Functions of 45° . Let ABC be a triangle right-angled at C , and let the angle A be equal to 45° . Then B is also equal to 45° , since the sum of the acute angles of a right triangle is equal to 90° . The triangle is therefore isosceles and $a = b$. Hence $c = \sqrt{a^2 + b^2} = \sqrt{2a^2} = a\sqrt{2}$. We therefore have the following values for the functions of 45° :

$$\sin 45^\circ = \frac{a}{c} = \frac{a}{a\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{1}{2}\sqrt{2},$$

$$\cos 45^\circ = \frac{b}{c} = \frac{a}{a\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{1}{2}\sqrt{2},$$

$$\tan 45^\circ = \frac{a}{b} = \frac{a}{a} = 1.$$

16. Functions of 30° and 60° . Let ABC be a triangle right-angled at C , and let the angle A be equal to 30° . Produce the line BC to D making $DC = CB$. The triangle ABD is

equilateral since each of its angles is equal to 60° . Hence

$$DB = AB = c, \text{ and } CB = a = \frac{c}{2}.$$

Also

$$b^2 = c^2 - \left(\frac{c}{2}\right)^2 = \frac{3c^2}{4}$$

and hence

$$b = \frac{c}{2} \sqrt{3}.$$

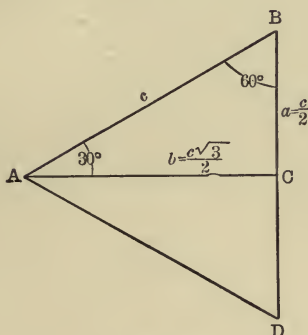


FIG. 12.

We therefore have

$$\sin A = \sin 30^\circ = \frac{a}{c} = \frac{1}{2}, \quad \sin B = \sin 60^\circ = \frac{b}{c} = \frac{1}{2} \sqrt{3},$$

$$\cos A = \cos 30^\circ = \frac{b}{c} = \frac{1}{2} \sqrt{3}, \quad \cos B = \cos 60^\circ = \frac{a}{c} = \frac{1}{2},$$

$$\tan A = \tan 30^\circ = \frac{a}{b} = \frac{1}{\sqrt{3}}, \quad \tan B = \tan 60^\circ = \frac{b}{a} = \sqrt{3}.$$

17. Exercise IV.

1. Find the values of the cotangent, secant, cosecant, and versed sine of each of the following angles: 30° , 45° , 60° .

2. Prove that $\cos 60^\circ = (\cos 30^\circ + \sin 30^\circ) (\cos 30^\circ - \sin 30^\circ)$.

3. Prove that $\sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ = \sqrt{\frac{1 - \cos 30^\circ}{2}}$.

4. Find the values of $\sin 120^\circ$, $\cos 210^\circ$, $\tan 135^\circ$, $\sin 300^\circ$, vers 330° .*

* Note that the reference triangles for these angles are, except for algebraic signs, the same as those for the angles 30° , 45° , and 60° .

5. Find the value of $\frac{\cos 120^\circ (\tan 225^\circ - \sin 330^\circ)}{\cos 300^\circ (\sin 240^\circ + \cos 150^\circ)}$.

6. Express each of the other functions of θ in terms of $\csc \theta$.

Hint. Construct a right triangle ABC in which the angle BAC equals θ . Then $\csc \theta = AC/BC$. Hence, if we take the length of BC as the unit of measure, $\csc \theta = AC$ and $AB = \sqrt{AC^2 - BC^2} = \sqrt{\csc^2 \theta - 1}$.*

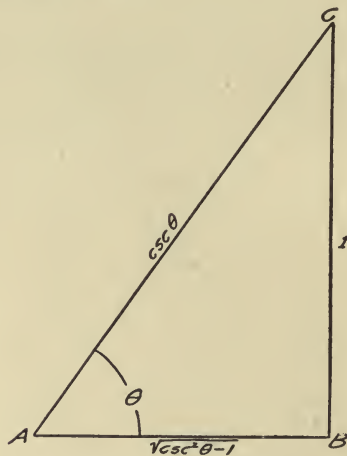


FIG. 13.

The lengths of the sides of ABC are then as denoted in the accompanying figure.

Hence,

$$\sin \theta = \frac{BC}{AC} = \frac{1}{\csc \theta}, \quad \cos \theta = \frac{AB}{AC} = \frac{\sqrt{\csc^2 \theta - 1}}{\csc \theta},$$

$$\tan \theta = \frac{BC}{AB} = \frac{1}{\sqrt{\csc^2 \theta - 1}}, \text{ etc.}$$

7. Express each of the other functions of θ in terms of (a) $\sin \theta$, (b) $\cos \theta$, (c) $\tan \theta$, (d) $\cot \theta$, (e) $\sec \theta$.

18. Polar Coördinates. Let $X'OX$ be a given straight line and O a given point on it. Then the position of any point P in the plane of the paper is determined when the angle XOP and the distance OP are given. If then we denote the angle

* The symbol $\csc^2 \theta$ means $[\csc \theta]^2$ and is read "cosecant squared of θ ." Similarly $\sin^2 \theta$ means the square of the sine of θ , etc.

XOP by θ and the distance OP by r , the position of P is known when the values of θ and r are given. The positive direction of OP ($= r$) is from O along the terminal line of the angle XOP ($= \theta$). Instead of de-

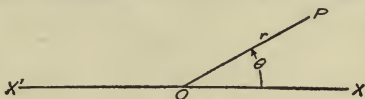


FIG. 14.

fining the position of a point by equations such as $\theta = 30^\circ$, $r = 2$, it is customary to speak of the point $(2, 30^\circ)$. The values of θ and r are called the polar coördinates of the point, and both this pair of values and the point which corresponds to them are denoted by the symbol (r, θ) . Fig. 15 shows the points $(2, 30^\circ)$ and $(3, 240^\circ)$. To construct the point $(-3, 120^\circ)$ we first draw the terminal line of the

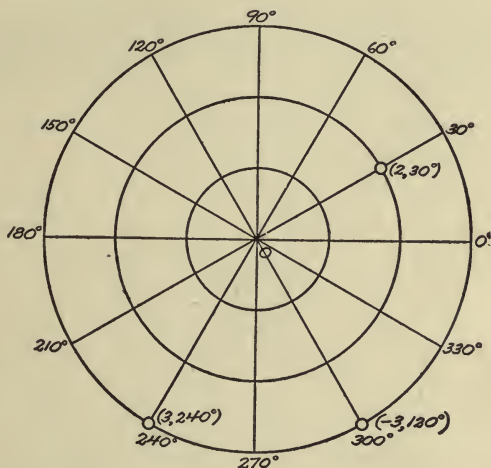


FIG. 15.

angle 120° and then measure a distance -3 upon this line. But the positive direction of this line extends into the second quadrant and hence a negative distance must be measured along the production of this terminal line into the fourth quadrant. Thus the point is constructed as in Fig. 15.

Example. Construct the following points: $(2, 65^\circ)$, $(-2, 25^\circ)$, $(3, 110^\circ)$, $(-2, 225^\circ)$, $(1, 240^\circ)$, $(-3, 330^\circ)$.

19. Numerical Values of Sine, Cosine, Tangent, Cotangent.

In the following table will be found the values correct to two

decimals of these four functions for every five degrees from 0° to 90° .

$^\circ$	Sin.	Tan.	Cot.	Cos.	
0	0	0	∞	1	90
5	0.09	0.09	11.43	0.996	85
10	0.17	0.18	5.67	0.98	80
15	0.26	0.27	3.73	0.97	75
20	0.34	0.36	2.75	0.94	70
25	0.42	0.47	2.14	0.91	65
30	0.50	0.58	1.73	0.87	60
35	0.57	0.70	1.43	0.82	55
40	0.64	0.84	1.19	0.77	50
45	0.71	1	1	0.71	45
	Cos.	Cot.	Tan.	Sin.	$^\circ$

For angles less than 45° the functions are found at the top of the table and the angles in the left hand column. For angles greater than 45° the functions are found at the bottom of the table and the angles in the right hand column. Thus we find from the table $\sin 15^\circ = 0.26$, $\cos 70^\circ = 0.34$, etc.

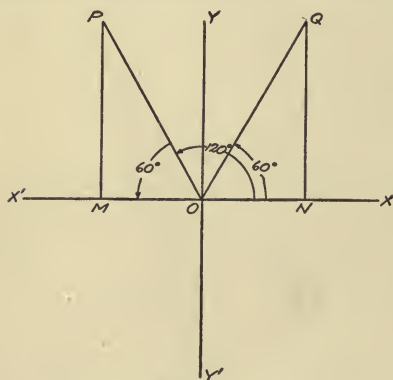


FIG. 16.

We may also find the values of the functions of angles greater than 90° in the following manner.* Let us be required to find the value of $\cos 120^\circ$. First construct the angle $XOP = 120^\circ$ and the angle $XOQ = POX' = 180^\circ - XOP = 60^\circ$. If then we take $OP = OQ$ and construct the reference triangles OMP and ONQ , it is evident that these triangles are equal in all respects except that ON is positive while OM is negative, so that we have $OP = OQ$, $MP = NQ$, $OM = -ON$. Hence $\cos 120^\circ = OM/OP = -ON/OQ = -\cos 60^\circ = -0.5$. Similarly $\sin 120^\circ = MP/OP = NQ/OQ = \sin 60^\circ = 0.87$ and $\tan 120^\circ = MP/OM = NQ/(-ON) = -NQ/ON = -\tan 60^\circ = -1.73$.

* For a more complete discussion of this problem see section 31.

In a similar manner the functions of angles in the third and fourth quadrants may be found. Thus the reference triangles for the angles 230° and 50° are equal except for the algebraic signs of the two legs, and hence the functions of 230° may be found from the corresponding functions of 50° .

20. Graphs of Equations in Polar Coördinates. Let us be given an equation involving r and θ , for example

$$r = 1 - 2 \cos \theta.$$

If we assign any value to θ , say 20° , the equation determines

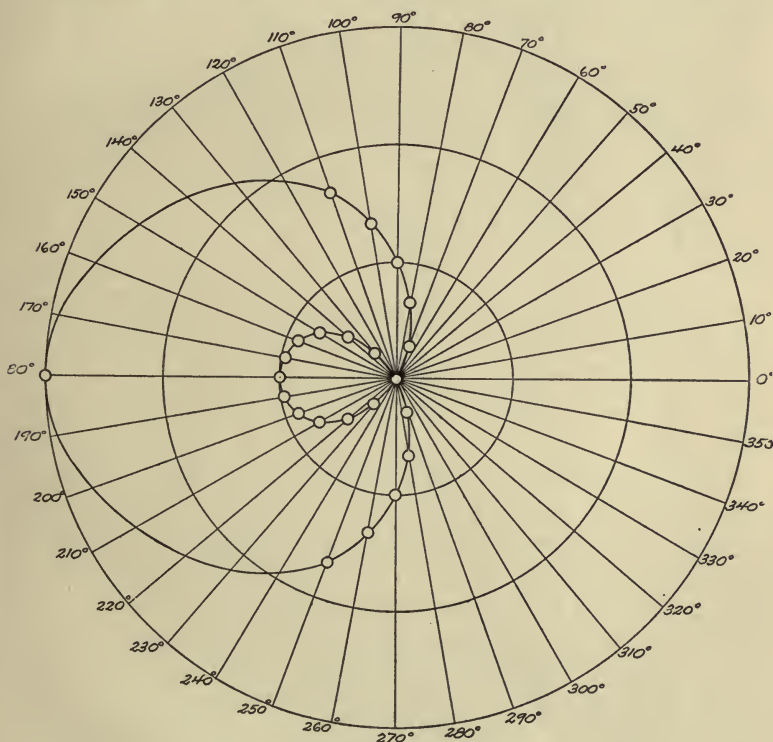


FIG. 17.

a corresponding value for r , that is, $1 - 2 \cos 20^\circ = 1 - 2(.94) = 1 - 1.88 = -.88$. Thus the pair of values $r = -.88$ $\theta = 20^\circ$ satisfy the given equation. To this pair of values there corresponds a point in Fig. 17. In a similar manner if we

let θ equal 70° , we find $r = 1 - 2(0.34) = 1 - 0.68 = 0.32$ and to this pair of values there corresponds another point in the figure. We may thus construct as many points as we choose, the coördinates of each of which satisfy the given equation. It will be found that all of these points lie upon a smooth curve and this curve is called the graph of the equation.

To find the graph of any equation it is well to first construct a table as follows. Let the equation be $r = 1 - 2 \cos \theta$.

θ	$\cos \theta$	$2 \cos \theta$	r	Point
0°	1	2	-1	$(-1, 0^\circ)$
10°	0.98	1.96	-0.96	$(-.96, 10^\circ)$
20°	0.94	1.88	-0.88	$(-.88, 20^\circ)$
30°	0.87	1.74	-0.74	$(-.74, 30^\circ)$
40°	0.77	1.54	-0.54	$(-.54, 40^\circ)$
50°	0.64	1.28	-0.28	$(-.28, 50^\circ)$
60°	0.5	1	0	$(0, 60^\circ)$
70°	0.34	0.68	0.32	$(0.32, 70^\circ)$
80°	0.17	0.34	0.66	$(0.66, 80^\circ)$
90°	0	0	1	$(1, 90^\circ)$
100°	-.17	-.34	1.34	$(1.34, 100^\circ)$
110°	-.34	-.68	1.68	$(1.68, 110^\circ)$
.	.	.	.	.
.	.	.	.	.
180°	-1	-2	3	$(3, 180^\circ)$

This table may be completed by the student for angles in the third and fourth quadrants. Each point should then be located, and finally a smooth curve drawn carefully through successive points.

21. Exercise V.

Draw the graphs of the following equations:

- | | | |
|-----------------------------|-------------------------------------|-------------------------------|
| 1. $r = a \sin \theta$ | 8. $r = 2 \sin 3\theta$ | 15. $r = \sec \theta$ |
| 2. $r = a \sin 2\theta$ | 9. $r = 1 - \sin 2\theta$ | 16. $r^2 = \sin \theta$ |
| 3. $r = a(1 + \sin \theta)$ | 10. $r = 1 + 2 \cos \theta$ | 17. $r^2 = 1 + \sin \theta$ |
| 4. $r = a(1 - \sin \theta)$ | 11. $r = \sin \theta + \cos \theta$ | 18. $r^2 = 1 - 2 \sin \theta$ |
| 5. $r = 2 - \cos \theta$ | 12. $r = \tan \theta$ | 19. $r^2 = 2 - \sin \theta$ |
| 6. $r = 2 \sin^2 \theta$ | 13. $r = 1 - \tan \theta$ | 20. $r^2 = \sin 2\theta$ |
| 7. $r = 1 - 2 \cos \theta$ | 14. $r = \sin \frac{\theta}{2}$ | |

22. Graph of $y = \sin x$. If $x = 0^\circ$, $y = \sin 0^\circ = 0$; similarly if $x = 30^\circ$, $y = \sin 30^\circ = \frac{1}{2}$. In this way we may obtain as many pairs of corresponding values of x and y as we wish. Take any convenient length on the X axis to represent 30° . Twice that length will be 60° and similarly any other angle may be represented by a length on the X axis. Plot the points (see Art. 6) $(0^\circ, 0)$, $(30^\circ, .5)$, $(45^\circ, .71)$, $(60^\circ, .87)$, $(90^\circ, 1)$, $(120^\circ, .87)$, $(135^\circ, .71)$, $(150^\circ, .5)$, $(180^\circ, 0)$, $(210^\circ, -.5)$ and so on through the

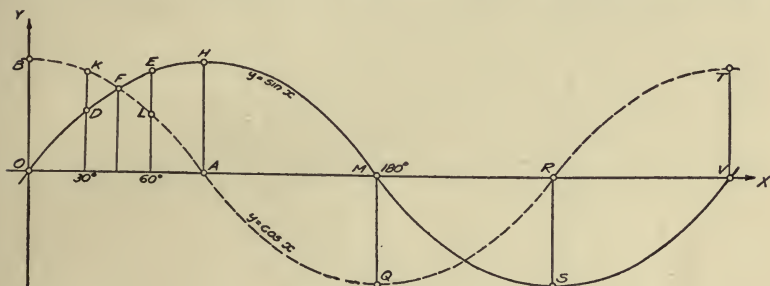


FIG. 18.

third and fourth quadrants. Draw a smooth curve through these points and we shall have the graph of $y = \sin x$. Evidently as x increases beyond 360° the values of y repeat themselves and the complete curve is an endless series of waves exactly like the one in the figure.

23. Doing the same thing with $y = \cos x$ we get its graph as in Fig. 18.

24. In graphing $y = \tan x$, we get the points $(0^\circ, 0)$, $(30^\circ, .58)$, $(45^\circ, 1)$, $(60^\circ, 1.73)$, $(90^\circ, \infty)$. The point $(90^\circ, \infty)$ raises a new difficulty, to get over which we must remember that when x is between 60° and 90° , $\tan x$ is positive. Hence the curve goes up as in Fig. 20, continually getting closer to the line AB but never actually touching it. If x is between 90° and 120° , $\tan x$ is negative; so the curve is below the X axis, passes upward through $(120^\circ, -1.73)$, $(135^\circ, -1)$, $(150^\circ, -.58)$, $(180^\circ, 0)$, $(210^\circ, .58)$, $(240^\circ, 1.73)$, $(270^\circ, \infty)$. When x is between 240° and 270° , $\tan x$ is positive, so that the curve in this region is above the X axis. When x is a little larger than 270° , $\tan x$ is negative; so the curve appears below the X axis and passes

upward through $(300^\circ, -1.73)$ $(330^\circ, -.58)$, $(360^\circ, 0)$, etc.

It will be noticed that the curve from 180° to 360° is exactly the same as it was between 0° and 180° . (See ex. 7, p. 36.)

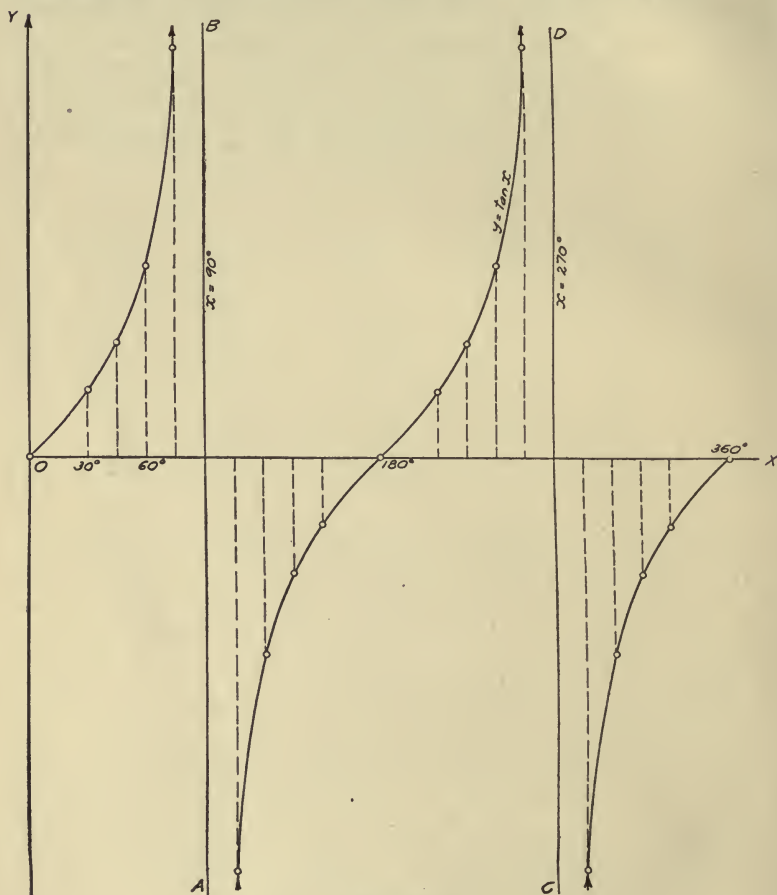


FIG. 20.

It is left as an exercise to the student to draw the part of these graphs for which x is negative.

CHAPTER II.

25. Solution of Right Triangles. We have seen in articles 15 and 16 that it is possible to compute the values of the trigonometric functions of 30° , 45° and 60° . By methods involving more knowledge than the student of elementary trigo-

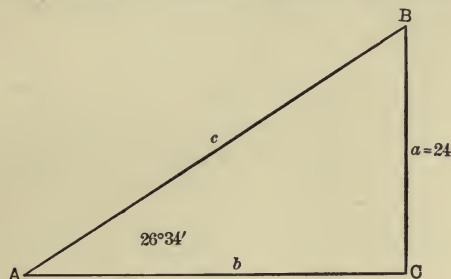


FIG. 21.

nometry possesses it is possible to compute the values of the trigonometric functions of any desired angle. These values are collected in tables so arranged that any function of a given angle may be readily found. These are called tables of the *Natural Functions*. However in the great majority of actual problems it is found that the logarithms of these functions are more easily used than the functions themselves. Tables have therefore been computed of the logarithms of each function of angles from 0° to 90° . By the symbol $\log \sin 39^\circ$ is meant the logarithm of that number which is found to be the value of $\sin 39^\circ$; and similarly for the logarithms of the other functions.

Let us now have a triangle ABC right-angled at C . In this triangle let us be given that the angle A is equal to $26^\circ 34'$, and that the length of the side a is 24 ft. It is required to find the values of the angle B and the sides b and c .

We have $B = 90^\circ - A = 90^\circ - 26^\circ 34' = 63^\circ 26'$. And

$$\sin A = \sin 26^\circ 34' = \frac{a}{c} = \frac{24}{c},$$

$$\tan A = \tan 26^\circ 34' = \frac{a}{b} = \frac{24}{b}.$$

Solving these equations for the required parts we have

$$c = \frac{24}{\sin 26^\circ 34'} \quad \text{and} \quad b = \frac{24}{\tan 26^\circ 34'}.$$

Referring to the tables of natural functions we find that $\sin 26^\circ 34' = 0.44724$ and $\tan 26^\circ 34' = 0.50004$. Hence upon performing the indicated divisions we have

$$c = 53.662 \text{ ft.} \quad \text{and} \quad b = 47.996 \text{ ft.}$$

This triangle may also be solved by logarithms.

Thus we find from the logarithmic tables

$$\begin{array}{ll} \log 24 = 11.38021 - 10 & \log 24 = 1.38021 \\ \log \sin 26^\circ 34' = 9.65054 - 10 & \log \tan 26^\circ 34' = 9.69900 \\ \text{therefore } \log c = 1.72967 & \text{and } \log b = 1.68121 \\ \text{and hence } c = 53.662 \text{ ft.} & \text{and } b = 47.996 \text{ ft.} \end{array}$$

This problem illustrates the method of solving a right triangle. From our study of geometry we know that if any two parts of one right triangle (one of which at least is a side) be equal to the two corresponding parts of another right triangle, the two triangles are in all respects equal. In other words if two such parts be given the remaining three parts are determined. One of the purposes of the study of trigonometry is to enable the student to compute the values of these three parts. Two types of problem only need be considered.

1. *Given an angle and a side*, for example A, a

The angle B may be found by the relation $B = 90^\circ - A$.

To find b make use of a function of the *given* angle which involves the *given side* a and the *required side* b , as $\tan A = a/b$. Then $b = a/\tan A$. Hence by aid of the logarithmic tables we may find the value of b .

To find the side c make use of a function of the *given angle* A which involves both the *given side* a and the *required side* c , as $\sin A = a/c$. Hence $c = a/\sin A$ and we may find the value of c .

In finding both b and c make use of the two *given parts* A, a . It would, of course, be possible to find c , after having found the value of b as above, by means of the relation $\sin B = b/c$. This method has, however, the serious disadvantage

that any error we may have made in finding b will also appear in c .

2. *Given two sides, for example a, b .*

The ratio of the *given sides* will be some function of one of the acute angles; in this case A may be found from $a/b = \tan A$. Having made sure that A is correct, the question is finished as in the first case.

26. Angles of Elevation and Depression. Let us have two points A and C in two different horizontal planes. For example let C be a point higher than A as in the figure. Draw the vertical line CC' through C and let this line cut the horizontal plane through A in C' , so that C' is the point vertically below C and at the same level as A . The angle $C'AC$ is called the *angle of elevation* of C from A . Similarly if AA' is the vertical line through A , and A' the point in which it cuts the horizontal plane through C , the angle $A'CA$ is called the *angle of depression*



FIG. 22.

of A from C . It is obvious that the angle of elevation of C from A is equal to the angle of depression of A from C . Let B be a third point and let the vertical line BB' meet the horizontal plane through A in B' . The plane $AB'C'$ is horizontal. The angle $B'AC'$ is defined as the *horizontal angle* at A subtended by B and C , or the horizontal angle at A of B and C .

By means of an instrument known as a transit surveyors are

enabled to measure horizontal angles and angles of elevation and depression.

By the *horizontal distance* between two points is meant the distance between the vertical lines through the two points. Thus in Fig. 22 the horizontal distance between B and C is $B'C'$, that between A and C is AC' , etc.

27. Exercise VI.

1. A flag-pole stands in a level field. The angle of elevation of the top of the pole from a point 200.64 feet from the base is found to be $19^\circ 28' 35''$. Find the height of the pole.

Let C be the base and B the top of the pole, and A the given point in the field. In the right triangle ABC ,

$$A = 19^\circ 28' 35'' \quad \text{and} \quad CA = b = 200.64 \text{ ft.}$$

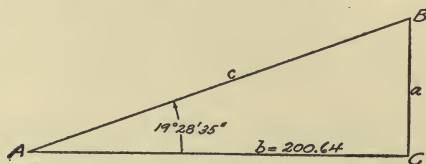


FIG. 23.

Our problem then consists in finding the value of $CB = a$ when given A and b . We have

$$\tan A = \frac{a}{b} \quad \text{and therefore} \quad a = b \tan A.$$

$$\log b = 2.30242$$

$$\log \tan A = 9.54858 - 10$$

$$\log a = 1.85100$$

Hence a = the height of the flag-pole = 70.958 ft.

2. From a point A on one bank of a river the angle of elevation of the top of a tree growing on the opposite bank is $5^\circ 53' 20''$. A horizontal line AB equal to 206.45 ft. is measured along the river bank at right angles to the line from A to the foot of the tree. The horizontal angle subtended at B by A and the tree is found to be $72^\circ 48' 5''$. Find the height of the tree and the distance from B to the foot of the tree.

Let C and D denote the foot and top of the tree respectively. We then have the two right triangles, ABC right-angled at A ,

and ACD right-angled at C . In the former triangle ABC , we know $AB = 206.45$, and the angle $ABC = 72^\circ 48' 5''$. We can therefore find the required distance BC . From this triangle we must also find the value of the side AC . Having found AC

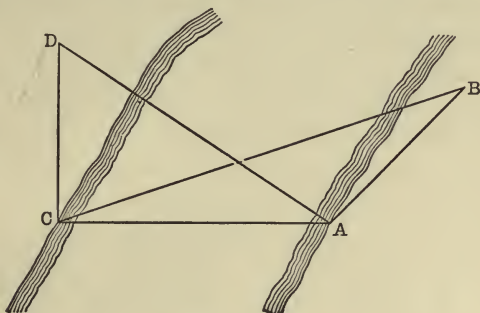


FIG. 24.

we shall have, in the triangle ACD , the side AC and the angle $CAD = 5^\circ 53' 20''$.

In the triangle ABC

$$\cos ABC = \frac{AB}{BC} \quad \text{and therefore} \quad BC = \frac{AB}{\cos ABC}.$$

Also

$$\tan ABC = \frac{AC}{AB} \quad \text{and therefore} \quad AC = AB \tan ABC.$$

$$\begin{array}{rcl} \log AB = 2.31482 & \log AB = 2.31482 & \\ \log \cos ABC = 9.47083 & \log \tan ABC = 0.50931 & \\ \hline \log BC = 2.84399 & \log AC = 2.82413^* & \end{array}$$

Hence BC = distance from B to the foot of the tree = 698.22 feet.

In the triangle ACD

$$\tan CAD = \frac{CD}{AC}, \quad \text{and therefore} \quad CD = AC \tan CAD.$$

$$\begin{array}{rcl} \log AC = 2.82413 & \text{(as found above)} & \\ \log \tan CAD = 9.01344 & & \\ \hline \log CD = 1.83757 & & \end{array}$$

Hence CD = height of the tree = 68.797 feet.

* Note that it is not necessary to find the value of AC , since this distance is not required, and the logarithm of AC only is needed in the rest of the work.

3. A tower stands on the shore of a river 206.8 feet wide. The angle of elevation of the top of the tower from the point on the other shore exactly opposite to the tower is $46^{\circ} 31' 25''$. Find the height of the tower.

4. The angle of elevation of the top of a mountain from a point of observation is found to be $41^{\circ} 54' 35''$. Find the height of the mountain if the horizontal distance from the given point to the top is known to be 5436.7 feet.

5. At 141.78 feet from the base of a tower, and on a level with the base, the angle of elevation of the top was found to be $54^{\circ} 22' 55''$. Find the height of the tower.

6. A rope is stretched from the top of a building to the ground. The rope is found to make an angle of $58^{\circ} 56' 10''$ with the horizontal, and the building is 61.004 feet high. Find the length of the rope.

7. From one end of a bridge the angle of depression of an object 200.87 feet down stream from the bridge and at the water line is found to be $21^{\circ} 43' 55''$. From the same point the angle of depression of an object at the water line exactly under the opposite end of the bridge is found to be $17^{\circ} 22' 25''$. Find the length of the bridge and its height above the river.

8. From a ship's masthead 157.87 feet high the angle of depression of a boat is $27^{\circ} 56' 25''$. Find the distance from the boat to the ship.

9. Two flag-poles are known to be 61.502 and 43.124 feet high respectively. A person moves about until he finds a position from which the top of the nearer pole just hides that of the farther. From this point the angle of elevation of the top of the nearer one is found to be $36^{\circ} 21' 10''$. Find the distance between the poles, and the distance from the observer to the nearer pole.

10. From the foot of a post 32.548 feet high the angle of elevation of the top of a steeple is $63^{\circ} 43' 50''$, and from the top of the post the angle of depression of the base of the steeple is $49^{\circ} 12' 35''$. Find the height of the steeple and its distance from the post.

11. The angles of depression of the top and foot of a building seen from the top of a tower 98.253 feet high are

$32^{\circ} 21' 55''$ and $59^{\circ} 56' 10''$ respectively. Find the height of the building.

12. From the top of a cliff 156.43 ft. high the angles of depression of two boats at sea, each due north of the observer, are $31^{\circ} 52' 55''$ and $17^{\circ} 23' 35''$. How far are the boats apart?

In the following problems ABC represents a triangle right-angled at C , and the letters a , b , and c represent the sides of the triangle opposite to the angles A , B , and C respectively.

In each problem find all of the parts which are not given.

13. $a = 235.61$, $b = 687.99$.

14. $b = 78352$, $A = 54^{\circ} 55' 25''$.

15. $c = 2138.7$, $A = 29^{\circ} 43' 45''$.

16. $c = 1.3276$, $a = 0.65998$.

17. $c = 78.657$, $B = 81^{\circ} 21' 25''$.

18. $a = 0.02659$, $A = 19^{\circ} 54' 35''$.

19. $b = 568.02$, $c = 603.87$.

20. $a = 528090$, $A = 63^{\circ} 27' 55''$.

CHAPTER III.

TRIGONOMETRIC ANALYSIS.

28. Relations among the Trigonometric Functions. Let θ be an angle in any quadrant, and draw the reference triangle OMP . By definition we have

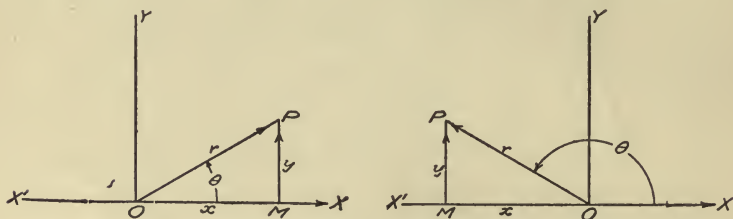


FIG. 25.

$$\sin \theta = \frac{y}{r} \quad \text{and} \quad \csc \theta = \frac{r}{y}.$$

Hence

$$\csc \theta = \frac{1}{\sin \theta}. \quad (1)$$

In the same way we may show that

$$\cot \theta = \frac{1}{\tan \theta} \quad (2)$$

and

$$\sec \theta = \frac{1}{\cos \theta}. \quad (3)$$

The sine and cosecant are said to be reciprocal functions, for either one of them is the reciprocal of the other. Similarly the tangent and cotangent, and the secant and cosine are reciprocal functions.

We also have

$$\frac{\sin \theta}{\cos \theta} = \frac{\frac{y}{r}}{\frac{x}{r}} = \frac{y}{x},$$

and hence

$$\frac{\sin \theta}{\cos \theta} = \tan \theta. \quad (4)$$

Similarly [or by aid of equations (2) and (4)] we have

$$\frac{\cos \theta}{\sin \theta} = \cot \theta. \quad (5)$$

Also in the triangle OMP we have $x^2 + y^2 = r^2$. If we divide this equation by r^2 we obtain

$$\left(\frac{x}{r}\right)^2 + \left(\frac{y}{r}\right)^2 = 1.$$

Hence,

$$\cos^2 \theta + \sin^2 \theta = 1. \quad (6)$$

If we divide the equation $x^2 + y^2 = r^2$ first by x^2 and then by y^2 we obtain the two following relations

$$1 + \tan^2 \theta = \sec^2 \theta, \quad (7)$$

$$\cot^2 \theta + 1 = \csc^2 \theta. \quad (8)$$

29. Identical Equations. If A denote the same expression as B , or one which can be transformed into B , we say that A is *identically equal* to B .

The notation $A \equiv B$ means A is identically equal to B . We call $A \equiv B$ an identical equation or an identity

Thus $(x + 2)^2 - 4 \equiv x(x + 4).$

For $(x + 2)^2 - 4 \equiv x^2 + 4x + 4 - 4,$
 $\equiv x^2 + 4x,$
 $\equiv x(x + 4).$

It follows that the given equality is an identity since the first expression $(x + 2)^2 - 4$ can be transformed into the second expression $x(x + 4)$.

The eight relations of the previous section are simple examples of trigonometric identities. For in each of these relations we have seen that the two members stand for the same expression.

Consider the following problem:

Prove that

$$\tan^2 \theta \equiv \frac{1}{\cos^2 \theta} - 1.$$

We have

$$\tan^2 \theta \equiv \sec^2 \theta - 1 \quad (7), \text{ p. 31}$$

$$\equiv \frac{1}{\cos^2 \theta} - 1. \quad (3)$$

The given relation is therefore proved true since the first expression can be transformed into the second expression.

This problem may also be proved as follows:

$$\begin{aligned} \frac{1}{\cos^2 \theta} - 1 &\equiv \frac{1 - \cos^2 \theta}{\cos^2 \theta} \\ &\equiv \frac{\sin^2 \theta}{\cos^2 \theta} \end{aligned} \quad (6)$$

$$\equiv \tan^2 \theta. \quad (4)$$

Hence the identity is a true one since the second expression may be transformed into the first expression.

It is not necessary to transform one of the given expressions directly into the other. We may prove the identity given above in the following way.

Adding 1 to each member of the assumed identity, we have

$$\tan^2 \theta + 1 \equiv \frac{1}{\cos^2 \theta}.$$

This is true since each member is identically equal to $\sec^2 \theta$, the first by (7) and the second by (3), section 28.

It must be observed that in this last method all we have really proved is that if $\tan^2 \theta \equiv 1/\cos^2 \theta - 1$, then $\sec^2 \theta \equiv \sec^2 \theta$. What we really wish to prove is that if $\sec^2 \theta \equiv \sec^2 \theta$ (which is obvious), then the given identity is a true one. That this is true follows from the fact that each step in our work may be *reversed*. In other words we may go *backwards* from the known relation $\sec^2 \theta \equiv \sec^2 \theta$ to the desired relation

$$\tan^2 \theta \equiv \frac{1}{\cos^2 \theta} - 1.$$

Thus $\sec^2 \theta \equiv \sec^2 \theta$ gives by aid of (7) and (3)

$$\tan^2 \theta + 1 \equiv \frac{1}{\cos^2 \theta},$$

and hence, upon subtracting 1 from each member,

$$\tan^2 \theta \equiv \frac{1}{\cos^2 \theta} - 1,$$

as required.

30. Exercise VII. Prove the following identities:

- (1) $\cos \theta \tan \theta \equiv \sin \theta$.
- (2) $\cos \theta \csc \theta \equiv \cot \theta$.
- (3) $\tan \theta + \cot \theta \equiv \sec \theta \csc \theta$.
- (4) $\cos^2 \theta (1 + \tan^2 \theta) \equiv 1$.
- (5) $\cos \theta + \tan \theta \sin \theta \equiv \sec \theta$.
- (6) $\sin \theta + \cos \theta \equiv \sin \theta \cos \theta (\sec \theta + \csc \theta)$.
- (7) $(1 + \tan \theta)(1 + \cot \theta) \sin \theta \cos \theta \equiv (\sin \theta + \cos \theta)^2$.
- (8) $(\cot \theta \sin \theta)^2 + \sin^2 \theta \equiv (\sin \theta + \cos \theta)^2 - 2 \sin \theta \cos \theta$.
- (9) $\sec^2 \theta - 1 \equiv \sin^2 \theta \sec^2 \theta$.
- (10) $\csc^2 \theta - 1 \equiv \cos^2 \theta \csc^2 \theta$.
- (11) $\sec x - 1 \equiv \sec x (1 - \cos x)$.
- (12) $\cos x \csc x \tan x \equiv \sin x \sec x \cot x$.
- (13) $(\tan \theta - \sin \theta)^2 + (1 - \cos \theta)^2 \equiv (\sec \theta - 1)^2$.
- (14) $(\cot \theta + \csc \theta)^2 \equiv \frac{1 + \cos \theta}{1 - \cos \theta}$.
- (15) $\sec^4 \theta - \tan^2 \theta \equiv \sec^2 \theta + \tan^4 \theta$.
- (16) $(\cos^2 \theta - 1)(\cot^2 \theta + 1) + 1 \equiv 0$.
- (17) $\sin x(1 + \tan x) + \cos x(1 + \cot x) \equiv \sec x + \csc x$.
- (18) $(\sin 60^\circ - \sin 45^\circ)(\cos 30^\circ + \cos 45^\circ) \equiv \sin^2 30^\circ$.
- (19) $(\cos 30^\circ - \sin 30^\circ)(\sin 60^\circ + \cos 60^\circ) \equiv \sin^2 45^\circ$.
- (20) If the lengths of the sides of a right triangle be in arithmetical progression, prove that the sines of its acute angles are $\frac{3}{5}$ and $\frac{4}{5}$ respectively.

(21) One leg of a right triangle is equal to half of the hypotenuse; prove that the acute angles are 30° and 60° respectively.

31. To Express the Functions of $90^\circ \pm \theta$, $180^\circ \pm \theta$, etc., in Terms of θ .

$90^\circ - \theta$. Let θ be any angle such as XOP ; then if $P'OY = \theta$, XOP' will be $90^\circ - \theta$. Draw the triangles of reference. By construction $P'OY = \theta$; therefore $OP'M' = MOP$. Hence the right triangles POM and $OP'M'$ are equal in all respects; in particular $OM' = MP$ and $M'P' = OM$. Then

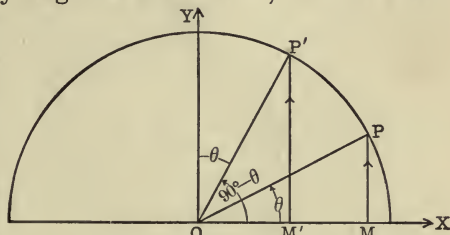


FIG. 26.

$$\sin (90^\circ - \theta) = \frac{M'P'}{OP'} = \frac{OM}{OP} = \cos \theta,$$

$$\cos (90^\circ - \theta) = \frac{OM'}{OP'} = \frac{MP}{OP} = \sin \theta,$$

$$\tan (90^\circ - \theta) = \frac{M'P'}{OM'} = \frac{OM}{MP} = \cot \theta.$$

Find $\cot (90^\circ - \theta)$, $\sec (90^\circ - \theta)$, and $\csc (90^\circ - \theta)$.

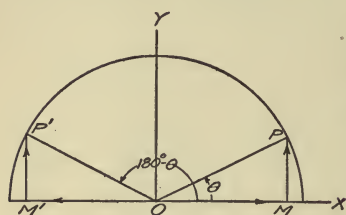


FIG. 27.

$180^\circ - \theta$. Let θ be the angle XOP ; then if $P'OM' = \theta$, XOP' will be $180^\circ - \theta$. Draw the triangles of reference. The right triangles OPM and $OP'M'$ are equal in all respects. That is, $M'P' = MP$ and $OM' = -OM$.

Then

$$\sin (180^\circ - \theta) = \frac{M'P'}{OP'} = \frac{MP}{OP} = \sin \theta,$$

$$\cos (180^\circ - \theta) = \frac{OM'}{OP'} = -\frac{OM}{OP} = -\cos \theta,$$

$$\tan (180^\circ - \theta) = \frac{M'P'}{OM'} = -\frac{MP}{OM} = -\tan \theta.$$

Find $\cot (180^\circ - \theta)$, $\sec (180^\circ - \theta)$ and $\csc (180^\circ - \theta)$.

$-\theta$. Again, in Fig. 28, the triangles of reference are equal in all respects and we have

$MP' = -MP$. Then

$$\begin{aligned} \sin (-\theta) &= \frac{MP'}{OP'} = -\frac{MP}{OP} \\ &= -\sin \theta, \end{aligned}$$

$$\begin{aligned} \cos (-\theta) &= \frac{OM}{OP'} = \frac{OM}{OP} \\ &= \cos \theta, \end{aligned}$$

$$\begin{aligned} \tan (-\theta) &= \frac{MP'}{OM} = -\frac{MP}{OM} \\ &= -\tan \theta. \end{aligned}$$

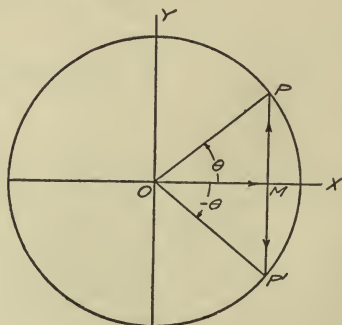


FIG. 28.

The solution of any question of this type depends on our ability to draw the angles and their triangles of reference and then to properly pair the sides of these triangles, attention being paid to the signs. Since logarithmic tables give only acute angles it is necessary to use one of these relations in order to find the logarithm of a function of an angle greater than 90° . The most useful ones are those for $180^\circ - \theta$. In the examples worked out θ was taken as an acute angle; the same results would be obtained no matter what the size of θ , the method of proof being the same in every case.

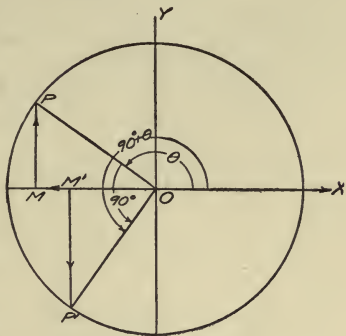


FIG. 29.

As an example, let us take $90^\circ + \theta$, θ being an obtuse angle such as XOP . Draw $POP' = 90^\circ$; then $XOP' = 90^\circ + \theta$. Since $POP' = 90^\circ$ and $MOP' + OP'M' = 90^\circ$, by taking away the common part $M'OP'$ we have that $POM = OP'M'$. Therefore the triangles OPM and $OP'M'$ are equal in all respects; in particular $OM' = -MP$ and $M'P' = OM$. Then

$$\sin (90^\circ + \theta) = \frac{M'P'}{OP'} = \frac{OM}{OP} = \cos \theta,$$

$$\cos (90^\circ + \theta) = \frac{OM'}{OP'} = -\frac{MP}{OP} = -\sin \theta, \text{ etc.}$$

The results of all these are included in the following rule:

Any function of $90^\circ \pm \theta$ (or $270^\circ \pm \theta$) is equal to its co-function of θ , and any function of $-\theta$, $180^\circ \pm \theta$ or $360^\circ \pm \theta$ is equal to the same function of θ , the proper sign being used in each case. To fix the sign, suppose θ to be acute and use the sign of the function in question for the quadrant in which the compound angle falls.

Thus to get $\tan (270^\circ + \theta)$, we say that with θ acute $270^\circ + \theta$ would be in the fourth quadrant, in which the tangent is negative. Hence $\tan (270^\circ + \theta) = -\cot \theta$.

32. Exercise VIII.

1. By a figure, prove that $\sin(90^\circ + \theta) = \cos \theta$ and that $\cos(90^\circ + \theta) = -\sin \theta$. Find $\tan(90^\circ + \theta)$.

2. Find $\sin(180^\circ + \theta)$, $\cos(180^\circ + \theta)$, and $\tan(180^\circ + \theta)$ in terms of θ .

3. Find $\tan 160^\circ 10'$ in terms of $19^\circ 50'$.

4. Find $\log \sin 132^\circ 20'$, $\log \tan 200^\circ 15'$, $\log \cos 312^\circ 18'$ and $\log \sin 150^\circ$.

5. Prove that $\sin 160^\circ = \cos 70^\circ$ and that $\cos 110^\circ = \sin 200^\circ$.

6. Given $\sin 140^\circ = a/b$ find $\sin 40^\circ$, $\cos 130^\circ$ and $\cos 310^\circ$.

7. Prove that the tangent and cotangent are periodic functions of period 180° [p. 12].

33. Inverse Functions. If $\sin \theta = a$ and it is desired to draw attention to θ , one would use the phrase " θ is an angle whose sine is a ." This is written $\theta = \arcsin a$.* Similarly $\theta = \arctan b$ means that θ is an angle whose tangent is b . Such functions are called inverse trigonometric functions; they are also called circular functions. For a given value of θ only one value of a is possible; but on the other hand for a given value of a , there are infinitely many values of θ . Thus if $\theta = \arcsin \frac{1}{2}$, θ is 30° or 150° or any angle obtained from these by adding multiples of 360° . Similarly $\arctan \sqrt{3}$ is 60° or any angle obtained from 60° by adding multiples of 180° . Of all these values, the smallest positive one is called the principal value and is the one to be used in the following examples:

1. If $\arcsin \frac{1}{3} = \theta$, find $\cot \theta$.

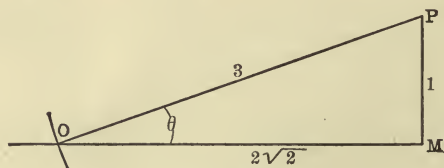


FIG. 30.

If $\arcsin \frac{1}{3} = \theta$, then $\sin \theta = \frac{1}{3}$. To construct θ , draw a

* The symbol $\sin^{-1} a$ is also commonly used for this purpose.

right angle and on one arm lay off $MP = 1$; with P as centre and radius 3 draw an arc cutting the other arm at O . Then $MOP = \theta$ and $OM = \sqrt{9 - 1} = 2\sqrt{2}$. Hence $\cot \theta = 2\sqrt{2}$.

2. If $\operatorname{arccot}(-\frac{1}{2}) = \theta$, find $\sin \theta$.

Since $\operatorname{arccot}(-\frac{1}{2}) = \theta$, then $\cot \theta = -\frac{1}{2}$ and θ must be an

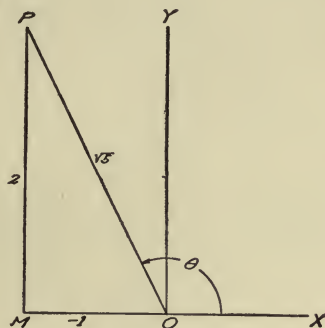


FIG. 31.

obtuse angle. To construct θ , lay off $OM = -1$ and at M erect $MP = 2$. Then $XOP = \theta$ and $OP = \sqrt{5}$. Hence $\sin \theta = 2/\sqrt{5}$.

34. Exercise IX.

1. Construct $\arccos(-\frac{4}{5})$, $\arctan \frac{2}{3}$, $\arcsin(-\frac{1}{2})$.
2. Prove $\arcsin \frac{2}{3} = \arctan 2/\sqrt{21}$.
3. Prove $\arccos(-\frac{4}{5}) = \arctan(-\sqrt{33}/4)$.
4. Find $\tan(\arcsin \frac{2}{3})$; $\cos[\arccos(-\frac{1}{4})]$.
5. If $\arccos(2x^2 - 2x) = 2\pi/3$, find x .
6. Find $\arcsin 0$, $\arctan 1$, $\arccos \frac{1}{2}$, $\arctan \infty$, $\arcsin(-\frac{1}{2}\sqrt{3})$, $\arccos 1$.

35. Orthogonal Projection. AB being any line segment and PQ any line, draw AA' and BB' perpendicular to PQ . Then $A'B'$ is the orthogonal projection of AB on PQ . Through A draw AE parallel to PQ and produce BA to meet PQ in L . Then $EAB = B'LB = \theta$, and $AE = A'B'$. Since $AE/AB = \cos \theta$ we have $AE = AB \cos \theta$ and therefore $A'B' = AB \cos \theta$. Hence the projection of any segment AB on a line is AB times the cosine of the angle between the lines.

Project the sides of the triangle ABC (in Fig. 33) on a line PQ . Since $A'B' + B'C' = A'C'$ and $A'C' + C'B' = A'B'$ we see

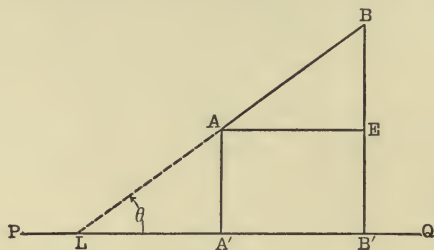


FIG. 32.

that the sum of the projections on any line of two sides of a triangle is equal to the projection on that line of the third side.

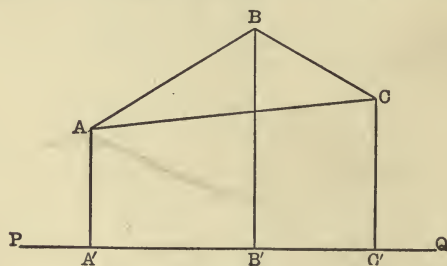


FIG. 33.

36. Functions of the Sum or Difference of Two Angles.

To prove

$$\sin (A+B)=\sin A \cos B+\cos A \sin B,$$

$$\cos (A+B)=\cos A \cos B-\sin A \sin B.$$

The following proofs are valid no matter what may be the size of A and B . For the student to see this, however, is harder than to limit the present proofs to the case where A and B are acute and later to show that they may be extended to cover all cases.

First Method. The following proofs apply whether $A+B$ is acute or obtuse.

Draw $XOX' = A$ and $X'OP = B$. Then $XOP = A+B$. From P draw PQR perpendicular to OX' .

By Art. 35, the projection on any line of OP is equal to the sum of the projections on that line of OQ and QP . Projecting on OX gives

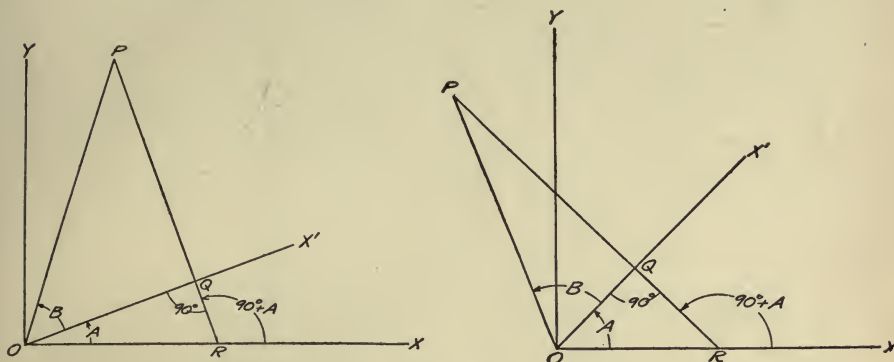


FIG. 34.

$$OP \cos (A+B) = OQ \cos A + QP \cos XRP.$$

But

$$\cos XRP = \cos (90^\circ + A) = -\sin A.$$

Hence

$$OP \cos (A+B) = OQ \cos A - QP \sin A.$$

Dividing by OP ,

$$\cos (A+B) = OQ/OP \cos A - QP/OP \sin A,$$

and therefore

$$\cos (A+B) = \cos B \cos A - \sin B \sin A. \quad \text{Q. E. D.}$$

In the second figure, projecting OP on OY gives

$$OP \cos [(A+B) - 90^\circ],$$

which is equal to $OP \cos [90^\circ - (A+B)]$. Hence in either figure, projecting OP , OQ and QP on OY gives

$$OP \cos [90^\circ - (A+B)] = OQ \cos (90^\circ - A) + QP \cos (XRP - 90^\circ)$$

Hence

$$OP \sin (A+B) = OQ \sin A + QP \cos A,$$

and dividing by OP

$$\sin (A+B) = \cos B \sin A + \sin B \cos A. \quad \text{Q. E. D.}$$

Second Method. Draw $XOX'' = A$ and $X''OP = B$; then $XOP = A+B$. To get triangles of reference for A , B and $A+B$, take any point P on the terminal line of $A+B$ and

draw PM perpendicular to OX . P being also on the terminal line of B , draw PQ perpendicular to OX'' ; and Q being on the

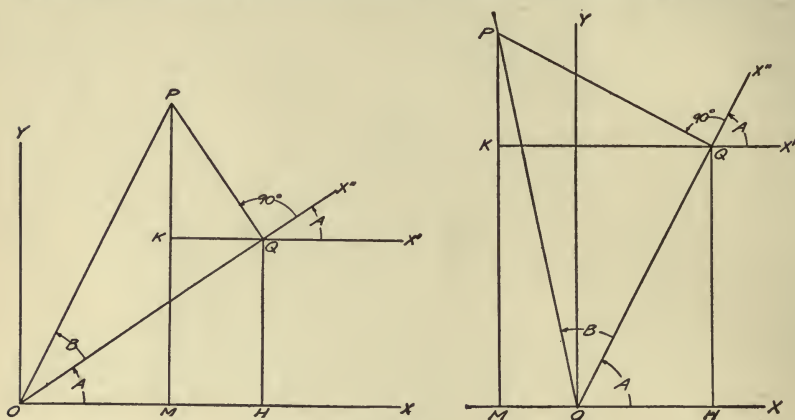


FIG. 35.

terminal line of A draw QH perpendicular to OX . Through Q draw KQX' parallel to OX . Then $X'QP$ is $90^\circ + A$.

$$\sin (A+B)=\frac{MP}{OP}=\frac{MK+KP}{OP}=\frac{HQ}{OP}+\frac{KP}{OP}.$$

But $HQ=OQ \sin A$, $KP=QP \sin (90^\circ+A)=QP \cos A$.

$$\therefore \sin (A+B)=\sin A \frac{OQ}{OP}+\cos A \frac{QP}{OP},$$

$$=\sin A \cos B+\cos A \sin B. \quad \text{Q. E. D.}$$

$$\text{Also } \cos (A+B)=\frac{OM}{OP}=\frac{OH+HM}{OP}=\frac{OH}{OP}+\frac{QK}{OP}.$$

But $OH=OQ \cos A$, $QK=QP \cos (90^\circ+A)=-QP \sin A$.

$$\therefore \cos (A+B)=\cos A \frac{OQ}{OP}-\sin A \frac{QP}{OP},$$

$$=\cos A \cos B-\sin A \sin B. \quad \text{Q. E. D.}$$

37. To prove

$$\sin (A-B)=\sin A \cos B-\cos A \sin B$$

$$\cos (A-B)=\cos A \cos B+\sin A \sin B.$$

Suppose $B+B'=90^\circ$, so that B' also is acute.

Then

$$\begin{aligned}
 \sin (A-B) &= \sin (A+B'-90^\circ) = -\sin (90-A-B') \\
 &= -\cos (A+B') \\
 &= -\cos A \cos B' + \sin A \sin B' \\
 &= -\cos A \cos (90^\circ-B) + \sin A \sin (90^\circ-B) \\
 &= -\cos A \sin B + \sin A \cos B. \qquad \text{Q. E. D.}
 \end{aligned}$$

Similarly

$$\begin{aligned}
 \cos (A-B) &= \cos (A+B'-90^\circ) \\
 &= \cos (90-A-B') \\
 &= \sin (A+B') \\
 &= \sin A \cos B' + \cos A \sin B' \\
 &= \sin A \cos (90^\circ-B) + \cos A \sin (90^\circ-B) \\
 &= \sin A \sin B + \cos A \cos B. \qquad \text{Q. E. D.}
 \end{aligned}$$

38. That these formulæ are true when A and B are not acute may be shown in any special case as follows:

Suppose A to be an angle in the third quadrant and B to be one in the second. Then A can be expressed in the form $180^\circ + A'$ where A' will be an acute angle. Similarly $B = 90^\circ + B'$, B' being acute.

Then

$$\begin{aligned}
 \sin (A+B) &= \sin (270^\circ + A' + B') \\
 &= -\cos (A' + B') \qquad \qquad \qquad \S 31. \\
 &= -\cos A' \cos B' + \sin A' \sin B' \\
 &= -\cos (A-180^\circ) \cos (B-90^\circ) \\
 &\quad + \sin (A-180^\circ) \sin (B-90^\circ) \\
 &= \cos A \sin B + \sin A \cos B.
 \end{aligned}$$

In like manner

$$\begin{aligned}
 \cos (A-B) &= \cos (90^\circ + A' - B') \\
 &= -\sin (A' - B') \\
 &= -\sin A' \cos B' + \cos A' \sin B' \\
 &= -\sin (A-180^\circ) \cos (B-90^\circ) \\
 &\quad + \cos (A-180^\circ) \sin (B-90^\circ) \\
 &= \sin A \sin B + \cos A \cos B.
 \end{aligned}$$

39. From the preceding formulæ we get

$$\tan (A+B) = \frac{\sin (A+B)}{\cos (A+B)} = \frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B - \sin A \sin B}.$$

Dividing numerator and denominator by $\cos A \cos B$, we obtain

$$\begin{aligned} & \frac{\sin A \cos B}{\cos A \cos B} + \frac{\cos A \sin B}{\cos A \cos B} \\ &= \frac{\sin A \cos B}{\cos A \cos B} - \frac{\sin A \sin B}{\cos A \cos B} \\ \therefore \tan (A + B) &= \frac{\tan A + \tan B}{1 - \tan A \tan B}. \end{aligned}$$

Similarly it can be proved that

$$\tan (A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}.$$

40. Exercise X.

Apply the formulæ of §§ 36-39 to the following examples, simplifying when possible:

1. $\sin (180^\circ - \theta)$.
2. $\cos (90^\circ + \theta)$.
3. $\tan (180^\circ + \theta)$.
4. $\sin (2A + B)$.
5. $\cos (90^\circ - \theta)$.
6. $\tan (0^\circ - \theta)$.
7. By expanding $\cos (\theta - \theta)$ find $\cos 0^\circ$.
8. Similarly find $\sin 0^\circ$ and $\tan 0^\circ$.
9. From $\tan (45^\circ + 45^\circ)$ find $\tan 90^\circ$.
10. From $\sin (45^\circ + 30^\circ)$ find $\sin 75^\circ$.
11. Find $\sin 15^\circ$ and $\cos 15^\circ$.
12. If $\sin A = \frac{3}{5}$ and $\cos B = \frac{5}{13}$ find $\sin (A + B)$, $\cos (A - B)$ and $\tan (A + B)$, A and B being acute.
13. If $\cos A = -\frac{2}{3}$ and $\sin B = \frac{1}{3}$ find $\cos (A + B)$, $\sin (A - B)$, and $\tan (A - B)$, A and B being obtuse.
14. If $A = \arcsin \frac{1}{4}$ and $B = \arccos (-\frac{3}{4})$ find $\sin (A - B)$.
15. Prove the formula for $\sin (A - B)$ when A and B are in the third quadrant.
16. Prove the formula for $\cos (A + B)$ when A is in the second quadrant and B in the fourth.

41. Functions of an Angle in Terms of Half the Angle.

$$\sin 2\theta = \sin (\theta + \theta) = \sin \theta \cos \theta + \cos \theta \sin \theta$$

$$\therefore \sin 2\theta = 2 \sin \theta \cos \theta.$$

$$\cos 2\theta = \cos (\theta + \theta) = \cos^2 \theta - \sin^2 \theta.$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}.$$

Similarly

$$\sin \theta = \sin \left(\frac{\theta}{2} + \frac{\theta}{2} \right) = 2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}.$$

$$\cos \theta = \cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2}.$$

$$\tan \theta = \frac{2 \tan \frac{\theta}{2}}{1 - \tan^2 \frac{\theta}{2}}.$$

42. Functions of an Angle in Terms of Double the Angle.

Since $\cos^2 \theta - \sin^2 \theta = \cos 2\theta$ and $\cos^2 \theta + \sin^2 \theta = 1$ we get by adding and subtracting

$$2 \cos^2 \theta = 1 + \cos 2\theta \quad \text{and} \quad 2 \sin^2 \theta = 1 - \cos 2\theta.$$

$$\text{From } \cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2} = \cos \theta \quad \text{and} \quad \cos^2 \frac{\theta}{2} + \sin^2 \frac{\theta}{2} = 1$$

we can get in the same way

$$2 \cos^2 \frac{\theta}{2} = 1 + \cos \theta \quad \text{and} \quad 2 \sin^2 \frac{\theta}{2} = 1 - \cos \theta.$$

Hence $\cos \frac{\theta}{2} = \pm \sqrt{\frac{1 + \cos \theta}{2}};$

$$\sin \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{2}};$$

and $\tan \frac{\theta}{2} = \frac{\sin \frac{\theta}{2}}{\cos \frac{\theta}{2}} = \pm \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}}.$

43. It is well to be able to state any of these formulæ in words. For example, the first one in Art. 41 is: the sine of any angle is twice the sine of half the angle times the cosine of half the angle. This enables us to write

$$\sin \theta = 2 \sin \frac{\theta}{2} \cos \frac{\theta}{2},$$

$$\sin (A + B) = 2 \sin \frac{A + B}{2} \cos \frac{A + B}{2}, \text{ etc.}$$

44. Factoring Formulæ. To prove

$$\sin \theta + \sin \phi = 2 \sin \frac{\theta + \phi}{2} \cos \frac{\theta - \phi}{2},$$

$$\sin \theta - \sin \phi = 2 \cos \frac{\theta + \phi}{2} \sin \frac{\theta - \phi}{2},$$

$$\cos \theta + \cos \phi = 2 \cos \frac{\theta + \phi}{2} \cos \frac{\theta - \phi}{2},$$

$$\cos \theta - \cos \phi = -2 \sin \frac{\theta + \phi}{2} \sin \frac{\theta - \phi}{2}.$$

From $\sin(A + B) = \sin A \cos B + \cos A \sin B$
 and $\sin(A - B) = \sin A \cos B - \cos A \sin B$
 we get $\sin(A + B) + \sin(A - B) = 2 \sin A \cos B$ (a)
 and $\sin(A + B) - \sin(A - B) = 2 \cos A \sin B$. (b)

Similarly

$$\cos(A + B) + \cos(A - B) = 2 \cos A \cos B, \quad (c)$$

$$\cos(A + B) - \cos(A - B) = -2 \sin A \sin B. \quad (d)$$

Put $A + B = \theta,$

$$A - B = \phi$$

Then $\frac{2A = \theta + \phi}{2A = \theta + \phi}$ and $A = \frac{\theta + \phi}{2},$

$$2B = \theta - \phi \quad \text{and} \quad B = \frac{\theta - \phi}{2}.$$

Using these values for A and B in (a), (b), (c) and (d) we get the desired formulæ.

45. Exercise XI.

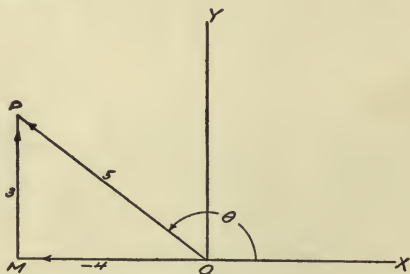


FIG. 36.

1. Given $\arctan(-\frac{3}{4}) = \theta$ find $\sin \theta/2$. Since $\arctan(-\frac{3}{4}) = \theta$, $\tan \theta = -\frac{3}{4}$ and θ must be in the second quadrant.

Cut off $OM = -4$ and draw $MP = 3$. Then $XOP = \theta$ and $OP = 5$. From Art. 42, $2 \sin^2 \theta/2 = 1 - \cos \theta = 1 - (-\frac{4}{5}) = \frac{9}{5}$. Hence $\sin \theta/2 = \pm \sqrt{\frac{9}{10}}$; but since θ lies between 90° and 180° , $\theta/2$ lies between 45° and 90° and the positive sign must be taken. So $\sin \theta/2 = \sqrt{\frac{9}{10}} = \frac{3}{\sqrt{10}}$.

2. Given $\theta = \arcsin(-\frac{2}{3})$ find $\sin 2\theta$, $\cos \theta/2$, $\sin \theta/2$.

3. If $A = \arctan \frac{2}{3}$ find $\sin 2A$ and $\tan 2A$.

4. By aid of Art. 42 find the functions of $22^\circ 30'$.

5. Given $\cos \theta = -\frac{3}{5}$, θ being in the third quadrant, find $\cos \theta/2$, $\tan \theta/2$, $\tan 2\theta$.

6. If $\arctan \frac{1}{2} = A$ and $\arcsin(-\frac{2}{3}) = B$, find $\sin(2A + B)$ and $\cos(2A - B)$.

7. If $\operatorname{arccot}(-\frac{5}{12}) = A$, find $\tan A/2$, $\sin A/2$ and $\tan 2A$.

8. If $\sin A = \frac{1}{\sqrt{2}}$ and $\cos B = -\frac{3}{5}$, A and B being in the second quadrant, find $\tan(A - B)$, $\cos A/2$, $\tan B/2$.

Factor: 9. $\sin 5A + \sin A$. 10. $\cos 3A + \cos A$.

11. $\sin 80^\circ - \sin 70^\circ$. 12. $\cos 40^\circ - \cos 10^\circ$.

13. $\cos(A + B) + \cos(A - B)$.

14. $\sin(3A + B) - \sin(A + 3B)$.

15. $\sin 55^\circ + \cos 65^\circ$. 16. $\cos 15^\circ - \sin 35^\circ$

17. $\cos(2\theta + \phi) + \cos(2\theta - \phi)$.

18. $\cos A - \cos 3A$.

19. $\sin \frac{3A + B}{2} - \sin \frac{A + 3B}{2}$.

20. $\sin(A + B - C) - \sin(A - B + C)$.

21. $\cos(A + B/4) + \cos(A/2 + 3B/4)$.

22. If $\alpha + \beta = 45^\circ$ prove that

$$(1 + \tan \alpha)(1 + \tan \beta) = 2;$$

also that

$$2 \sin \alpha \cos \alpha = (\cos \beta + \sin \beta)(\cos \beta - \sin \beta).$$

23. If $\tan \theta = \frac{1}{7}$ and $\cot \phi = \frac{1}{2}$ prove that

$$\tan(2\theta + \phi) = \frac{1}{2}.$$

24. If $x = \arcsin 1/\sqrt{5}$ and $\cot y = 3$ prove that

$$x + y = 45^\circ.$$

46. Identical Equations.

1. Prove

$$\tan A + \tan B \equiv \frac{\sin(A + B)}{\cos A \cos B}.$$

Changing to sines and cosines

$$\frac{\sin A}{\cos A} + \frac{\sin B}{\cos B} \equiv \frac{\sin (A + B)}{\cos A \cos B}.$$

Clearing of fractions

$$\sin A \cos B + \cos A \sin B \equiv \sin (A + B).$$

But this is true by Art. 36; and since the steps may be reversed the original relation is also true.

2. Prove $\tan 2\theta \equiv \tan \theta + \tan \theta \sec 2\theta$

Changing to sines and cosines

$$\frac{\sin 2\theta}{\cos 2\theta} \equiv \frac{\sin \theta}{\cos \theta} + \frac{\sin \theta}{\cos \theta} \cdot \frac{1}{\cos 2\theta}.$$

Clearing of fractions

$$\sin 2\theta \cos \theta \equiv \sin \theta \cos 2\theta + \sin \theta.$$

Transposing

$$\sin 2\theta \cos \theta - \sin \theta \cos 2\theta \equiv \sin \theta$$

But the left side is $\sin (2\theta - \theta)$. Hence the identity has been proved.

3. Prove

$$\frac{\sin 5\theta - 2 \sin 3\theta + \sin \theta}{\cos 5\theta - 2 \cos 3\theta + \cos \theta} \equiv \tan 3\theta.$$

The left side is equal to

$$\begin{aligned} \frac{(\sin 5\theta + \sin \theta) - 2 \sin 3\theta}{(\cos 5\theta + \cos \theta) - 2 \cos 3\theta} &\equiv \frac{2 \sin 3\theta \cos 2\theta - 2 \sin 3\theta}{2 \cos 3\theta \cos 2\theta - 2 \cos 3\theta} \\ &\equiv \frac{2 \sin 3\theta [\cos 2\theta - 1]}{2 \cos 3\theta [\cos 2\theta - 1]} \\ &\equiv \frac{\sin 3\theta}{\cos 3\theta} \equiv \tan 3\theta. \quad \text{Q. E. D.} \end{aligned}$$

4. If A , B , and C are the angles of a triangle prove

$$\sin A + \sin B + \sin C \equiv 4 \cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}.$$

Since

$$\frac{A+B}{2} + \frac{C}{2} \equiv 90^\circ, \sin \frac{A+B}{2} \equiv \cos \frac{C}{2} \text{ and } \cos \frac{A+B}{2} \equiv \sin \frac{C}{2}.$$

$$(\sin A + \sin B) + \sin C \equiv 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2} + 2 \sin \frac{C}{2} \cos \frac{C}{2}$$

$$\begin{aligned}
&\equiv 2 \cos \frac{C}{2} \cos \frac{A-B}{2} + 2 \cos \frac{A+B}{2} \cos \frac{C}{2} \\
&\equiv 2 \cos \frac{C}{2} \left[\cos \frac{A-B}{2} + \cos \frac{A+B}{2} \right] \\
&\equiv 2 \cos \frac{C}{2} \left[2 \cos \frac{A}{2} \cos \frac{B}{2} \right] \quad (\text{by } \S 44) \\
&\equiv 4 \cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}. \quad \text{Q. E. D.}
\end{aligned}$$

47. Exercise XII.

Prove the following identities:

1. $\tan A - \tan B \equiv \sin(A - B)/\cos A \cos B.$
2. $\sin(A + B) \cos(A - B)$
 $\quad + \cos(A + B) \sin(A - B) \equiv \sin 2A.$
3. $\sin(A + B) \sin(A - B) \equiv \sin^2 A - \sin^2 B.$
4. $\cos(n + 1)A \cos A + \sin(n + 1)A \sin A \equiv \cos nA.$
5. $\cos(A + B) \cos(A - B) \equiv \cos^2 A - \sin^2 B.$
6. $\frac{\tan(A + B) + \tan(A - B)}{1 - \tan(A + B) \tan(A - B)} \equiv \tan 2A.$
7. $\sin n\theta \cos \theta \equiv \sin(n + 1)\theta - \cos n\theta \sin \theta.$
8. $\tan 2x + \sec 2x \equiv (\cos x + \sin x)/(\cos x - \sin x).$
9. $\frac{\sin 3A + \sin 2A + \sin A}{\cos 3A + \cos 2A + \cos A} \equiv \tan 2A.$
10. $(\cot \theta + 1)/(\cot \theta - 1) \equiv (1 + \sin 2\theta)/\cos 2\theta.$
11. $\sin 3\theta/\sin \theta - \cos 3\theta/\cos \theta \equiv 2.$
12. $2 \sin 3A \cos A \equiv \sin 4A + \sin 2A.$
13. $\cos A - \cos 3A \equiv 2 \sin 2A \sin A.$
14. $\cos 3\theta \equiv 2 \cos 4\theta \cos \theta - \cos 5\theta.$
15. $\frac{\sin 8A - \sin 6A + \sin 4A - \sin 2A}{\cos 8A - \cos 6A + \cos 4A - \cos 2A} \equiv -\cot 5A.$
16. $\sin 2\theta \equiv 2 \tan \theta/(1 + \tan^2 \theta).$
17. $\cos \theta \equiv (1 - \tan^2 \theta/2)/(1 + \tan^2 \theta/2).$
18. $(\tan A + \tan B)/(\cot A + \cot B) \equiv \tan A \tan B.$
19. $(\tan A - \tan B)/(\cot A + \tan B) \equiv \tan A \tan(A - B).$
20. $1 + \tan A \tan B \equiv \cos(A - B)/\cos A \cos B.$
21. $\tan(A + B) \tan(A - B) \equiv \frac{\tan^2 A - \tan^2 B}{1 - \tan^2 A \tan^2 B}.$
22. $\tan(\pi/4 + \theta) \equiv (1 + \tan \theta)/(1 - \tan \theta).$
23. $\cot(\pi/4 + \theta) \equiv (\cot \theta - 1)/(\cot \theta + 1).$

24. $\tan^2 (\pi/4 - A) \equiv (1 - \sin 2A)/(1 + \sin 2A).$
 25. $1 + \tan 2\theta \tan \theta \equiv \sec 2\theta.$
 26. $\cos^2 A + \cos^2 (A + 60^\circ) + \cos^2 (A - 60^\circ) \equiv 3/2.$
 27. $\frac{\tan \theta}{\tan \theta - \tan 3\theta} + \frac{\cot \theta}{\cot \theta - \cot 3\theta} \equiv 1.$
 28. $\frac{1}{\tan 3\theta + \tan \theta} - \frac{1}{\cot 3\theta + \cot \theta} \equiv \cot 4\theta.$
 29. $\sin 3\theta \cos \theta + \cos 5\theta \sin 3\theta \equiv \cos 3\theta \sin \theta + \sin 5\theta \cos 3\theta.$
 30. $2 \sin (\pi/4 + A) \sin (\pi/4 - A) \equiv 1 - \text{vers } 2A.$
 31. $(\cos 2\theta + \cos 6\theta)/(\text{vers } 6\theta - \text{vers } 2\theta) \equiv \cot 2\theta \csc 4\theta - 1.$
 32. $1 - \text{vers } 2\theta \equiv (\cot^2 \theta - 1)/(\cot^2 \theta + 1).$
 33. $\frac{2 + \sin \theta - \text{vers } \theta}{\sin \theta + \text{vers } \theta} \equiv \cot \frac{\theta}{2}.$
 34. $\frac{\text{vers } 7\theta - \text{vers } \theta}{\text{vers } 5\theta - \text{vers } 3\theta} - \frac{\cos \theta + \cos 7\theta}{\cos 3\theta + \cos 5\theta} \equiv 2.$
 35. $\frac{\text{vers } 2\theta \text{ vers } \theta}{\sin 2\theta \sin \theta} \equiv \tan \theta \tan \frac{\theta}{2}.$
 36. $\tan \frac{A+B}{2} - \tan \frac{A-B}{2} \equiv 2 \sin B/(\cos A + \cos B).$
 37. $\frac{\sin (\alpha - \beta)}{\sin \alpha \sin \beta} + \frac{\sin (\beta - \gamma)}{\sin \beta \sin \gamma} + \frac{\sin (\gamma - \alpha)}{\sin \gamma \sin \alpha} \equiv 0.$
 38. $\tan \alpha/(1 - \cot 2\alpha \tan \alpha) \equiv \sin 2\alpha.$
 39. $\frac{\sin (\alpha + \beta) + \sin (\alpha - \beta)}{\sin (\alpha + \beta) - \sin (\alpha - \beta)} \equiv \frac{\tan \alpha}{\tan \beta}.$
- In examples 40-49 A , B , and C are the angles of a triangle.
40. $\sin (A + B) \equiv \sin C.$
 41. $\cos (A + B) \equiv -\cos C.$
 42. $\sin \frac{A+B}{2} \equiv \cos \frac{C}{2}.$
 43. $\cos \frac{B+C}{2} \equiv \sin \frac{A}{2}.$
 44. $\sin 2A + \sin 2B + \sin 2C \equiv 4 \sin A \sin B \sin C.$
 45. $\sin A + \sin B - \sin C \equiv 4 \sin \frac{A}{2} \sin \frac{B}{2} \cos \frac{C}{2}.$
 46. $\cos A + \cos B + \cos C \equiv 4 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2} + 1.$
 47. $\tan A + \tan B + \tan C \equiv \tan A \tan B \tan C.$
 48. $\tan \frac{A}{2} \tan \frac{B}{2} + \tan \frac{B}{2} \tan \frac{C}{2} + \tan \frac{C}{2} \tan \frac{A}{2} \equiv 1.$

49. If $\sin A \equiv 2 \cos B \sin C$ prove that the triangle is isosceles.

48. Conditional Equations.

Throughout the work equalities have been met with which are true for any value of the angles involved. These have varied from simple relations like $\tan \theta \equiv \sin \theta / \cos \theta$ and $\sin^2 \theta + \cos^2 \theta \equiv 1$ to the more complicated ones of the previous article. They are *identical equations* or more briefly *identities*. Equalities which are true only for special values of the angles are *conditional equations*, often called merely *equations*. To solve these is the present problem.

Find all the values of θ between 0° and 360° which satisfy:

1. $\sin^2 \theta + \cos \theta = 1$.

Changing into one function we get

$$1 - \cos^2 \theta + \cos \theta = 1,$$

$$-\cos^2 \theta + \cos \theta = 0,$$

$$-\cos \theta (\cos \theta - 1) = 0.$$

$$\therefore \cos \theta = 0, \text{ and } \theta = 90^\circ \text{ or } 270^\circ,$$

or $\cos \theta = 1, \text{ and } \theta = 0^\circ.$

2. $2 \cos \theta = \sin \theta + 1$.

Substituting for $\cos \theta$ its value in terms of $\sin \theta$

$$2\sqrt{1 - \sin^2 \theta} = \sin \theta + 1,$$

$$4(1 - \sin^2 \theta) = \sin^2 \theta + 2 \sin \theta + 1,$$

$$5 \sin^2 \theta + 2 \sin \theta - 3 = 0,$$

$$(5 \sin \theta - 3)(\sin \theta + 1) = 0,$$

$$\therefore \sin \theta = -1 \text{ and } \theta = 270^\circ,$$

or $\sin \theta = \frac{3}{5} \text{ and } \theta = 36^\circ 52' 12'' \text{ or } 143^\circ 7' 48''.$

Since both sides of this equation were squared extraneous roots were introduced. It is necessary to test the angles found to see which suit the original equation. It is seen immediately that $143^\circ 7' 48''$ does not satisfy the equation, the left side being negative and the right positive in that case.

3. $\sin 2\theta \cos \theta = \sin \theta$

Hence

$$2 \sin \theta \cos^2 \theta = \sin \theta,$$

$$\therefore \sin \theta = 0 \text{ and } \theta = 0^\circ \text{ or } 180^\circ,$$

or

$$2 \cos^2 \theta = 1, \cos \theta = \pm 1/\sqrt{2} \text{ and } \theta = 45^\circ, 135^\circ, 225^\circ, 315^\circ.$$

4. $\cos 3\theta + \cos \theta = \sin 3\theta + \sin \theta$.

By Art. 44

$$2 \cos 2\theta \cos \theta = 2 \sin 2\theta \cos \theta.$$

$$\therefore \cos \theta = 0 \quad \text{and} \quad \theta = 90^\circ \quad \text{or} \quad 270^\circ,$$

or $\cos 2\theta = \sin 2\theta$, that is, $\tan 2\theta = 1$.

Hence $2\theta = 45^\circ, 225^\circ, 405^\circ, 585^\circ$.

$$\theta = 22^\circ 30', 112^\circ 30', 202^\circ 30', 292^\circ 30'.$$

The above examples illustrate some of the methods of solving trigonometric equations. In all of them by factoring the equation values of some function of an angle are obtained. In the first two, everything is expressed in terms of a single function. This is the most common method. Having in any way found the value of one function it remains to find the values of the angle. For this it is very useful to remember that any function of θ has the same numerical value as the same function of $180^\circ \pm \theta$ and $360^\circ - \theta$, two of the four angles giving positive values, the remaining two negative ones. Thus if $\cos \theta = -\frac{1}{2}$, we look for the acute angle whose cosine is $\frac{1}{2}$. This is 60° . Since $\cos \theta$ is negative, θ must be in the second or third quadrants; it is therefore $180^\circ \pm 60^\circ$, that is, 120° or 240° .

The student must guard against cancelling out a factor without keeping account of the corresponding roots. Thus in example 3 a common error is to merely cancel out $\sin \theta$ and reduce the equation to $2 \cos^2 \theta = 1$. Also if it be found necessary to square both sides of an equation $A = B$, thus obtaining $A^2 = B^2$, the solutions will include those of $A = -B$, in addition to the ones desired. In this case the angles should be substituted in the original equation and those discarded which do not satisfy it. In example (2) an instance of this was shown.

49. Exercise XIII. Find all values of θ between 0° and 360° which satisfy:

1. $\cos \theta + \sec \theta = \frac{5}{2}$.

2. $\sin \theta = 1 - \text{vers } \theta$.

3. $\sin \theta = \tan \theta$.

4. $2 \cos \theta = 3 \tan \theta$.

5. $\tan^2 \theta + 3 \cot^2 \theta = 4$.

6. $3 \cos \theta = 2 \sin^2 \theta$.

7. $\cos 2\theta + \sin^2 \theta = \frac{3}{4}$.

8. $\sin 5\theta + \sin 3\theta = \cos \theta$.
9. $\sin 2\theta = 3 \tan \theta - 3 \tan \theta \text{ vers } 2\theta$.
10. $\sin 6\theta = \sin 4\theta - \sin 2\theta$.
11. $\cos 3\theta + \cos 2\theta + \cos \theta = 0$.
12. $\cot 2\theta + \tan \theta + 1 = 0$.
13. $\sin 2\theta + \sin \theta = \cos 2\theta + \cos \theta$.
14. $2 \sec \theta = \tan \theta + \cot \theta$.
15. $6 \tan^2 \theta - 4 \sin^2 \theta = 1$.
16. $\frac{\sin \theta}{\text{vers } \theta} = 2 + \cot \theta$.
17. $\tan \theta - \cot \theta = \cot 2\theta$.
18. $\sin 3\theta - \sin \theta = \cos 3\theta + \cos \theta$.
19. $\sin \theta + \sin 2\theta = \sqrt{3} \cos \frac{\theta}{2}$.
20. $\sin \theta + \sin 2\theta + \sin 3\theta = 1 + \cos \theta + \cos 2\theta$.
21. $\sin \theta - \cos \theta = \sqrt{2}$.
22. $\text{vers } \theta = \text{vers } 2\theta$.
23. $\sec^2 \theta + \csc^2 \theta = \frac{1}{8}$.
24. $\tan \theta + 2 \cot \theta = \frac{5}{2} \csc \theta$.
25. $2 \sin 3\theta \cos \theta = 2 \sin \theta \cos 3\theta - 1$.
26. $\tan 2\theta - 2 \text{ vers } \theta + 2 = 0$.
27. $\cos 2\theta + \sin \theta = 2 \text{ vers } 2\theta$.
28. $\sin 2\theta = 2 \sin \theta + \text{vers } \theta$.

Are the following identical or conditional equations? If the former, prove the identities; if the latter, solve the equations.

29. $\sin \theta + \csc \theta = \frac{5}{2}$.
30. $\tan 2x - \tan x = \tan x \sec 2x$.
31. $\cot A - \cot 2A = \csc 2A$.
32. $\sin 2A = 2 \sin^2 A$.
33. $\sin 3\theta + \sin 2\theta + \sin \theta = 0$.
34. $\cos^2 \theta = 1 - \sin \theta$.
35. $\frac{\cot 2\theta}{\cot 2\theta + \tan \theta} = \cos 2\theta$.
36. $\cos A + \cos 3A + \cos 5A + \cos 7A$
 $= 4 \cos A \cos 2A \cos 4A$.
37. $\text{vers } 2\theta = 2 \sin \theta$.
38. $\frac{\cos (\theta + \pi/4)}{\cos (\theta - \pi/4)} = \sec 2\theta - \tan 2\theta$.

39. $\cot 2\theta = \tan \theta - 1.$

40. $\cot^2 \theta (2 \csc \theta - 3) + 3 (\csc \theta - 1) = 0.$

41. $3 \tan^2 \theta + 8 \cos^2 \theta = 7.$

42. $\tan \theta + \cot \theta = 4.$

43. $\sin \theta + \sqrt{3} \cos \theta = 1.$

CHAPTER IV.

50. Solution of Triangles. In geometry it is shown that a triangle can be constructed when any three of its parts are given, provided that one of the three be a side. Four cases arise:

1. One side and two angles;
2. Two sides and an angle opposite one of them;
3. Two sides and the included angle;
4. Three sides.

In the second case two triangles may sometimes be found having the required parts.

We proceed to derive formulæ for computing the unknown parts.

51. Law of Sines. The sines of the angles of any triangle are proportional to the opposite sides.

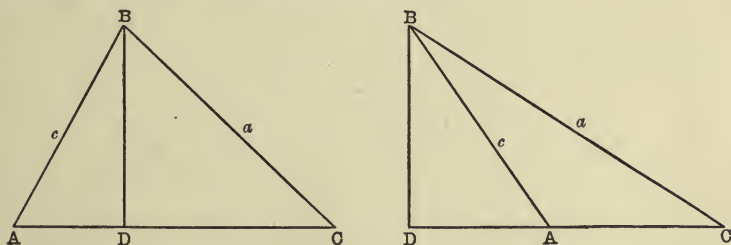


FIG. 37.

First proof. In the triangle ABC draw BD perpendicular to AC . Then in both figures

$$\sin A = \frac{DB}{c}, \quad \text{and} \quad \frac{DB}{a} = \sin C.$$

Multiplying one of these equations by the other and cancelling DB , we get

$$\frac{\sin A}{a} = \frac{\sin C}{c}.$$

In like manner by dropping from C a perpendicular to AB , it may be proved that

$$\frac{\sin A}{a} = \frac{\sin B}{b}.$$

Hence

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.$$

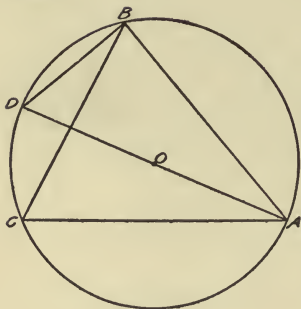


FIG. 38.

Second proof. Circumscribe a circle about ABC and draw the diameter through A . Join BD . Then, since the angle C is equal to the angle D , $\sin C = \sin D = c/AD = c/d$, where d is the diameter of the circumcircle. Hence

$$\frac{\sin C}{c} = \frac{1}{d}.$$

Similarly $\frac{\sin A}{a} = \frac{1}{d}$ and $\frac{\sin B}{b} = \frac{1}{d}$.

Therefore $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = \frac{1}{d}$.

52. Since $\sin A/\sin B = a/b$, we have by composition and division

$$\frac{a-b}{a+b} = \frac{\sin A - \sin B}{\sin A + \sin B} = \frac{2 \sin \frac{A-B}{2} \cos \frac{A+B}{2}}{2 \cos \frac{A-B}{2} \sin \frac{A+B}{2}},$$

$$\therefore \frac{a-b}{a+b} = \frac{\tan \frac{1}{2}(A-B)}{\tan \frac{1}{2}(A+B)},$$

Similarly

$$\frac{b-c}{b+c} = \frac{\tan \frac{1}{2}(B-C)}{\tan \frac{1}{2}(B+C)},$$

and

$$\frac{c-a}{c+a} = \frac{\tan \frac{1}{2}(C-A)}{\tan \frac{1}{2}(C+A)}.$$

53. Law of Cosines. The square on one side of any triangle is equal to the sum of the squares on the other two sides diminished by twice the product of these sides times the cosine of the angle between them.

To prove $a^2 = b^2 + c^2 - 2bc \cos A$.

Two cases arise, one when A is acute and the other when A is obtuse. In each, from C draw CD perpendicular to AB .

$$a^2 = \overline{DC}^2 + \overline{DB}^2,$$

$$DB = DA + AB = AB - AD = c - AD.$$

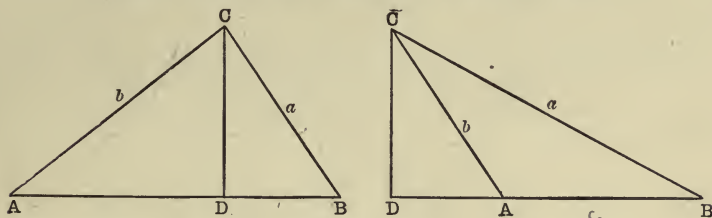


FIG. 39.

Hence

$$\begin{aligned} a^2 &= \overline{DC}^2 + (c - AD)^2 = \overline{DC}^2 + c^2 - 2c \cdot AD + \overline{AD}^2 \\ &= b^2 + c^2 - 2c \cdot AD. \end{aligned}$$

But $AD = b \cos A$ (being negative when A is obtuse)

$$\therefore a^2 = b^2 + c^2 - 2bc \cos A.$$

This may be written $\cos A = \frac{b^2 + c^2 - a^2}{2bc}.$

Similarly $\cos B = \frac{c^2 + a^2 - b^2}{2ca},$

and $\cos C = \frac{a^2 + b^2 - c^2}{2ab}.$

54. Law of Tangents. These equations are not in form for logarithmic work but from them we can get others that are.

$$\begin{aligned} \tan \frac{A}{2} &= \sqrt{\frac{1 - \cos A}{1 + \cos A}} = \sqrt{\frac{1 - \frac{b^2 + c^2 - a^2}{2bc}}{1 + \frac{b^2 + c^2 - a^2}{2bc}}} \\ &= \sqrt{\frac{a^2 - (b^2 - 2bc + c^2)}{(b^2 + 2bc + c^2) - a^2}} \\ &= \sqrt{\frac{(a - b + c)(a + b - c)}{(a + b + c)(b + c - a)}}. \end{aligned}$$

Putting

$$a + b + c \equiv 2s,$$

then

$$a + b - c \equiv 2(s - c), \text{ etc.}$$

Hence
$$\tan \frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$$

$$\therefore \tan \frac{A}{2} = \frac{1}{s-a} \sqrt{\frac{(s-a)(s-b)(s-c)}{s}}.$$

Putting (see Art. 55)

$$r = \sqrt{\frac{(s-a)(s-b)(s-c)}{s}}$$

we get

$$\tan \frac{A}{2} = \frac{r}{s-a}.$$

Similarly

$$\tan \frac{B}{2} = \frac{r}{s-b},$$

and

$$\tan \frac{C}{2} = \frac{r}{s-c}.$$

55. Radius of Inscribed Circle. Draw the bisectors of the angles of the triangle ABC . These meet at O , the centre of the inscribed circle; from O drop perpendiculars to the sides of the triangle. Since $BC_1 = BA_1$, $AC_1 = AB_1$ and $CA_1 = CB_1$,

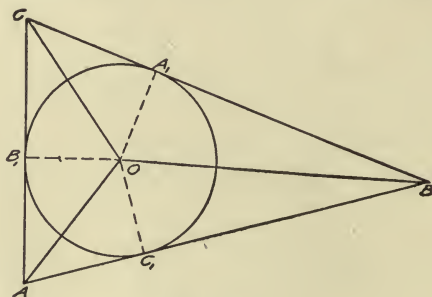


FIG. 40.

we have $BC_1 + AC_1 + CA_1 = BA_1 + AB_1 + CB_1$. Each side of this equation must therefore be half the sum of the sides. Hence

$$s = BC_1 + AC_1 + CA_1 = c + CA_1, \text{ or } CA_1 = s - c.$$

But
$$\tan OCA_1 = \tan \frac{C}{2} = \frac{A_1O}{CA_1} = \frac{A_1O}{s-c}.$$

Comparing this with the last equation of the preceding article shows that $A_1O = r$. Hence the radius of the inscribed circle is given by

$$r = \sqrt{\frac{(s-a)(s-b)(s-c)}{s}}.$$

56. Area. Drop a perpendicular from B on the opposite side, produced if necessary.

The area ABC is $\frac{1}{2}b \cdot DB$; but since

$$\frac{DB}{a} = \sin C,$$

we have on replacing DB by its value

$$\text{area} = \frac{1}{2}ab \sin C. \quad (1)$$

This formula gives the area in terms of two sides and the included angle.

From the law of sines,

$$a = \frac{c \sin A}{\sin C}$$

and

$$b = \frac{c \sin B}{\sin C}.$$

Using these in (1) gives

$$\text{area} = \frac{1}{2} \frac{c^2 \sin A \sin B}{\sin C}. \quad (2)$$

This formula may be used when one side and two angles are given.

Finally, in Fig. 40, $ABC = AOB + BOC + COA = \frac{1}{2}cr + \frac{1}{2}ar + \frac{1}{2}br = rs$. Using the value of r obtained in the preceding article we get

$$\text{area} = \sqrt{s(s-a)(s-b)(s-c)}, \quad (3)$$

a formula giving the area in terms of the sides.

57. Case I. Given One Side and Two Angles. The third angle may be found from the relation

$$A + B + C = 180^\circ;$$

the remaining sides may be found from the law of sines.

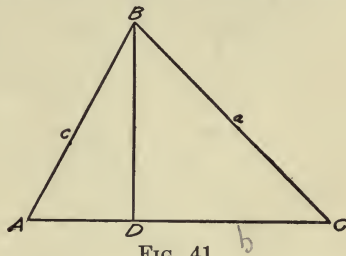


FIG. 41.

Example. Solve the triangle ABC in which

$$B = 100^\circ 21' 10'', \quad C = 58^\circ 17' 20'' \quad \text{and} \quad a = 31.656.$$

$$\text{Formulae} \begin{cases} A = 180^\circ - (B + C) = 21^\circ 21' 30'', \\ b = a \sin B / \sin A, \\ c = a \sin C / \sin A. \end{cases}$$

Taking logarithms:

$$\begin{array}{rcl} \log a & = & 11.50045 - 10 \\ - \log \sin A & = & 9.56134 - 10 \\ & & \hline & & 1.93911 \\ + \log \sin B^* & = & 9.99287 - 10 & + \log \sin C = 9.92978 - 10 \\ & & \hline \log b & = & 1.93198 & \log c = 1.86889 \\ \therefore b & = & 85.502 & \therefore c = 73.942 \end{array}$$

Hence the solution is

$$A = 21^\circ 21' 30'', \quad b = 85.502, \quad c = 73.942.$$

58. Case II. Given Two Sides and the Angle Opposite One of Them. If a , b and B are known A may be found from the law of sines, $\sin A = a \sin B / b$.† But since it is $\sin A$ that is given by this equation, A may be either an acute angle or its supplement, and in general there will be two values of A . Having found the values of A , C may be found from $A + B + C = 180^\circ$ and finally c may be found from the law of sines.

Example. Solve the triangle in which

$$a = 413.28, \quad b = 378.19, \quad \text{and} \quad B = 50^\circ 16' 25''.$$

From the law of sines $\sin A = a \sin B / b$.

$$\begin{array}{rcl} \log a & = & 2.61625 \\ + \log \sin B & = & 9.88599 - 10 \\ & & \hline & & 12.50224 - 10 \\ - \log b & = & 2.57771 \\ & & \hline \log \sin A & = & 9.92453 - 10 \\ \therefore A & = & 57^\circ 11' 30'' \quad \text{or} \quad 122^\circ 48' 30''. \end{array}$$

Call the obtuse angle A' . Then

$$\begin{array}{rcl} C & = & 180 - (A + B), & C' & = & 180 - (A' + B), \\ & = & 72^\circ 32' 5'', & & = & 6^\circ 55' 5''. \end{array}$$

* See Art. 31.

† If $a \sin B / b > 1$ the triangle is impossible since $\sin A$ cannot be greater than 1.

From the law of sines

$$c = \frac{b \sin C}{\sin B},$$

$$c' = \frac{b \sin C'}{\sin B}.$$

$$\begin{aligned} \log b &= 12.57771 - 10 \\ - \log \sin B &= \underline{9.88599 - 10} \end{aligned}$$

$$\begin{aligned} &2.69172 \\ + \log \sin C &= \underline{9.97950 - 10} \end{aligned}$$

$$\log c = 2.67122$$

$$\therefore c = 469.05$$

$$\begin{aligned} &2.69172 \\ + \log \sin C' &= \underline{9.08081 - 10} \end{aligned}$$

$$\log c' = 1.77253$$

$$\therefore c' = 59.229$$

Hence the solutions are

$$\left. \begin{aligned} A &= 57^\circ 11' 30'' \\ C &= 72^\circ 32' 5'' \\ c &= 469.05 \end{aligned} \right\} \text{ or } \left. \begin{aligned} A' &= 122^\circ 48' 30'' \\ C' &= 6^\circ 55' 5'' \\ c' &= 59.229. \end{aligned} \right\}$$

In the above example then, there are two triangles which satisfy the conditions. The solution of every problem under this case proceeds as this one did till we try to get C' . In certain examples $A' + B$ will be found to be greater than 180° . It is evidently impossible to find C' in that case and there is only one solution. *This will occur when the angle given is opposite the larger of the sides given.*

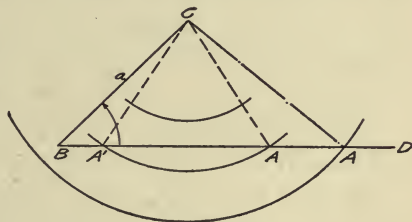


FIG. 42.

To recall the geometric construction in this case will be a help.

Construct $\angle B$ and on one arm lay off $BC = a$. Then with C as centre and b as radius draw a circle to cut BD . If $b < a$ two points A and A' will be found (provided b is long enough to reach BD . See footnote, p. 58). If $b > a$ only one point A will be found.

59. Case III. Given Two Sides and the Included Angle. If a , b , and C be known we can find $a - b$, $a + b$ and $\frac{1}{2}(A + B)$ and then by aid of a formula of Art. 52, $\frac{1}{2}(A - B)$ may be calculated. This with $\frac{1}{2}(A + B)$ will give A and B . c may then be found by the law of sines.

Example. Given $a = 42.38$, $b = 35$ and $C = 43^\circ 14' 40''$ solve the triangle.

Formulae:

$$\begin{aligned} \tan \frac{A-B}{2} &= \frac{a-b}{a+b} \tan \frac{A+B}{2}, & c &= \frac{a \sin C}{\sin A}. \\ a &= 42.38 & \log a &= 11.62716 - 10 \\ b &= 35 & - \log \sin A &= 9.99565 - 10 \\ \hline a-b &= 7.38 & & 1.63151 \\ a+b &= 77.38 & + \log \sin C &= 9.83576 - 10 \\ A+B &= 180 - C = 136^\circ 45' 20'' & \log c &= 1.46727 \\ \frac{1}{2}(A+B) &= 68^\circ 22' 40'' & \therefore c &= 29.327 \\ \log(a-b) &= 0.86806 \\ + \log \tan \frac{1}{2}(A+B) &= 0.40189 \\ \hline & 11.26995 - 10 \\ - \log(a+b) &= 1.88863 \\ \log \tan \frac{1}{2}(A-B) &= 9.38132 - 10 \\ \therefore \frac{1}{2}(A-B) &= 13^\circ 31' 45'' \\ \frac{1}{2}(A+B) &= 68^\circ 22' 40'' \\ \text{Adding,} & A = 81^\circ 54' 25'' \\ \text{Subtracting,} & B = 54^\circ 50' 55'' \end{aligned}$$

Hence the solution is $A = 81^\circ 54' 25''$, $B = 54^\circ 50' 55''$ and $c = 29.327$.

60. Case IV. Given Three Sides. If a , b , and c are given $A/2$, $B/2$ and $C/2$ may be found from the formulæ of Art. 54.

> Example. Given $a = 0.312$, $b = 0.423$ and $c = 0.342$, find A , B and C .

Formulae:

$$\begin{aligned} \tan \frac{A}{2} &= \frac{r}{s-a}, & \tan \frac{B}{2} &= \frac{r}{s-b}, & \tan \frac{C}{2} &= \frac{r}{s-c}. \\ a &= 0.312 & \log(s-a) &= 9.35507 - 10 \\ b &= 0.423 & + \log(s-b) &= 9.06258 - 10 \\ c &= 0.342 & + \log(s-c) &= 9.29336 - 10 \\ 2s &= 1.077 & & 27.71101 - 30 \\ s &= 0.5385 & - \log s &= 9.73119 - 10 \\ s-a &= 0.2265 & & 2 \overline{17.97982 - 20} \\ s-b &= 0.1155 & \log r &= 8.98991 - 10 \\ s-c &= 0.1965 & & \\ s &= 0.5385 & & \end{aligned}$$

$$\begin{aligned}
 \log r &= 18.98991 - 20 \\
 - \log (s - a) &= \frac{9.35507 - 10}{9.63484 - 10} \\
 \log \tan A/2 &= \\
 \therefore A/2 &= 23^\circ 20' \\
 A &= 46^\circ 40', \\
 \log r &= 18.98991 - 20 \\
 \log (s - b) &= \frac{9.06258 - 10}{9.92733 - 10} \\
 \log \tan B/2 &= \\
 \therefore B/2 &= 40^\circ 13' 45'' \\
 B &= 80^\circ 27' 30'', \\
 \log r &= 18.98991 - 20 \\
 \log (s - c) &= \frac{9.29336 - 10}{9.69655 - 10} \\
 \log \tan C/2 &= \\
 \therefore C/2 &= 26^\circ 26' 15'' \\
 C &= 52^\circ 52' 30''.
 \end{aligned}$$

61. Example 1. Each of two ships A and B , 415 yards apart, measures the horizontal angle subtended by a cliff and the other ship; the angles are $48^\circ 17'$ and $110^\circ 10'$ respectively. If the angle of elevation of the cliff from A is $15^\circ 24'$ what is the height of the cliff?

C is the top of the cliff and CC' a vertical line, C' being at the water level. Then it is given that $AB = 415$, $\angle C'AB = 48^\circ 17'$, $\angle C'BA = 110^\circ 10'$, and $\angle CAC' = 15^\circ 24'$. It is required to find CC' .

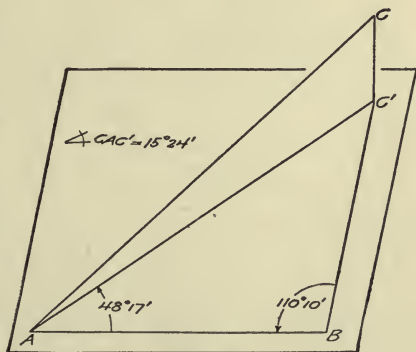


FIG. 43.

In the right triangle ACC' we can find CC' if AC' be known. But AC' may be found from the triangle ABC' since three parts of this triangle are given. Use the law of sines $\sin B/AC' = \sin C'/AB$ remembering that $C' = 180 - (A + B) = 21^\circ 33'$.

Solving,

$$AC' = \frac{AB \sin B}{\sin C'} = \frac{415 \sin 110^\circ 10'}{\sin 21^\circ 33'}.$$

$$\begin{aligned}
 \log 415 &= 12.61805 - 10 \\
 - \log \sin 21^\circ 33' &= \frac{9.56504 - 10}{3.05301} \\
 + \log \sin 110^\circ 10' &= \frac{9.97252 - 10}{\therefore \log AC' = 3.02553}
 \end{aligned}$$

In the right triangle ACC'

$$\frac{CC'}{AC'} = \tan 15^\circ 24'.$$

$$\therefore CC' = AC' \tan 15^\circ 24'.$$

$$\log AC' = 3.02553$$

$$+ \log \tan 15^\circ 24' = \frac{9.44004 - 10}{\log CC' = 2.46557}$$

$$\therefore CC' = 292.13.$$

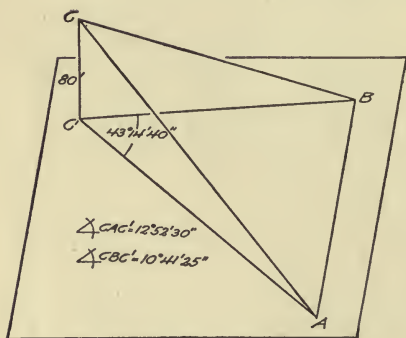


FIG. 44.

Example 2. From a tower 80 ft. high, objects A and B in a plane are found to have angles of depression $12^\circ 52' 30''$ and $10^\circ 41' 25''$ respectively; the horizontal angle at C subtended by A and B is $43^\circ 14' 40''$. Find the distance between A and B .

C being the top of the tower and C' the foot, we

have given

$$CC' = 80, \angle CAC' = 12^\circ 52' 30''.$$

$$\angle CBC' = 10^\circ 41' 25'' \text{ and } \angle AC'B = 43^\circ 14' 40''.$$

In the right triangles ACC' and BCC'

$$AC' = 80 \cot 12^\circ 52' 30'', \quad BC' = 80 \cot 10^\circ 41' 25''.$$

$$\log 80 = 1.90309$$

$$+ \log \cot 12^\circ 52' 30'' = \frac{0.64098}{\log AC' = 2.54407}$$

$$\log AC' = 2.54407$$

$$\therefore AC' = 350.$$

$$\log 80 = 1.90309$$

$$+ \log \cot 10^\circ 41' 25'' = \frac{0.72405}{\log BC' = 2.62714}$$

$$\log BC' = 2.62714$$

$$\therefore BC' = 423.78.$$

Now in the triangle ABC' we know three parts. Hence AB may be found as in Art. 59.

62. Exercise XIV. Solve the triangles in which the following parts are given:

1. $A = 40^\circ 20' 25''$, $B = 75^\circ 15' 35''$, $a = 315.7$.
2. $A = 52^\circ 18' 25''$, $C = 62^\circ 18' 50''$, $b = 42.72$.
3. $A = 57^\circ 18'$, $a = 3.715$, $b = 4.285$.
4. $a = 87.24$, $b = 73.58$, $C = 48^\circ 17'$.
5. $a = 73.53$, $c = 81.27$, $B = 72^\circ 19'$.
6. $a = 763.2$, $b = 653.5$, $c = 827.2$.
7. $a = 41.38$, $b = 51.71$, $c = 47.82$.

8. A and B are two points 258 ft. apart and C a point such that at A the angle subtended by BC is $41^\circ 18'$, at B the angle subtended by AC is $72^\circ 21' 5''$; find the distance between A and C .

9. A and B are points 400 ft. apart taken on the edge of a river and C is a stone on the opposite edge. If the angles CAB and CBA are $72^\circ 20' 10''$ and $65^\circ 10' 40''$ respectively find the width of the river.

10. The angle of elevation of an object from the foot of a hill is $32^\circ 15'$; after going 317 yds. up the hill away from the object the observer finds himself on a level with it. If the slope of the hill is 15° , find the distance from the foot of the hill to the object.

11. A cliff known to be 450 ft. high is observed to be due north of a boat and at an elevation of 30° . After going a distance northeast the boat found the elevation to be 35° . How far did it go?

12. From a point on one side of a muskeg a man measures 450 yds. east; then northwest a distance of 652 yds. How far is he from the starting place?

13. The sides AB , BC and CA of a triangle are 250, 300 and 350 ft. respectively. BC is produced 275 ft. to the point D . What is the angle at A subtended by CD ?

14. At each of two places 400 ft. apart the elevation of a kite is found to be $27^\circ 15'$. The horizontal angle at one place subtended by the kite and the other is $50^\circ 20'$. Find the height of the kite.

15. At noon the shadow of a cloud falls 450 ft. to the north of an observer; if the elevation of the cloud (which is south of the observer) be $75^{\circ} 12'$ and of the sun be $62^{\circ} 8'$ what is the vertical height of the cloud?

16. In 15 what would be the height were the cloud north of the observer?

17. A lighthouse is 75 ft. high and has an elevation $25^{\circ} 19'$ from one point; from another point 120 ft. from the first the elevation is $28^{\circ} 23' 40''$. What is the horizontal angle at the lighthouse subtended by the two points?

18. The elevation of a balloon due north of a station is 30° ; at a place one mile southeast of the first the horizontal angle subtended by the balloon and the first place is $20^{\circ} 15'$. Find the height of the balloon.

19. The elevation of a rock is 47° ; after walking 1,000 ft. toward it up a hill inclined at 32° to the level the elevation is 77° . Find the height of the rock above the first point.

20. From the foot of a tree in a level field a line 200 ft. is measured and the elevation of the top of the tree is found to be $18^{\circ} 20'$. From this point a line 325 ft. long is measured and the elevation of the top is then found to be $22^{\circ} 30' 25''$. Find the angle between the two lines which were measured.

21. A triangular piece of ground is found to be 37 ft. 6 in. by 42 ft. 3 in. by 58 ft. 3 in. Find the angle opposite the shortest side.

22. From a tower 75 ft. high the angles of depression of two objects are $24^{\circ} 29'$ and $31^{\circ} 58'$; the horizontal angle subtended by the objects is $51^{\circ} 42'$. Find the distance between the objects (supposed to be level with the foot of the tower).

23. From a mountain top 3,200 ft. above sea level ships are observed, one east, the other southeast; the angles of depression are $12^{\circ} 43'$ and $15^{\circ} 37'$. Find the distance between the ships, and the direction from one ship to the other.

24. Two towns A and B are four miles apart; from a balloon above A the depression of B is $15^{\circ} 6'$ and when the balloon is above B the depression of A is $19^{\circ} 30'$. Find how much the balloon has risen.

25. The horizontal distance between two points A and B

is a mile and a half. The horizontal angle at A subtended by B and the top of a mountain is $65^{\circ} 18' 25''$ and that at B subtended by A and the top is $74^{\circ} 29' 40''$. The angle of elevation at A of the top of the mountain is $18^{\circ} 28' 55''$. If the height above sea level of A is 2,500 ft. find the height of the mountain.

26. A flagstaff 78 ft. high stands on the face of a hill whose inclination to the horizon is 34° . At a point down the hill from the flagstaff, the angle of elevation of its top is $57^{\circ} 28'$; find the distance from the observer to the foot of the flagstaff.

27. At a point up the hill from the flagstaff in (26), the depression of its top is $18^{\circ} 53'$. Find the distance of the observer from the top of the flagstaff.

28. Two stations A and B on a level plane are 785.4 ft. apart. At A the elevation of an aeroplane C is $38^{\circ} 19' 25''$, and the horizontal angle at A subtended by B and C is $41^{\circ} 30' 20''$. At B the horizontal angle subtended by A and C is $63^{\circ} 18' 20''$. Find the height of the aeroplane.

29. The observer at B in (28) makes the elevation of C $46^{\circ} 49'$ at the same time that the other observations were taken. Find the height of C using this observation.

CHAPTER V.

SPHERICAL TRIGONOMETRY.

INTRODUCTORY REVIEW.

63. Those definitions and theorems of solid geometry which are essential to the study of spherical trigonometry will be stated or briefly discussed in this chapter.

64. Dihedral Angles. Let ABC and ABD be two planes intersecting in the line AB . The figure formed by these planes at their intersection is called a dihedral angle. The line AB is called the edge of the dihedral angle. Let P be any point on the edge, and let PE and PF be lines drawn perpendicular to AB in the planes ABC and ABD respectively. The angle EPF

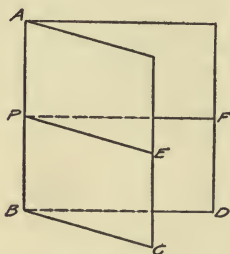


FIG. 45.

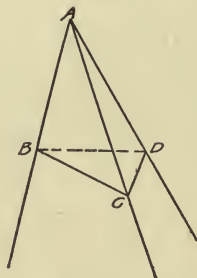


FIG. 46.

is called the plane angle of the dihedral angle, and is the measure of the dihedral angle.

65. Trihedral Angles. Let ABC , ACD , and ADB be three planes meeting in the common point A . The figure formed by these planes is called a trihedral angle. The point A is called the vertex; the intersections of the planes AB , AC , AD are called the edges; the portions of the planes between the edges are called the faces; the angles formed at the vertex by the edges BAC , CAD , DAB are called the face angles; and the dihedral angles formed at the edges are called the dihedral angles of the trihedral angle.

66. The Sphere. A spherical surface is a surface all points

of which are equidistant from a point called the centre. A sphere is a solid bounded by a spherical surface.

Every plane section of a sphere is a circle. A great circle on a sphere is a circle whose plane passes through the centre of the sphere. All other circles on a sphere are called small circles.

Let A and B be two points on the surface of a sphere, and let AB be the shorter arc of the great circle passing through these points. Then AB is the shortest path that can be drawn on the surface of the sphere between A and B . The length of this arc is defined as the distance from A to B measured on the surface of the sphere. Let O be the centre of the sphere, and draw OA and OB . Then, since on the same or equal circles equal arcs subtend equal angles at the centre, the angle AOB may be taken as the measure of the arc AB . The angle AOB is called the *angular distance* between A and B . Produce BO to meet the sphere again in B_1 . Let the radius of the sphere be r , and let the angle AOB (measured in degrees) be θ° . Then, from plane geometry, we have

$$\text{arc } AB : \text{arc } BB_1 = \theta : 180$$

But

$$\text{arc } BB_1 = \text{a semicircle} = \pi r.$$

Therefore

$$\text{arc } AB = \frac{\pi r \theta}{180}.$$

Thus, given the angular distance θ° and the radius of the sphere, the actual distance on the surface of the sphere may be computed.

Two points are said to be at a quadrant's distance when their angular distance is 90° .

The poles of any circle on a sphere are the extremities of the diameter of the sphere drawn perpendicular to the plane of the circle. Each pole of a great circle is at a quadrant's distance from every point of the circle.

67. Spherical Angles. The angle between two circles on the

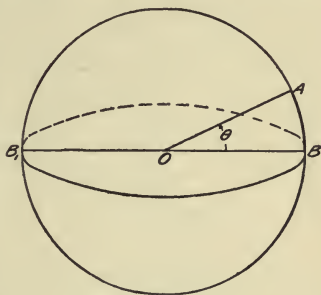


FIG. 47.

surface of a sphere is defined as the angle between the tangents to the circles at their point of intersection. The angle formed by the arcs of two great circles is called a spherical angle.

Let AB and AC be arcs of two great circles on a sphere whose centre is at O , and let AS and AT be the tangents to these circles at A .

Then by definition the spherical angle is measured by the angle TAS . But since a tangent to a circle is perpendicular

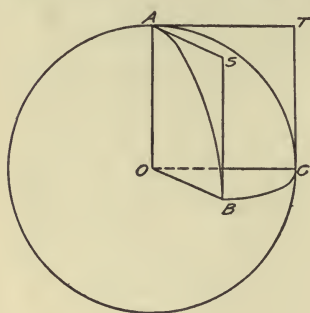


FIG. 48.

to the radius drawn to the point of contact, AT and AS are perpendicular to OA , and hence TAS is the measure of the diedral angle $T-OA-S$ or $B-OA-C$. A spherical angle is therefore measured by the plane angle of the diedral angle formed by the planes of the circles.

Draw OB and OC perpendicular to OA at O in the planes AOB and AOC respectively. Then the

angles TAS and BOC are equal. But BOC is measured by the great circle arc BC . Hence, since A is the pole of the great circle arc BC , a spherical angle is measured by the arc of the great circle of which the vertex is a pole, and which is intercepted between the sides of the angle.

68. Spherical Triangles. The figure bounded by the arcs of three great circles is called a spherical triangle. Let ABC be a spherical triangle on a sphere whose centre is at O . Draw the radii OA , OB and OC . Then the figure formed by the planes OAB , OAC , and OBC is a triedral angle. The angles of the spherical triangle are measured by the plane angles of the diedral angles of this triedral angle. Also the sides of the spherical triangle are measured by the face angles of the triedral angles. For, by definition the angular

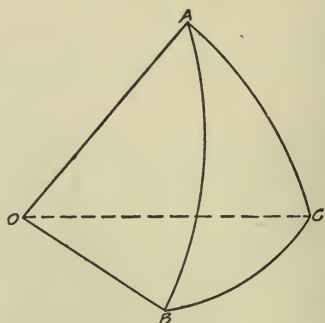


FIG. 49

distance AB is the angle AOB , subtended by AB at the centre. Hence with any spherical triangle there is associated a triedral angle at the centre of the sphere, such that the angles of the triangle are measured by the diedral angles of the triedral angle, and the sides of the triangle by the face angles of the triedral angle. This complete correspondence between the spherical triangle and the triedral angle at the centre of the sphere is of fundamental importance in what follows.

69. It is usual to assume that each side of a spherical triangle is less than 180° . This assumption causes no loss of generality. For let $AB(D)C$ be a spherical triangle in which the side BC is greater than 180° . Complete the great circle $BDCE$. Then in the triangle $AB(E)C$ each of the sides is less than 180° . We may then consider this triangle instead of the original one, and at the end of our investigations we may return to the triangle $AB(D)C$ by taking the supplements of the angles B and C of $AB(E)C$, for the corresponding angles of $AB(D)C$, and by taking for BDC the difference between 360° and BEC .

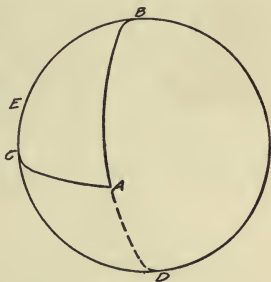


FIG. 50.

It is easily shown (by producing BA to meet BDC in D) that the angle A is greater than 180° if the side opposite to it, BDC , is greater than 180° , and conversely.

The sum of any two sides of a spherical triangle is greater than the third side. This follows from the corresponding theorem concerning the face angles of a convex triedral angle.

The sum of the sides of a spherical triangle is less than four right angles, since the sum of the face angles of a convex triedral angle is less than four right angles.

70. **Polar Triangles.** Let arcs of great circles be drawn with the vertices of a spherical triangle ABC as poles, and let C' be that intersection of the circles drawn with A and B as poles which lies on the same side of the arc AB as does C , and similarly for the other two intersections B' and A' . The triangle $A'B'C'$ is said to be the polar triangle of ABC .

If the first of two spherical triangles is the polar of the second, then the second is also the polar of the first.

In two polar triangles each angle of one is the supplement of the side opposite to it in the other.

The latter statement may be proved as follows.

Let ABC and $A'B'C'$ be two polar triangles.

To prove that $A = 180^\circ - a'$, etc.

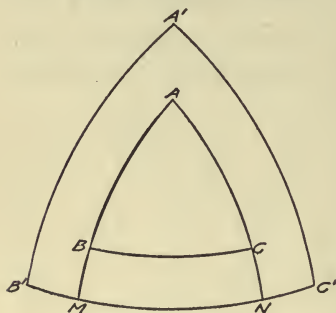


FIG. 51.

Produce the arcs AB and AC to meet $B'C'$ in M and N respectively. Since B' is the pole of the arc ACN , $B'N$ is a quadrant. Similarly MC' is a quadrant. Hence $B'N + MC' = 180^\circ$. That is, $B'N + NC' + MN = 180^\circ$. But $B'N + N'C' = B'C' = a'$, and MN is the measure of the angle A , (page 68). Therefore $a' + A = 180^\circ$,

or $A = 180^\circ - a'$, etc.

71. The sum of the angles of a spherical triangle is greater than two and less than six right angles.

Let ABC be a spherical triangle.

To prove that $180^\circ < A + B + C < 540^\circ$.

Construct the polar triangle. Then,

$$A = 180^\circ - a', \quad B = 180^\circ - b', \quad C = 180^\circ - c'.$$

Hence, upon addition,

$$A + B + C = 540^\circ - (a' + b' + c').$$

Now

$$0^\circ < a' + b' + c' < 360^\circ.$$

Hence

$$A + B + C = 540^\circ - [\text{something less than } 360^\circ],$$

that is,

$$A + B + C > 180^\circ.$$

Similarly,

$$A + B + C = 540^\circ - [\text{something greater than } 0^\circ],$$

that is,

$$A + B + C < 540^\circ.$$

Therefore

$$180^\circ < A + B + C < 540^\circ.$$

The *spherical excess* of a triangle is defined as the difference between the sum of the angles of the triangle and 180° .

72. Lunes. The figure formed by joining the extremities of a diameter of a sphere by the arcs of two great circles is called a lune. The extremities of the diameter are called the vertices of the lune, and the spherical angles formed at the vertices by the great circle arcs are called the angles of the lune. Thus in Fig. 50 $BADC B$ is a lune with vertices at B and D_1 .

73. Spherical Degrees. Let A and B be the ends of a diameter of a sphere. With A and B as vertices construct a lune whose angle is 1° . Then the area of this lune is one three hundred and sixtieth of the area of the sphere. If now we draw with A and B as poles the great circle arc CD , the lune $ACBD$ is divided into two equal triangles. The area of one of these triangles is defined as a spherical degree of area, or as a spherical degree. The total surface of the sphere then contains 720 spherical degrees.

Since the areas of two lunes on the same or equal spheres are proportional to their angles, a lune of angle A° contains $2A$ spherical degrees.

74. Areas of Spherical Triangles. We shall now show that the area of a spherical triangle, measured in spherical degrees, is equal to the spherical excess of that triangle.

Let ABC be a spherical triangle. Complete the great circle arcs AB , BC , CA , the two circles through A meeting again in A_1 , etc.

The arcs AC_1 and A_1C are equal since each is equal to a semicircle diminished by the arc AC . Similarly, $BC = B_1C_1$, and $AB_1 = A_1B$. Then the two triangles A_1BC and AB_1C_1 are equivalent.

Hence, the area of the lune $ABA_1C = ABC + AB_1C_1 = 2A$ spherical degrees.

Also, the area of the lune $BAB_1C = ABC + AB_1C = 2B$ spherical degrees.

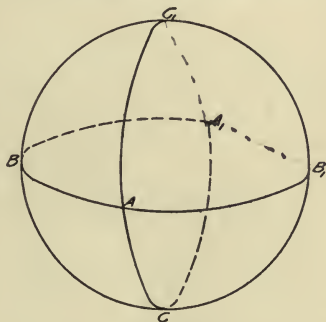


FIG. 52.

And the area of the lune $CAC_1B = ABC + AC_1B = 2C$ spherical degrees. Hence, on addition,

$$2ABC + \text{the hemisphere } CBC_1B_1 = 2(A + B + C)$$

or

the area of $ABC = A + B + C - 180$ spherical degrees.

Hence the area of a spherical triangle is measured by its spherical excess.

The total area of a sphere of radius r is $4\pi r^2$. Also there are 720 spherical degrees in the surface of a sphere. Hence the area of one spherical degree is equal to $4\pi r^2/720$ or $\pi r^2/180$. It follows then that the area of a spherical triangle is given by the formula

$$\text{area } ABC = \frac{(A + B + C - 180)\pi r^2}{180}.$$

CHAPTER VI.

RELATIONS BETWEEN THE SIDES AND ANGLES OF A SPHERICAL TRIANGLE.

75. The Law of Cosines. Let ABC be a spherical triangle on a sphere whose centre is at O , and let the sides b and c be acute.

At any point, A_1 , on OA draw a plane perpendicular to OA meeting the planes OAC , OAB , and OBC in A_1L , A_1M , and ML respectively.

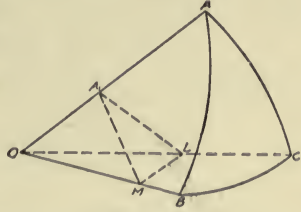


FIG. 53.

Then MA_1L is the measure of the dihedral angle $B-OA-C$, and therefore measures the spherical angle A [Art. 67]. Also the angles OA_1M and OA_1L are right angles by construction.

In the triangle MA_1L ,

$$ML^2 = MA_1^2 + LA_1^2 - 2MA_1 \cdot LA_1 \cos A,$$

and in the triangle MOL ,

$$ML^2 = MO^2 + LO^2 - 2MO \cdot LO \cos a.$$

Hence, upon subtraction and transposition,

$$2MO \cdot LO \cos a = MO^2 - MA_1^2 + LO^2 - LA_1^2 + 2MA_1 \cdot LA_1 \cdot \cos A. \quad (1)$$

But, since the triangles MOA_1 and LOA_1 are right triangles,

$$MO^2 - MA_1^2 = OA_1^2 \quad \text{and} \quad LO^2 - LA_1^2 = OA_1^2.$$

The equation (1) may then be written

$$MO \cdot LO \cos a = OA_1^2 + MA_1 \cdot LA_1 \cdot \cos A,$$

or

$$\cos a = \frac{OA_1}{MO} \cdot \frac{OA_1}{LO} + \frac{MA_1}{MO} \cdot \frac{LA_1}{LO} \cos A.$$

But

$$\frac{OA_1}{MO} = \cos MOA_1 = \cos c, \quad \frac{MA_1}{MO} = \sin MOA_1 = \sin c,$$

$$\frac{OA_1}{LO} = \cos LOA_1 = \cos b, \quad \frac{LA_1}{LO} = \sin LOA_1 = \sin b.$$

Hence $\cos a = \cos b \cos c + \sin b \sin c \cos A.$ (2)

It was necessary to assume that the sides b and c were acute in order to draw the right triangles LOA_1 and MOA_1 as in the figure. We shall now show that the formula (2) is true for any spherical triangle.

Case (1). Let ABC be a spherical triangle in which b and c are both obtuse.

Produce the arcs AB and AC to form the lune AA' . Then in the triangle A_1BC , b_1 and c_1 are acute and equal to $180^\circ - b$

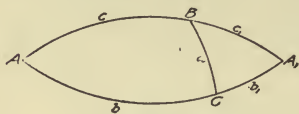


FIG. 54.

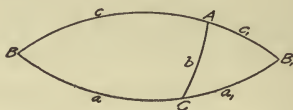


FIG. 55.

and $180^\circ - c$ respectively. Then the formula (2) is true for this triangle. That is,

$$\cos a = \cos (180^\circ - b) \cos (180^\circ - c) + \sin (180^\circ - b) \sin (180^\circ - c) \cos A$$

or

$$\cos a = \cos b \cos c + \sin b \sin c \cos A.$$

Case (2). Let ABC be a spherical triangle in which b is acute and c obtuse. Produce the arcs BA and BC to form the lune BB_1 . Then in the triangle AB_1C , b and c_1 are acute.

Hence,

$$\cos a_1 = \cos b \cos c_1 + \sin b \sin c_1 \cos CAB_1.$$

But

$$a_1 = 180^\circ - a, \quad c_1 = 180^\circ - c.$$

and

$$CAB_1 = 180^\circ - CAB = 180^\circ - A.$$

Therefore

$$\cos a = \cos b \cos c + \sin b \sin c \cos A.$$

Hence in any spherical triangle

$$\cos a = \cos b \cos c + \sin b \sin c \cos A,$$

and similarly $\cos b = \cos c \cos a + \sin c \sin a \cos B,$

$$\text{and } \cos c = \cos a \cos b + \sin a \sin b \cos C.$$

I

76. Since the equations (I) are true for any triangle, they are true for the polar triangle $A'B'C'$ of a given triangle ABC .

Hence,

$$\cos a' = \cos b' \cos c' + \sin b' \sin c' \cos A'.$$

But

$$a' = 180^\circ - A, \quad b' = 180^\circ - B, \quad c' = 180^\circ - C.$$

and

$$A' = 180^\circ - a.$$

Therefore,

$$-\cos A = (-\cos B)(-\cos C) + \sin B \sin C(-\cos a)$$

or

$$\cos A = -\cos B \cos C + \sin B \sin C \cos a,$$

$$\text{similarly, } \cos B = -\cos C \cos A + \sin C \sin A \cos b, \quad \text{II}$$

$$\text{and } \cos C = -\cos A \cos B + \sin A \sin B \cos c.$$

77. The Sine Proportion. From the first equation of (I), Art. 75, we have

$$\cos A = \frac{\cos a - \cos b \cos c}{\sin b \sin c}.$$

Hence

$$1 - \cos^2 A = 1 - \frac{(\cos a - \cos b \cos c)^2}{\sin^2 b \sin^2 c}$$

or

$$\begin{aligned} \sin^2 A &= \frac{\sin^2 b \sin^2 c - (\cos a - \cos b \cos c)^2}{\sin^2 b \sin^2 c} \\ &= \frac{(1 - \cos^2 b)(1 - \cos^2 c) - (\cos a - \cos b \cos c)^2}{\sin^2 b \sin^2 c} \end{aligned}$$

Therefore,

$$\frac{\sin^2 A}{\sin^2 a} = \frac{1 - \cos^2 a - \cos^2 b - \cos^2 c + 2 \cos a \cos b \cos c}{\sin^2 a \sin^2 b \sin^2 c}$$

or

$$\frac{\sin A}{\sin a} = \frac{\sqrt{1 - \cos^2 a - \cos^2 b - \cos^2 c + 2 \cos a \cos b \cos c}}{\sin a \sin b \sin c},$$

the positive sign being taken since A and a are each less than 180° .

The expression on the right is *symmetric* in the letters a , b , and c . That is, if any two of the letters be interchanged, the expression is unaltered. Then, if we proceed as above with the second and third equations of (I), page 74, we shall obtain on the left $\sin B/\sin b$, and $\sin C/\sin c$, and on the right the same expression as above. Hence,

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}. \quad \text{III}$$

78. Law of Tangents. From Art. 42, page 43, we have

$$\tan \frac{A}{2} = \sqrt{\frac{1 - \cos A}{1 + \cos A}}, \quad (1)$$

and from (I), page 74,

$$\cos A = \frac{\cos a - \cos b \cos c}{\sin b \sin c}. \quad (2)$$

Substituting in (1) the value of $\cos A$ found from (2), we obtain

$$\begin{aligned} \tan \frac{A}{2} &= \frac{\sqrt{1 - \frac{\cos a - \cos b \cos c}{\sin b \sin c}}}{\sqrt{1 + \frac{\cos a - \cos b \cos c}{\sin b \sin c}}} \\ &= \sqrt{\frac{\sin b \sin c - \cos a + \cos b \cos c}{\sin b \sin c + \cos a - \cos b \cos c}} \\ &= \sqrt{\frac{\cos b \cos c + \sin b \sin c - \cos a}{\cos a - (\cos b \cos c - \sin b \sin c)}} \\ &= \sqrt{\frac{\cos (b - c) - \cos a}{\cos a - \cos (b + c)}} \\ &= \sqrt{\frac{\cos a - \cos (b - c)}{\cos (b + c) - \cos a}}. \end{aligned}$$

Or, upon making use of a factor formula (page 44),

$$\tan \frac{A}{2} = \sqrt{\frac{\sin \frac{1}{2}(a + b - c) \sin \frac{1}{2}(a - b + c)}{\sin \frac{1}{2}(b + c + a) \sin \frac{1}{2}(b + c - a)}}.$$

Let

$$a + b + c = 2s.$$

Then

$$-a + b + c = 2s - 2a,$$

and so

$$\frac{1}{2}(-a + b + c) = s - a,$$

and

$$\frac{1}{2}(a - b + c) = s - b,$$

and

$$\frac{1}{2}(a + b - c) = s - c.$$

Then

$$\begin{aligned} \tan \frac{A}{2} &= \sqrt{\frac{\sin (s - b) \sin (s - c)}{\sin s \sin (s - a)}}, \\ &= \frac{1}{\sin (s - a)} \sqrt{\frac{\sin (s - a) \sin (s - b) \sin (s - c)}{\sin s}}. \end{aligned}$$

Now let

$$K = \sqrt{\frac{\sin(s-a) \sin(s-b) \sin(s-c)^*}{\sin s}}.$$

We then have, observing that K is symmetric in a, b, c ,

$$\tan \frac{A}{2} = \frac{K}{\sin(s-a)}$$

$$\text{similarly} \quad \tan \frac{B}{2} = \frac{K}{\sin(s-b)} \quad \text{IV}$$

$$\text{and} \quad \tan \frac{C}{2} = \frac{K}{\sin(s-c)}.$$

79. Napier's Analogies. From the equations above we have

$$\tan \frac{A}{2} \tan \frac{B}{2} = \frac{K^2}{\sin(s-a) \sin(s-b)}.$$

Or, upon replacing each tangent by the ratio of the sine to the cosine and substituting for K^2 its value,

$$\frac{\sin \frac{A}{2} \sin \frac{B}{2}}{\cos \frac{A}{2} \cos \frac{B}{2}} = \frac{\sin(s-c)}{\sin s}. \quad (1)$$

Subtracting each side from unity and simplifying,

$$\frac{\cos \frac{A}{2} \cos \frac{B}{2} - \sin \frac{A}{2} \sin \frac{B}{2}}{\cos \frac{A}{2} \cos \frac{B}{2}} = \frac{\sin s - \sin(s-c)}{\sin s}.$$

Making use of the formula for the cosine of the sum of two angles, page 38, and that for the difference between the sines of two angles, page 44, the equation above becomes

$$\frac{\cos \frac{1}{2}(A+B)}{\cos \frac{A}{2} \cos \frac{B}{2}} = \frac{2 \cos \frac{1}{2}(2s-c) \sin \frac{c}{2}}{\sin s}. \quad (2)$$

If now we add unity to each side of (1) and simplify the result in a manner precisely similar to the above, we shall obtain

* For the geometrical interpretation of K see page 101.

$$\frac{\cos \frac{1}{2}(A - B)}{\cos \frac{A}{2} \cos \frac{B}{2}} = \frac{2 \sin \frac{1}{2}(2s - c) \cos \frac{c}{2}}{\sin s}. \quad (3)$$

Upon dividing (2) by (3) and noting that

$$2s - c = (a + b + c) - c = a + b,$$

we have

$$\begin{aligned} \frac{\cos \frac{1}{2}(A + B)}{\cos \frac{1}{2}(A - B)} &= \frac{\cos \frac{1}{2}(a + b) \sin \frac{c}{2}}{\sin \frac{1}{2}(a + b) \cos \frac{c'}{2}} \\ &= \cot \frac{1}{2}(a + b) \tan \frac{c}{2}. \end{aligned}$$

Therefore

$$\frac{\cos \frac{1}{2}(A + B)}{\cos \frac{1}{2}(A - B)} = \frac{\tan \frac{c}{2}}{\tan \frac{1}{2}(a + b)}. \quad V(a)$$

From (IV) we also have

$$\frac{\tan \frac{A}{2}}{\tan \frac{B}{2}} = \frac{\sin(s - b)}{\sin(s - a)}.$$

By replacing each tangent by the ratio of the sine to the cosine, this equation becomes

$$\frac{\sin \frac{A}{2} \cos \frac{B}{2}}{\cos \frac{A}{2} \sin \frac{B}{2}} = \frac{\sin(s - b)}{\sin(s - a)}. \quad (4)$$

If now we apply to (4) the same series of operations that we have above applied to (1), we obtain

$$\frac{\sin \frac{1}{2}(A + B)}{\sin \frac{1}{2}(A - B)} = \frac{\tan \frac{c}{2}}{\tan \frac{1}{2}(a - b)}. \quad V(b)$$

The details of this reduction are left for the student.

Since $V(a)$ is true for any spherical triangle, it is true for the polar triangle $A'B'C'$ of a given triangle ABC . That is,

$$\frac{\cos \frac{1}{2}(A' + B')}{\cos \frac{1}{2}(A' - B')} = \frac{\tan \frac{c'}{2}}{\tan \frac{1}{2}(a' + b')}. \quad (5)$$

But,

$$\frac{1}{2}(A' + B') = \frac{1}{2}(180^\circ - a + 180^\circ - b) = 180^\circ - \frac{1}{2}(a + b),$$

$$\frac{1}{2}(A' - B') = \frac{1}{2}(180^\circ - a - 180^\circ + b) = -\frac{1}{2}(a - b),$$

$$\frac{1}{2}(a' + b') = \frac{1}{2}(180^\circ - A + 180^\circ - b) = 180^\circ - \frac{1}{2}(A + B),$$

and

$$\frac{c'}{2} = \frac{1}{2}(180^\circ - C) = 90^\circ - \frac{C}{2}.$$

Substituting these values in (5) we have

$$\frac{\cos \frac{1}{2}(a + b)}{\cos \frac{1}{2}(a - b)} = \frac{\cot \frac{C}{2}}{\tan \frac{1}{2}(A + B)}. \quad V(c)$$

Similarly, by aid of the polar triangle, V(b) gives rise to

$$\frac{\sin \frac{1}{2}(a + b)}{\sin \frac{1}{2}(a - b)} = \frac{\cot \frac{C}{2}}{\tan \frac{1}{2}(A - B)}. \quad V(d)$$

The four equations found in this article are known as Napier's analogies. Each of these equations, upon permuting the letters, gives rise to two new equations. In all, then, there are twelve such equations of which a , b , c and d are the types.

80. On Species. Two angles are said to be of the same species if they are both acute or both obtuse, and of different species if one is acute and the other obtuse.

The following law of species is of great importance and should be carefully memorized.

One half the sum of any two sides of a spherical triangle and one half the sum of the two opposite angles are of the same species. For, from V(a),

$$\frac{\cos \frac{1}{2}(A + B)}{\cos \frac{1}{2}(A - B)} = \frac{\tan \frac{c}{2}}{\tan \frac{1}{2}(a + b)}.$$

Since each side and angle of the spherical triangle is less than 180° [Art. 69, page 69], $\frac{1}{2}(A + B)$ and $\frac{1}{2}(a + b)$ are each less than 180° , and $\frac{1}{2}(A - B)$ and $c/2$ are each less than 90° . Hence, in the above equation, the denominator on the left and the numerator on the right are each positive. Then $\cos \frac{1}{2}(A + B)$ and $\tan \frac{1}{2}(a + b)$ are either both positive or both negative, and therefore $\frac{1}{2}(A + B)$ and $\frac{1}{2}(a + b)$ are of the same species.

CHAPTER VII.

THE RIGHT SPHERICAL TRIANGLE.

81. A spherical triangle one of whose angles is a right angle is called a right spherical triangle.

We shall prove [Art. 85] that if, in addition to the right angle, two other parts be given the triangle can be solved. For this purpose we deduce from formulæ of the preceding chapter ten relations each involving a different set of three parts. These formulæ can be conveniently recalled by means of *Napier's rule of circular parts*.

By the circular parts of a right spherical triangle we mean the two sides about the right angle and the complements of the hypotenuse and the other two angles. Place these five

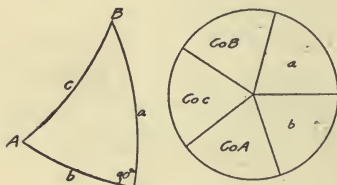


FIG. 56.

parts as in the accompanying figure, being careful to arrange them in the order in which they occur in the triangle. It will be noticed that on this figure any three parts can be classed as either a middle and two adjacent parts, or a middle and two opposite parts.

82. Napier's Rule. *The sine of the middle part is equal to the product of the tangents of the adjacent parts, and to the product of the cosines of the opposite parts.*

Since there are ten combinations of five things taken three at a time, this rule will give us ten formulæ. For example, let ABC be a right spherical triangle right-angled at C . Then taking b , $\text{co-}A$, $\text{co-}B$ the middle part is $\text{co-}B$ and the other two are the opposite parts. Therefore,

$$\sin (90^\circ - B) = \cos (90^\circ - A) \cos b,$$

or

$$\cos B = \sin A \cos b.$$

Also taking a , $\text{co-}c$, $\text{co-}B$, the middle part is $\text{co-}B$ and the others are the adjacent parts. Therefore,

$$\sin (90^\circ - B) = \tan a \tan (90^\circ - c),$$

or

$$\cos B = \tan a \cot c$$

The following table contains all the formulæ so obtained:

$\cos c = \cos a \cos b,$ (1)	$\cos c = \cot A \cot B,$ (2)
$\cos A = \sin B \cos a,$ (3)	$\cos B = \sin A \cos b,$ (4)
$\sin a = \sin c \sin A,$ (5)	$\sin b = \sin c \sin B,$ (6)
$\sin a = \tan b \cot B,$ (7)	$\sin b = \tan a \cot A,$ (8)
$\cos A = \tan b \cot c,$ (9)	$\cos B = \tan a \cot c.$ (10)

83. These relations may be deduced as follows:

Upon setting $C = 90^\circ$, the last equation of (I), page 74, becomes

$$\cos c = \cos a \cos b. \quad (1)$$

The last equation of (II), page 75, gives

$$0 = -\cos A \cos B + \sin A \sin B \cos c,$$

and therefore,

$$\cos c = \cot A \cot B. \quad (2)$$

The first two equations of (II), page 75, give

$$\cos A = \sin B \cos a, \quad (3)$$

and

$$\cos B = \sin A \cos b \quad (4)$$

The sine proportion (III), Art. 77, becomes

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{1}{\sin c},$$

and hence

$$\sin a = \sin c \sin A, \quad (5)$$

$$\sin b = \sin c \sin B, \quad (6)$$

and

$$\sin a = \frac{\sin b \sin A}{\sin B}.$$

Or, upon substituting for $\sin A$ its value found from (4),

$$\sin a = \tan b \cot B, \quad (7)$$

and similarly

$$\sin b = \tan a \cot A. \quad (8)$$

From 8,

$$\tan a = \sin b \tan A$$

or

$$\frac{\sin a}{\cos a} = \frac{\sin b \sin A}{\cos A}.$$

Substituting in this equation the values of $\sin a$ and $\cos a$ found from 5 and 1, we have

$$\frac{\sin c \sin A \cos b}{\cos c} = \frac{\sin b \sin A}{\cos A}$$

or

$$\cos A = \tan b \cot c, \quad (9)$$

and similarly

$$\cos B = \tan a \cot c. \quad (10)$$

84. Laws of Species for the right Spherical Triangle. We shall prove the following two laws of species for the right spherical triangle.

1. *A side of a right spherical triangle and the angle opposite it are of the same species.*

2. *If the hypotenuse of a right spherical triangle be acute, the two other sides are of the same species, and if the hypotenuse be obtuse, the two other sides are of different species.*

Proof of 1.

$$\sin b = \tan a \cot A. \quad (8)$$

Since b is less than 180° , $\sin b$ is positive. Then $\tan a$ and $\cot A$ are of the same sign, and hence a and A are of the same species.

Proof of 2.

$$\cos c = \cos a \cos b. \quad (1)$$

If c is less than 90° , $\cos c$ is positive. Then $\cos a$ and $\cos b$ are of the same sign, and hence a and b are of the same species. On the other hand, if c is greater than 90° , $\cos c$ is negative, then $\cos a$ and $\cos b$ are of opposite signs, and hence a and b are of different species.

85. The Solution of the right Spherical Triangle. When two parts of a right spherical triangle are given the remaining parts may be computed by means of Napier's rule of circular parts and the laws of species. The method in detail will be illustrated by the following examples.

When a relation involves a trigonometric function of an obtuse

angle we replace the angle by its supplement. This substitution in any one of the formulæ (1)–(10) will at most give the wrong sign to the desired function, thus leaving it uncertain as to whether the desired angle is acute or obtuse. The laws of species are intended to remove this ambiguity

Ex. 1.

$$\begin{aligned} a &= 47^\circ 30' 40'', \\ c &= 120^\circ 20' 30''. \end{aligned}$$

Applying Napier's rule to acA , acB , acb and solving each equation for the unknown part, we have

$$\sin A = \frac{\sin a}{\sin c}, \quad \cos B = \tan a \cot c, \quad \cos b = \frac{\cos c}{\cos a}.$$

Since c is obtuse a and b are of different species. Then b and B are obtuse, and A is acute.

$$\begin{array}{lll} \log \sin a = 9.86771 & \log \tan a = 0.03812 & \log \cos c = 9.70342 \\ \log \sin c = 9.93603 & \log \cot c = 9.76740 & \log \cos a = 9.82959 \\ \log \sin A = 9.93168 & \log \cos B = 9.80552 & \log \cos b = 9.87383 \\ A = 58^\circ 41' 55'' & B = 180^\circ - 50^\circ 16' 50'' & b = 180^\circ - 41^\circ 35' 35'' \\ & = 129^\circ 43' 10'' & = 138^\circ 24' 25'' \end{array}$$

Check:* Napier's rules applied to A , B , b give

$$\cos B = \cos b \sin A.$$

$$\log \cos b = 9.87383$$

$$\log \sin A = 9.93168$$

$$\log \cos B = 9.80551$$

Ex. 2. $a = 103^\circ 12', \quad A = 97^\circ 24'.$

Napier's rules give

$$\sin b = \tan a \cot A, \quad \sin B = \frac{\cos A}{\cos a}, \quad \sin c = \frac{\sin a}{\sin A}.$$

$$\begin{array}{lll} \log \tan a = 0.62977 & \log \cos A = 9.10990 & \log \sin a = 9.98837 \\ \log \cot A = 9.11353 & \log \cos a = 9.35860 & \log \sin A = 9.99637 \\ \log \sin b = 9.74330 & \log \sin B = 9.75130 & \log \sin c = 9.99200 \end{array}$$

Since a is obtuse it follows that, if b be also obtuse, c must

* To test the accuracy of the arithmetical work it is necessary to use as a check some formula which has not already been used. In any right spherical triangle the most convenient relation for this purpose is that one which involves the three required parts.

be acute, and, if b be acute, c must be obtuse. The laws of species give no means of telling whether b is acute or obtuse. In fact there are two triangles with the given parts a and A , which together form a lune of angle A , as in the accompanying figure.

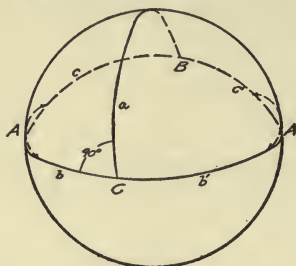


FIG. 57.

The two solutions are then

- (1) $b = 33^\circ 37' 25''$,
 $B = 34^\circ 20' 5''$,
 $c = 100^\circ 58'$,
- (2) $b' = 180^\circ - b = 146^\circ 22' 35''$,
 $B' = 180^\circ - B = 145^\circ 39' 55''$,
 $c' = 180^\circ - c = 79^\circ 2'$.

Check:

$$\begin{aligned}\sin b &= \sin c \sin B. \\ \log \sin c &= 9.99200 \\ \log \sin B &= 9.75130 \\ \hline \log \sin b &= 9.74330\end{aligned}$$

86. Exercise XV.

Solve the following spherical triangles in which $C = 90^\circ$:

1. $a = 53^\circ 48' 10''$,
 $b = 73^\circ 12' 20''$.
2. $a = 120^\circ 40' 5''$,
 $c = 78^\circ 22' 25''$.
3. $a = 23^\circ 59' 55''$,
 $c = 133^\circ 24' 50''$.
4. $A = 27^\circ 54' 5''$,
 $b = 100^\circ 58' 10''$.
5. $b = 27^\circ 50' 30''$,
 $A = 114^\circ 14' 25''$.
6. $B = 78^\circ 53' 55''$,
 $b = 40^\circ 12' 10''$.
7. $A = 53^\circ 54' 55''$,
 $c = 127^\circ 12' 10''$.
8. $a = 127^\circ 54' 10''$,
 $A = 105^\circ 50' 25''$.
9. $A = 110^\circ 13' 20''$,
 $B = 73^\circ 13' 35''$.
10. $b = 35^\circ 55' 30''$,
 $c = 143^\circ 14' 10''$.
11. $a = 29^\circ 13' 35''$,
 $b = 54^\circ 14' 25''$.
12. $a = 154^\circ 42' 10''$,
 $c = 148^\circ 14' 20''$.

87. Quadrantal Triangles. A spherical triangle is said to be quadrantal if one of its sides is equal to a quadrant, that is, equal to 90° .

In the spherical triangle ABC let $c = 90^\circ$. Then in the polar triangle $A'B'C'$ we have $C' = 90^\circ$. When any two parts of the quadrantal triangle in addition to c are given, this triangle may be solved by the methods explained in the foregoing

articles; we may find the unknown parts of the (right) polar triangle, and, by subtracting these from 180° , find the required parts of the quadrantal triangle.

Quadrantal triangles may also be solved directly as follows.

Napier's rule, as stated for the right triangle, also holds for quadrantal triangles provided we take as the circular parts $A, B, 90^\circ - a, - (90^\circ - C), 90^\circ - b$.

The laws of species for quadrantal triangles are:

1. *A side of a quadrantal triangle and the angle opposite it are of the same species.*

2. *If the angle opposite the quadrant be acute, the two other angles are of different species, and if this angle be obtuse the two other angles are of the same species.*

The proofs of these laws of species and of the application of Napier's rule to quadrantal triangles are left as exercises to the student.

88. Exercise XVI.

Solve the following spherical triangles in which $c = 90^\circ$:

- | | |
|------------------------------|-------------------------------|
| 1. $a = 50^\circ 49' 25''$, | 2. $A = 103^\circ 25' 55''$, |
| $B = 73^\circ 12' 35''$. | $C = 79^\circ 15' 5''$. |
| 3. $b = 18^\circ 28' 10''$, | 4. $a = 100^\circ 13' 20''$, |
| $C = 55^\circ 58' 30''$ | $A = 123^\circ 16' 45''$. |

CHAPTER VIII.

THE SOLUTION OF OBLIQUE TRIANGLES.

89. In the study of solid geometry we have learned that two triangles are equal or symmetrical if

1) Three sides of one are equal respectively to three sides of the other.

2) Three angles of one are equal respectively to three angles of the other.

3) Two sides and the included angle of one are equal respectively to two sides and the included angle of the other.

4) A side and the adjacent angles of one are equal respectively to a side and the adjacent angles of the other.

That is, if the three parts mentioned in any one of these cases be given, the remaining parts are fully determined. It is then natural to suppose that, given such a set of three parts, the remaining three parts may be computed. In addition to these four cases we shall find that, when given two sides and an angle opposite one of them, or two angles and a side opposite one of them, there will be, at most, two distinct triangles having these parts, and that the remaining parts of each triangle may be computed.

90. We shall now show how to find the unknown parts of a spherical triangle of which we are given

1) The three sides, a, b, c .

2) The three angles, A, B, C .

3) Two sides and the included angle, e. g., a, b, C .

4) A side and the two adjacent angles, e. g., c, A, B .

5) Two sides and the angle opposite one of them, e. g., a, b, A .

6) Two angles and the side opposite one of them, e. g., A, B, a .

91. Case (1). **Given the Three Sides, a, b, c .**

The half angles may be determined by the formulæ (IV), page 77. It is important to observe that, since the angles are less than 180° , each of the half angles is less than 90° . It therefore is not necessary to use the laws of species. After

finding the three angles the accuracy of the results should be tested by substituting the angles in some relation that has not already been used. For this the sine proportion (III), page 75, is the most convenient.

Ex. 1. $a = 58^\circ$, $b = 80^\circ$, $c = 96^\circ$.

Formulæ:

$$\tan \frac{A}{2} = \frac{K}{\sin (s-a)}, \quad \tan \frac{B}{2} = \frac{K}{\sin (s-b)}, \quad \tan \frac{C}{2} = \frac{K}{\sin (s-c)},$$

$$K = \sqrt{\frac{\sin (s-a) \sin (s-b) \sin (s-c)}{\sin s}}.$$

$a = 58^\circ$			
$b = 80^\circ$	$\log \sin (s-a) = 9.93307$		
$c = 96^\circ$	$\log \sin (s-b) = 9.77946$	$\log \tan \frac{A}{2} = 9.72542$	
$2s = 234^\circ$	$\log \sin (s-c) = 9.55433$		
$s = 117^\circ$	$\frac{9.26686}{\log \sin s = 9.94988}$	$\log \tan \frac{B}{2} = 9.87903$	
$s-a = 59^\circ$	$\log K^2 = 9.31698$	$\log \tan \frac{C}{2} = 0.10416$	
$s-b = 37^\circ$	$\log K = 9.65849$		
$s-c = 21^\circ$			
$s = 117^\circ$			

$$\frac{A}{2} = 27^\circ 59' 10'', \quad A = 55^\circ 58' 20'',$$

$$\frac{B}{2} = 37^\circ 7' 20'', \quad B = 74^\circ 14' 40'',$$

$$\frac{C}{2} = 51^\circ 48' 20'', \quad C = 103^\circ 36' 40''.$$

Check: $d = \frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}.$

$\log \sin A = 9.91843$	$\log \sin B = 9.98337$	$\log \sin C = 9.98763$
$\log \sin a = 9.92842$	$\log \sin b = 9.99335$	$\log \sin c = 9.99761$
$\log d = 9.99001$	$\log d = 9.99001$	$\log d = 9.99002$

Ex. 2. $a = 53^\circ 12' 35''$, $b = 75^\circ 14' 25''$, $c = 69^\circ 27' 20''$.

92. Case (2). Given the Three Angles, A , B , C .

Since the angles of a spherical triangle are the supplements of the corresponding sides of the polar triangle, we may find the sides of the polar triangle. Then, by the method of case (1), we may compute the angles of the polar triangle. The supplements of these angles will then be the required sides of the original triangle.

Ex. 1. $A = 78^\circ 40'$	$\log \sin (s - a)' = 9.98338$
$B = 63^\circ 50'$	$\log \sin (s - b)' = 9.93495$
$C = 46^\circ 20'$	$\log \sin (s - c)' = 9.82481$
	$\hline 9.74314$

In the polar triangle

$a' = 101^\circ 20'$	$\log \sin s' = 8.88654$
$b' = 116^\circ 10'$	$\log K'^2 = 0.85660$
$c' = 133^\circ 40'$	$\log K' = 0.42830$
$2s' = 350^\circ 70'$	
$s' = 175^\circ 35'$	$\log \tan \frac{A'}{2} = 0.44492,$
$(s - a)' = 74^\circ 15'$	$\log \tan \frac{B'}{2} = 0.49335,$
$(s - b)' = 59^\circ 25'$	
$(s - c)' = 41^\circ 55'$	$\log \tan \frac{C'}{2} = 0.60349.$
$\hline s' = 175^\circ 35'$	

$$\frac{A'}{2} = 70^\circ 15' 10''-, \quad A' = 140^\circ 30' 20''-,$$

$$\frac{B'}{2} = 72^\circ 11' 50''+, \quad B' = 144^\circ 23' 40''+,$$

$$\frac{C'}{2} = 76^\circ 0' 30''-, \quad C' = 152^\circ 1'-.$$

Hence, in the original triangle,

$$a = 39^\circ 29' 40'', \quad b = 35^\circ 36' 20'', \quad c = 27^\circ 59'.$$

Check: $\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}.$

$\log \sin A = 9.99145$	$\log \sin B = 9.95304$	$\log \sin C = 9.85936$
$\log \sin a = 9.80346$	$\log \sin b = 9.76507$	$\log \sin c = 0.67137$
$\hline 0.18799$	$\hline 0.18797$	$\hline 0.18798$

Ex. 2. $A = 105^\circ 14' 20'', B = 80^\circ 0' 10'', C = 68^\circ 23' 35''.$

93. Case (3). Given Two Sides and the Included Angle, e. g., $a, b, C.$

Solving the relations (c), (d), and (b), Art. 79, for $\tan \frac{1}{2}(A + B)$, $\tan \frac{1}{2}(A - B)$, and $\tan c/2$ respectively, we have

$$\tan \frac{1}{2}(A + B) = \frac{\cos \frac{1}{2}(a - b) \cot \frac{C}{2}}{\cos \frac{1}{2}(a + b)},$$

$$\tan \frac{1}{2}(A - B) = \frac{\sin \frac{1}{2}(a - b) \cot \frac{C}{2}}{\sin \frac{1}{2}(a + b)},$$

and
$$\tan \frac{c}{2} = \frac{\sin \frac{1}{2}(A + B) \tan \frac{1}{2}(a - b)}{\sin \frac{1}{2}(A - B)}.$$

By the first two of these relations we may compute half the sum, and half the difference of the unknown angles, and so can find these angles. This being done, the third side, c , may be found by means of the last relation above. The accuracy of the results may then be tested by the sine proportion.

Observe that $\frac{1}{2}(A - B)$ and $c/2$ are always acute, and that therefore the law of species need not be used in finding the values of these angles. On the other hand, $\frac{1}{2}(A + B)$ may be obtuse. In fact, this angle is acute or obtuse according as $\frac{1}{2}(a + b)$ is acute or obtuse, page 79.

Ex. 1:

$$\begin{array}{ll} c = 40^\circ 20' & \frac{1}{2}(a + c) = 70^\circ 25' \\ a = 100^\circ 30' & \frac{1}{2}(a - c) = 30^\circ 5' \\ B = 46^\circ 40' & B/2 = 23^\circ 20' \end{array}$$

Formulæ:

$$\tan \frac{1}{2}(A + C) = \frac{\cos \frac{1}{2}(a - c) \cot \frac{B}{2}}{\cos \frac{1}{2}(a + c)},$$

$$\tan \frac{1}{2}(A - C) = \frac{\sin \frac{1}{2}(a - c) \cot \frac{B}{2}}{\sin \frac{1}{2}(a + c)},$$

$$\tan \frac{b}{2} = \frac{\sin \frac{1}{2}(A + C) \tan \frac{1}{2}(a - c)}{\sin \frac{1}{2}(A - C)}.$$

$$\log \cos \frac{1}{2}(a - c) = 9.93717 \quad \log \sin \frac{1}{2}(a - c) = 9.70006$$

$$\log \cot B/2 = 0.36516 \quad 0.36516$$

$$0.30233 \quad 0.06522$$

$$\log \cos \frac{1}{2}(a + c) = 9.52527 \quad \log \sin \frac{1}{2}(a + c) = 9.97412$$

$$\log \tan \frac{1}{2}(A + C) = 0.77706 \quad \log \tan \frac{1}{2}(A - C) = 0.09110$$

$$\frac{1}{2}(A + C) = 80^\circ 30' 50'' +. \quad \frac{1}{2}(A - C) = 50^\circ 58' -.$$

Hence,

$$A = 131^\circ 28' 50'',$$

$$C = 29^\circ 32' 50''.$$

$$\log \sin \frac{1}{2}(A + C) = 9.99402$$

$$\log \tan \frac{1}{2}(a - c) = 9.76290$$

$$9.75692$$

$$\log \sin \frac{1}{2}(A - C) = 9.89030$$

$$\log \tan b/2 = 9.86662$$

and hence,
$$\begin{aligned} b/2 &= 36^\circ 20' 15'' - \\ b &= 72^\circ 40' 30'' - . \end{aligned}$$

Check:
$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}.$$

$$\begin{array}{lll} \log \sin A = 9.87459 & \log \sin B = 9.86176 & \log \sin C = 9.69297 \\ \log \sin a = \frac{9.99267}{9.88192} & \log \sin b = \frac{9.97984}{9.88192} & \log \sin c = \frac{9.81106}{9.88191} \end{array}$$

Ex. 2. $b = 109^\circ 12' 10''$, $c = 131^\circ 18' 25''$, $A = 46^\circ 14' 55''$.

94. Case (4). Given a Side and the Two Adjacent Angles,
e. g., c , A , B .

The solution in this case is exactly similar to that of III.

Ex. 1: Formulæ:

$$\begin{aligned} C &= 110^\circ 40' \\ B &= 100^\circ 36' \\ a &= 76^\circ 38' \end{aligned} \quad \tan \frac{1}{2}(c+b) = \frac{\cos \frac{1}{2}(C-B) \tan \frac{a}{2}}{\cos \frac{1}{2}(C+B)},$$

$$\begin{aligned} \frac{1}{2}(C+B) &= 105^\circ 38' \\ \frac{1}{2}(C-B) &= 5^\circ 2' \end{aligned} \quad \tan \frac{1}{2}(c-b) = \frac{\sin \frac{1}{2}(C-B) \tan \frac{a}{2}}{\sin \frac{1}{2}(C+B)},$$

$$\frac{a}{2} = 38^\circ 19' \quad \cot \frac{A}{2} = \frac{\sin \frac{1}{2}(c+b) \tan \frac{1}{2}(C-B)}{\sin \frac{1}{2}(c-b)}.$$

$$\begin{array}{ll} \log \cos \frac{1}{2}(C-B) = 9.99832 & \log \sin \frac{1}{2}(C-B) = 8.94317 \\ \log \tan a/2 = \frac{9.89775}{9.89607} & \frac{9.89775}{8.84092} \end{array}$$

$$\begin{array}{ll} \log \cos \frac{1}{2}(C+B) = \frac{9.43053}{0.46554} & \log \sin \frac{1}{2}(C+B) = \frac{9.98363}{8.85729} \\ \log \tan \frac{1}{2}(c+b) = 0.46554 & \log \tan \frac{1}{2}(c-b) = \frac{8.85729}{8.85729} \end{array}$$

$$\begin{aligned} \frac{1}{2}(c+b) &= 180^\circ - 71^\circ 6' 5'' + \frac{1}{2}(c-b) = 4^\circ 7' 5'' - \\ &= 108^\circ 53' 55'' - \end{aligned}$$

$$c = 113^\circ 1' - , \quad b = 104^\circ 46' 50''.$$

$$\log \sin \frac{1}{2}(c+b) = 9.97593$$

$$\log \tan \frac{1}{2}(C-B) = \frac{8.94485}{8.92078}$$

$$\log \sin \frac{1}{2}(c-b) = 8.85617$$

$$\log \cot A/2 = 0.06461$$

$$\frac{A}{2} = 40^\circ 45' 15'' - , \quad A = 81^\circ 30' 30'' - .$$

Check:
$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}.$$

$$\begin{array}{lll} \log \sin A = 9.99521 & \log \sin B = 9.99252 & \log \sin C = 9.97111 \\ \log \sin a = 9.98807 & \log \sin b = 9.98539 & \log \sin c = 9.96397 \\ \hline & 0.00714 & 0.00713 & 0.00714 \end{array}$$

Ex. 2. $A = 59^\circ 19' 15''$, $B = 76^\circ 14' 15''$, $c = 130^\circ 14' 50''$.

95. Case (5). Given two Sides and the Angle Opposite One of Them, e. g., a , b , A .

From III, page 75,

$$\sin B = \frac{\sin b \sin A}{\sin a}.$$

We may then compute the value of $\log \sin B$. But, since $\sin B = \sin (180^\circ - B)$, we must apply the law of species to tell whether B is acute or obtuse. Denote the acute angle, found in the tables from the value of $\log \sin B$, by B , and the corresponding obtuse angle ($= 180^\circ - B$) by B' . If B is to be a possible solution, the law of species, page 79, states that $\frac{1}{2}(a + b)$ and $\frac{1}{2}(A + B)$ must be of the same species. Similarly, if B' is a possible solution, $\frac{1}{2}(a + b)$ and $\frac{1}{2}(A + B')$ must be of the same species. Hence, this law will show whether there be one solution, two solutions, or no solution.

Having found B , we may compute the remaining parts, c and C , by Napier's analogies (b) and (d), page 78. We may then check by the sine proportion.

Ex. 1. $a = 30^\circ 20'$, $b = 46^\circ 30'$, $A = 36^\circ 40'$.

$$\begin{aligned} \text{Formulae:} \quad \sin B &= \frac{\sin b \sin A}{\sin a}, \\ \cot \frac{C}{2} &= \frac{\sin \frac{1}{2}(b + a) \tan \frac{1}{2}(B - A)}{\sin \frac{1}{2}(b - a)}, \\ \tan \frac{c}{2} &= \frac{\sin \frac{1}{2}(B + A) \tan \frac{1}{2}(b - a)}{\sin \frac{1}{2}(B - A)}. \end{aligned}$$

$$\log \sin b = 9.86056$$

$$\log \sin A = 9.77609$$

$$\hline 9.63665$$

$$\log \sin a = 9.70332$$

$$\log \sin B = 9.93333$$

$$B = 59^\circ 3' 30'',$$

$$\begin{aligned} B' &= 180^\circ - 59^\circ 3' 30'' \\ &= 120^\circ 56' 30''. \end{aligned}$$

$$\frac{1}{2}(b + a) = 38^\circ 25' \qquad \frac{1}{2}(B - A) = 11^\circ 11' 45''$$

$$\frac{1}{2}(b - a) = 8^\circ 5', \qquad \frac{1}{2}(B' + A) = 78^\circ 48' 15''$$

$$\frac{1}{2}(B + A) = 47^\circ 51' 45'', \qquad \frac{1}{2}(B' - A) = 42^\circ 8' 15''$$

There are two solutions since both $\frac{1}{2}(B + A)$ and $\frac{1}{2}(B' + A)$ are of the same species as $\frac{1}{2}(b + a)$.

First Solution.

$$\log \sin \frac{1}{2}(b + a) = 9.79335$$

$$\log \tan \frac{1}{2}(B - A) = 9.29651$$

$$\hline 9.08986$$

$$\log \sin \frac{1}{2}(b - a) = 9.14803$$

$$\log \cot C/2 = 9.94183$$

Second Solution.

$$9.79335$$

$$9.95653$$

$$\hline 9.74988$$

$$9.14803$$

$$\hline 0.60185$$

$$\frac{C}{2} = 48^\circ 49' 30'' \qquad \frac{C'}{2} = 14^\circ 2' 35'' -$$

$$C = 97^\circ 39' \qquad C' = 28^\circ 5' 10'' -$$

$$\log \sin \frac{1}{2}(B + A) = 9.87013 \qquad 9.99166$$

$$\log \tan \frac{1}{2}(b - a) = 9.15236 \qquad 9.15236$$

$$\hline 9.02249 \qquad \hline 9.14402$$

$$\log \sin \frac{1}{2}(B - A) = 9.28817 \qquad 9.82666$$

$$\log \tan c/2 = 9.73432 \qquad 9.31736$$

$$\frac{c}{2} = 28^\circ 28' 30'' \qquad \frac{c'}{2} = 11^\circ 43' 55'' -$$

$$c = 56^\circ 57' \qquad c' = 23^\circ 27' 50'' -.$$

Hence the solutions are

$$(1) B = 59^\circ 3' 30''$$

$$C = 97^\circ 39'$$

$$c = 56^\circ 57' +$$

$$(2) B' = 120^\circ 56' 30''$$

$$C' = 28^\circ 5' 10'' -$$

$$c' = 23^\circ 27' 50'' -$$

Check:*

$$\frac{\sin B}{\sin b} = \frac{\sin C}{\sin c} = \frac{\sin C'}{\sin c'}.$$

$$\log \sin B = 9.93333 \quad \log \sin C = 9.99612 \quad \log \sin C' = 9.67284$$

$$\log \sin b = 9.86056 \quad \log \sin c = 9.92335 \quad \log \sin c' = 9.60007$$

$$\hline 0.07277$$

$$\hline 0.07277$$

$$\hline 0.07277$$

$$\text{Ex. 2. } b = 32^\circ 18' 10'', c = 50^\circ 14' 15'', C = 48^\circ 12' 10''.$$

96. Case (6). Given Two Angles and the Side Opposite One of Them, e. g., A, B, a . The solution of this case is exactly similar to that of Case V.

* It is not necessary to use the ratio $\sin A/\sin a$ since this was used in the solution.

Ex. 1. $B = 69^\circ$, $C = 132^\circ$, $b = 65^\circ$.

Formulae: $\sin c = \frac{\sin C \sin b}{\sin B}$,
 $\cot \frac{A}{2} = \frac{\sin \frac{1}{2}(c+b) \tan \frac{1}{2}(C-B)}{\sin \frac{1}{2}(c-b)}$,
 $\tan \frac{a}{2} = \frac{\sin \frac{1}{2}(C+B) \tan \frac{1}{2}(c-b)}{\sin \frac{1}{2}(C-B)}$.

$$\log \sin C = 9.87107$$

$$\log \sin b = 9.95728$$

$$\hline 9.82835$$

$$\log \sin B = 9.97015$$

$$\log \sin c = 9.85820$$

Then,

$$c = 46^\circ 10' 25'' \quad c' = 180^\circ - 46^\circ 10' 25''$$

$$= 133^\circ 49' 35''$$

and

$$\frac{1}{2}(C+B) = 100^\circ 30' \quad \frac{1}{2}(c'+b) = 99^\circ 24' 50'' -$$

$$\frac{1}{2}(C-B) = 31^\circ 30' \quad \frac{1}{2}(c'-b) = 34^\circ 24' 50'' -$$

$$\frac{1}{2}(c+b) = 55^\circ 35' 15'' -$$

Of the two values, c and c' , the latter alone is a solution since it only obeys the law of species. Then, as there can be but one solution, we may omit accents and take $c = 133^\circ 49' 35''$.

Then,

$$\frac{1}{2}(C+B) = 100^\circ 30' \quad \frac{1}{2}(c+b) = 99^\circ 24' 50'' -$$

$$\frac{1}{2}(C-B) = 31^\circ 30' \quad \frac{1}{2}(c-b) = 34^\circ 24' 50'' -$$

$$\log \sin \frac{1}{2}(c+b) = 9.99411 \quad \log \sin \frac{1}{2}(C+B) = 9.99267$$

$$\log \tan \frac{1}{2}(C-B) = 9.78732 \quad \log \tan \frac{1}{2}(c-b) = 9.83573$$

$$\hline 9.78143 \quad \hline 9.82840$$

$$\log \sin \frac{1}{2}(c-b) = 9.75218 \quad \log \sin \frac{1}{2}(C-B) = 9.71809$$

$$\log \cot A/2 = 0.02925 \quad \log \tan a/2 = 0.11031$$

$$\frac{A}{2} = 43^\circ 4' 20'', \quad \frac{a}{2} = 52^\circ 12' -,$$

$$A = 86^\circ 8' 40'', \quad a = 104^\circ 24' -.$$

Check:

$$\frac{\sin A}{\sin a} = \frac{\sin C}{\sin c}.$$

$$\log \sin A = 9.99902 \quad \log \sin C = 9.87107$$

$$\log \sin a = 9.98614 \quad \log \sin c = 9.85820$$

$$\hline 0.01288 \quad \hline 0.01287$$

Ex. 2. $A = 70^\circ 14' 15''$, $B = 59^\circ 12' 25''$, $b = 51^\circ 18' 35''$.

97. **Exercise XVI.** Solve the following spherical triangles.

- | | |
|------------------------------|------------------------------|
| 1. $a = 48^\circ 35'$ | 2. $A = 105^\circ 24' 10''$ |
| $b = 77^\circ 23'$ | $b = 110^\circ 5' 15''$ |
| $c = 80^\circ 56'$ | $c = 80^\circ 37' 35''$ |
| 3. $A = 120^\circ 56' 25''$ | 4. $a = 29^\circ 15' 25''$ |
| $B = 80^\circ 32' 10''$ | $c = 50^\circ 29' 55''$ |
| $C = 100^\circ 27' 50''$ | $B = 47^\circ 48' 50''$ |
| 5. $a = 103^\circ 22' 50''$ | 6. $a = 58^\circ 32' 50''$ |
| $B = 76^\circ 13' 25''$ | $b = 42^\circ 23' 10''$ |
| $C = 37^\circ 58' 55''$ | $A = 60^\circ 36' 35''$ |
| 7. $a = 102^\circ 14' 10''$ | 8. $A = 70^\circ 25' 10''$ |
| $b = 74^\circ 14' 35''$ | $B = 50^\circ 46' 25''$ |
| $c = 118^\circ 29' 25''$ | $C = 80^\circ 39' 55''$ |
| 9. $A = 105^\circ 35' 35''$ | 10. $A = 123^\circ 34' 50''$ |
| $B = 120^\circ 23' 10''$ | $B = 78^\circ 22' 35''$ |
| $c = 74^\circ 24' 25''$ | $b = 98^\circ 29' 25''$ |
| 11. $c = 73^\circ 14' 10''$ | 12. $A = 58^\circ 58' 55''$ |
| $b = 60^\circ 12' 10''$ | $B = 36^\circ 49' 45''$ |
| $B = 40^\circ 28' 35''$ | $C = 96^\circ 37' 35''$ |
| 13. $A = 167^\circ 43' 35''$ | 14. $a = 76^\circ 56' 55''$ |
| $B = 103^\circ 25' 25''$ | $c = 48^\circ 48' 50''$ |
| $c = 70^\circ 45' 50''$ | $B = 39^\circ 32' 35''$ |
| 15. $A = 80^\circ 54' 55''$ | 16. $a = 47^\circ 54' 55''$ |
| $B = 90^\circ$ | $b = 36^\circ 25' 50''$ |
| $c = 47^\circ 16' 10''$ | $c = 80^\circ 19' 10''$ |
| 17. $b = 54^\circ 51' 35''$ | 18. $B = 54^\circ 51' 25''$ |
| $a = 79^\circ 22' 25''$ | $A = 79^\circ 22' 25''$ |
| $C = 28^\circ 29' 55''$ | $c = 28^\circ 29' 55''$ |

98. **Geographical Problems.**

The position of a point on the earth's surface is determined if the latitude and longitude of the point be given. By the aid of spherical trigonometry we may find the distance between two points whose latitudes and longitudes are known, and also the bearing of either point from the other. Since the shortest distance between two points is along the arc of a great circle, a navigator sails as nearly as possible upon the great circle arc from the point of departure to the desired destination. The

course, when sailing upon a great circle arc, is continually changing except in the special cases of sailing along the equator or a meridian. If, on the other hand, a ship kept upon the same course, for example N. 25° E., it would sail in a spiral slowly approaching one of the poles. Such a path upon the earth's surface is known as a loxodrome or rhumb line.

99. In the following table the latitudes and longitudes of several cities are given:

Baltimore	$39^{\circ} 17' \text{ N.}, 76^{\circ} 37' \text{ W.}$
Boston	$42^{\circ} 21' \text{ N.}, 71^{\circ} 4' \text{ W.}$
Cape Town	$33^{\circ} 56' \text{ S.}, 18^{\circ} 26' \text{ E.}$
Halifax	$44^{\circ} 40' \text{ N.}, 63^{\circ} 35' \text{ W.}$
Honolulu	$21^{\circ} 18' \text{ N.}, 157^{\circ} 55' \text{ W.}$
Liverpool	$53^{\circ} 24' \text{ N.}, 3^{\circ} 4' \text{ W.}$
New York	$40^{\circ} 43' \text{ N.}, 74^{\circ} \text{ W.}$
San Francisco	$37^{\circ} 48' \text{ N.}, 122^{\circ} 24' \text{ W.}$

100. **Exercise XVII.** In the following problems the earth is assumed to be spherical, and the radius is taken as 3960 miles.

1. Find the distance from Halifax to Liverpool, and the bearing of each place from the other. Also find the course of a ship sailing from Halifax to Liverpool when it crosses the meridian 55° W., the latitude of this point, and the distance the ship has sailed.

Let G , L and H denote respectively Greenwich, Liverpool, and Halifax. Then

$$\begin{aligned} GNL &= 3^{\circ} 4', & LN &= 90^{\circ} - 53^{\circ} 24' = 36^{\circ} 36', \\ GNH &= 63^{\circ} 35', & HN &= 90^{\circ} - 44^{\circ} 40' = 45^{\circ} 20'. \\ && \therefore LNH &= 60^{\circ} 31'. \end{aligned}$$

Then we are given two sides and the included angle of the triangle HNH . We can then solve for the angles NHL and

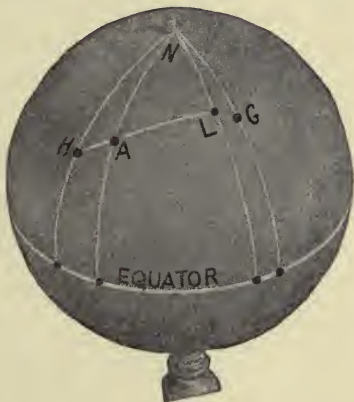


FIG. 58.

HLN and for the side HL by the method of Art. 93. This gives

$$\begin{aligned} NHL &= 54^\circ 54' 20'', \\ HLN &= 77^\circ 25' 25'', \\ HL &= 39^\circ 22' 5'' = 39.368^\circ. \end{aligned}$$

Then the distance from Halifax to Liverpool is

$$d = \frac{39.368 \times \pi \times 3960}{180} = 2721 \text{ miles.} \quad [\text{Art. 66}]$$

The bearing of Halifax from Liverpool is N. $77^\circ 25' 25''$, W., and that of Liverpool from Halifax is N. $54^\circ 54' 20''$ E.

To solve the last part of the problem consider the triangle HNA , where AN is the meridian of 55° W.

In this triangle

$$\begin{aligned} HN &= 45^\circ 20', \\ NHA &= 54^\circ 54' 20'', \\ HNA &= 63^\circ 35' - 55^\circ = 8^\circ 35'. \end{aligned}$$

We have then a side and the two adjacent angles, and can therefore solve for the remaining parts by the method of Art. 94. Hence.

$$\begin{aligned} AN &= 41^\circ 38' 20'', \\ HAN &= 118^\circ 51' 25'', \\ HA &= 6^\circ 57' 40'' = 6.961^\circ. \end{aligned}$$

Therefore the latitude of the point is $48^\circ 21' 40''$, the course is N. $180^\circ - 118^\circ 51' 25''$ E., or N. $61^\circ 8' 35''$ E., and the distance sailed is given by

$$d = \frac{6.961 \times \pi \times 3960}{180} = 481 + \text{miles.}$$

2. A ship sails from Halifax to Liverpool along the arc of a great circle. Find her latitude and longitude after she has sailed one thousand miles. Find also her course at this point.

3. Find the distance from New York to San Francisco.

4. A ship sails from San Francisco to Honolulu. Find the course she is steering when she has covered half the distance.

5. A ship sails from Boston to Cape Town. Find the total distance sailed, and her course when 1500 miles from Cape Town.

6. After sailing 1000 miles from Halifax a ship crosses the parallel of latitude of 40° . What will be the position of the ship (that is, the latitude and longitude) after she has sailed 800 miles further on the same great circle.

7. Find the area in square miles of the triangle whose vertices are Baltimore, Cape Town, and Liverpool.

CHAPTER IX.

OTHER FORMULÆ RELATING TO SPHERICAL TRIANGLES.

101. From IV, page 77, we have

$$\tan \frac{A}{2} = \frac{K}{\sin(s-a)}. \quad (1)$$

Since this relation is true for any spherical triangle it is true for the polar triangle of a given triangle. We may then replace A' by $180^\circ - a$, a' by $180^\circ - A$, etc.

Let $2S = A + B + C$, so that $s' = \frac{1}{2}(540^\circ - [A + B + C]) = 270^\circ - S$.

Then $\frac{1}{2}(-A + B + C) = S - A$, etc.

Hence

$$s' - a' = 90^\circ - (S - A),$$

Similarly

$$s' - b' = 90^\circ - (S - B),$$

and

$$s' - c' = 90^\circ - (S - C).$$

Therefore

$$\begin{aligned} K' &= \sqrt{\frac{\sin(s' - a') \sin(s' - b') \sin(s' - c')}{\sin s'}} \\ &= \sqrt{\frac{\cos(S - A) \cos(S - B) \cos(S - C)}{-\cos S}} \end{aligned}$$

Now let

$$k = \sqrt{\frac{-\cos S}{\cos(S - A) \cos(S - B) \cos(S - C)}}$$

so that

$$K' = \frac{1}{k}.$$

The equation (1) then becomes

$$\cot \frac{a}{2} = \frac{1}{k \cos(S - A)},$$

or $\tan \frac{a}{2} = k \cos (S - A).$

Similarly $\tan \frac{b}{2} = k \cos (S - B),$ VI

and $\tan \frac{c}{2} = k \cos (S - C).$

By means of these equations a triangle in which the three angles are given may be solved directly.

102. Radius of the Circumscribed Circle. Let A_1 , B_1 , and C_1 be the middle points of the sides of the spherical triangle ABC . At A_1 draw the arc of a great circle perpendicular to the side BC , and similarly draw arcs through B_1 and C_1 perpendicular to the corresponding sides of ABC .

These three arcs will meet in a point O . [This may be

proved in a manner entirely analogous to that employed for the corresponding theorem in plane geometry.]

Draw the great circle arcs OA , OB and OC . The right triangles OA_1B and OA_1C are symmetrical since $BA_1 = A_1C$ and OA_1 is common to the two triangles. Hence the angles A_1BO and A_1CO

are equal. Similarly $B_1AO = B_1CO$, and $C_1AO = C_1BO$.

Now

$$\begin{aligned} 2S &= A + B + C, \\ &= (OAB_1 + OAC_1) + (OBA_1 + OBC_1) + (OCA_1 + OCB_1), \\ &= (OAB_1 + OCB_1) + (OAC_1 + OBC_1) + (OBA_1 + OCA_1), \\ &= 2OAB_1 + 2OAC_1 + 2OBA_1, \\ &= 2A + 2OBA_1. \end{aligned}$$

Therefore

$$OBA_1 = S - A.$$

Then in the right triangle OBA_1 we have $OBA_1 = S - A$, $BA_1 = a/2$ and $OB = r$, where r is the radius of the circumscribed circle. Applying Napier's rule to this triangle, we have

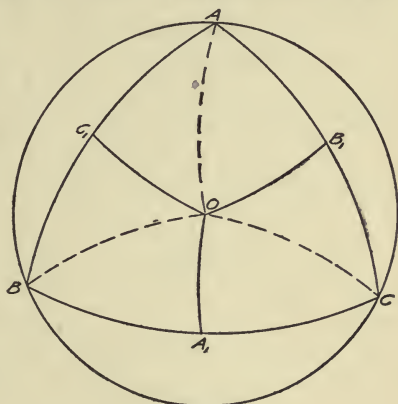


FIG. 59.

$$\cos OBA_1 = \cot OB \tan A_1B,$$

or,
$$\cos (S - A) = \cot r \tan \frac{a}{2}.$$

Substituting for $\tan a/2$ its value from VI, Art. 101, we have

$$\cos (S - A) = \cot r \cdot k \cos (S - A).$$

Hence it follows that

$$\tan r = k = \sqrt{\frac{-\cos S}{\cos (S - A) \cos (S - B) \cos (S - C)}}. \quad \text{VII}$$

By this formula we may find the radius r , that is the polar distance, of the small circle circumscribed to the spherical triangle.

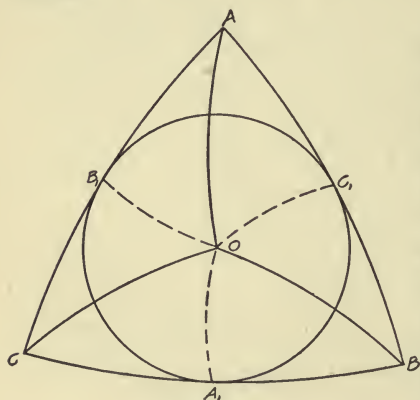


FIG. 60.

103. Radius of the Inscribed Circle. Draw arcs of great circles bisecting the angles of the spherical triangle. These arcs meet in a point O . Through O draw the great circle arc OA_1 perpendicular to the side BC , and similarly OB_1 and OC_1 perpendicular to CA and AB respectively. The right triangles OAC_1 and OAB_1 are equal since the angles OAC_1 and OAB_1 are equal and

the hypotenuse OA is common. Then $AB_1 = AC_1$. Similarly $BC_1 = BA_1$ and $CA_1 = CB_1$.

Now,

$$\begin{aligned} 2s &= a + b + c, \\ &= (BA_1 + A_1C) + (CB_1 + B_1A) + (AC_1 + C_1B), \\ &= (BA_1 + BC_1) + (A_1C + CB_1) + (B_1A + C_1A), \\ &= 2BA_1 + 2A_1C + 2B_1A, \\ &= 2a + 2B_1A. \end{aligned}$$

Then

$$B_1A = s - a.$$

We then have in the right triangle OB_1A , $B_1A = s - a$, $OAB_1 = A/2$ and $OB_1 = R$, where R is the radius of the inscribed

circle. Applying Napier's rule to this triangle we have

$$\sin (s-a) = \tan R \cot \frac{A}{2},$$

or

$$\tan R = \sin (s-a) \tan \frac{A}{2}, \quad (1)$$

$$= \sin (s-a) \frac{K}{\sin (s-a)}.$$

Therefore,

$$\tan R = K = \sqrt{\frac{\sin (s-a) \sin (s-b) \sin (s-c)}{\sin s}}. \quad \text{VIII}$$

104. Escribed Circles. Let us produce the sides AB and AC of the spherical triangle ABC to meet again in A_1 . The circle inscribed in the triangle A_1BC is said to be an escribed circle of ABC . Denote the radius of this circle by R_a . Obviously there are two other escribed circles of ABC . The radii of these circles will be denoted by R_b and R_c .

Applying equation (1) of the last article to the triangle A_1BC , we have

$$\tan R_a = \tan \frac{A}{2} \sin [\tfrac{1}{2}(a + 180^\circ - b + 180^\circ - c) - a],$$

$$= \frac{K}{\sin (s-a)} \sin [180^\circ - \tfrac{1}{2}(a + b + c)],$$

$$= \frac{K \sin s}{\sin (s-a)}.$$

Hence,

$$\tan R_a = \frac{K \sin s}{\sin (s-a)},$$

$$\tan R_b = \frac{K \sin s}{\sin (s-b)}, \quad \text{IX.}$$

$$\tan R_c = \frac{K \sin s}{\sin (s-c)}.$$

TABLE I.

LOGARITHMS
OF
NUMBERS FROM
1 TO 10,000.

N.	L.	0	1	2	3	4	5	6	7	8	9	P.P.					
100		00	000	043	087	130	173	217	260	303	346	389					
101			432	475	518	561	604	647	689	732	775	817		44	43	42	
102	00		860	903	945	988	*030	*072	*115	*157	*199	*242		1	4.4	4.3	4.2
103	01		284	326	368	410	452	494	536	578	620	662		2	8.8	8.6	8.4
104	02		703	745	787	828	870	912	953	995	*036	*078		3	13.2	12.9	12.6
105	03		119	160	202	243	284	325	366	407	449	490		4	17.6	17.2	16.8
106	04		531	572	612	653	694	735	776	816	857	898		5	22.0	21.5	21.0
107	05		938	979	*019	*060	*100	*141	*181	*222	*262	*302		6	26.4	25.8	25.2
108	06		342	383	423	463	503	543	583	623	663	703		7	30.8	30.1	29.4
109	07		743	782	822	862	902	941	981	*021	*060	*100		8	35.2	34.4	33.6
110	08		139	179	218	258	297	336	376	415	454	493		9	39.6	38.7	37.8
111			532	571	610	650	689	727	766	805	844	883					
112	04		922	961	999	*038	*077	*115	*154	*192	*231	*269		41	40	39	
113	05		308	346	385	423	461	500	538	576	614	652		1	4.1	4.0	3.9
114	06		690	729	767	805	843	881	918	956	994	*032		2	8.2	8.0	7.8
115	07		070	108	145	183	221	258	296	333	371	408		3	12.3	12.0	11.7
116	08		446	483	521	558	595	633	670	707	744	781		4	16.4	16.0	15.6
117	09		819	856	893	930	967	*004	*041	*078	*115	*151		5	20.5	20.0	19.5
118	10		188	225	262	298	335	372	408	445	482	518		6	24.6	24.0	23.4
119	11		555	591	628	664	700	737	773	809	846	882		7	28.7	28.0	27.3
120	12		918	954	990	*027	*063	*099	*135	*171	*207	243		8	32.8	32.0	31.2
121			279	314	350	386	422	458	493	529	565	600		9	36.9	36.0	35.1
122	08		636	672	707	743	778	814	849	884	920	955					
123	09		991	*026	*061	*096	*132	*167	*202	*237	*272	*307		38	37	36	
124	10		342	377	412	447	482	517	552	587	621	656		1	3.8	3.7	3.6
125	11		691	726	760	795	830	864	899	934	968	*003		2	7.6	7.4	7.2
126	12		037	072	106	140	175	209	243	278	312	346		3	11.4	11.1	10.8
127	13		380	415	449	483	517	551	585	619	653	687		4	15.2	14.8	14.4
128	14		721	755	789	823	857	890	924	958	992	*025		5	19.0	18.5	18.0
129	15		059	093	126	160	193	227	261	294	327	361		6	22.8	22.2	21.6
130			394	428	461	494	528	561	594	628	661	694		7	26.6	25.9	25.2
131	11		727	760	793	826	860	893	926	959	992	*024		8	30.4	29.6	28.8
132	12		057	090	123	156	189	222	254	287	320	352		9	34.2	33.3	32.4
133	13		385	418	450	483	516	548	581	613	646	678					
134	14		710	743	775	808	840	872	905	937	969	*001		35	34	33	
135	15		033	066	098	130	162	194	226	258	290	322		1	3.5	3.4	3.3
136	16		354	386	418	450	481	513	545	577	609	640		2	7.0	6.8	6.6
137	17		672	704	735	767	799	830	862	893	925	956		3	10.5	10.2	9.9
138	18		988	*019	*051	*082	*114	*145	*176	*208	*239	*270		4	14.0	13.6	13.2
139	19		301	333	364	395	426	457	489	520	551	582		5	17.5	17.0	16.5
140			613	644	675	706	737	768	799	829	860	891		6	21.0	20.4	19.8
141	14		922	953	983	*014	*045	*076	*106	*137	*168	*198		7	24.5	23.8	23.1
142	15		229	259	290	320	351	381	412	442	473	503		8	28.0	27.2	26.4
143	16		534	564	594	625	655	685	715	746	776	806		9	31.5	30.6	29.7
144	17		836	866	897	927	957	987	*017	*047	*077	*107					
145	18		137	167	197	227	256	286	316	346	376	406		32	31	30	
146	19		435	465	495	524	554	584	613	643	673	702		1	3.2	3.1	3.0
147			732	761	791	820	850	879	909	938	967	997		2	6.4	6.2	6.0
148	17		026	056	085	114	143	173	202	231	260	289		3	9.6	9.3	9.0
149			319	348	377	406	435	464	493	522	551	580		4	12.8	12.4	12.0
150			609	638	667	696	725	754	782	811	840	869		5	16.0	15.5	15.0
														6	19.2	18.6	18.0
														7	22.4	21.7	21.0
														8	25.6	24.8	24.0
														9	28.8	27.9	27.0
N.	L.	0	1	2	3	4	5	6	7	8	9	P.P.					

N.	L.	0	1	2	3	4	5	6	7	8	9	P.P.
150	17	609	638	667	696	725	754	782	811	840	869	
151	17	898	926	955	984	*013	*041	*070	*099	*127	*156	29 28
152	18	184	213	241	270	298	327	355	384	412	441	1 2.9 2.8
153	18	469	498	526	554	583	611	639	667	696	724	2 5.8 5.6
154	19	752	780	808	837	865	893	921	949	977	*005	3 8.7 8.4
155	19	033	061	089	117	145	173	201	229	257	285	4 11.6 11.2
156		312	340	368	396	424	451	479	507	535	562	5 14.5 14.0
157		590	618	645	673	700	728	756	783	811	838	6 17.4 16.8
158	19	866	893	921	948	976	*003	*030	*058	*085	*112	7 20.3 19.6
159	20	140	167	194	222	249	276	303	330	358	385	8 23.2 22.4
160		412	439	466	493	520	548	575	602	629	656	9 26.1 25.2
161	20	683	710	737	763	790	817	844	871	898	925	
162	21	952	978	*005	*032	*059	*085	*112	*139	*165	*192	27 26
163	21	219	245	272	299	325	352	378	405	431	458	1 2.7 2.6
164		484	511	537	564	590	617	643	669	696	722	2 5.4 5.2
165		748	775	801	827	854	880	906	932	958	985	3 8.1 7.8
166	22	011	037	063	089	115	141	167	194	220	246	4 10.8 10.4
167		272	298	324	350	376	401	427	453	479	505	5 13.5 13.0
168		531	557	583	608	634	660	686	712	737	763	6 16.2 15.6
169	22	789	814	840	866	891	917	943	968	994	*019	7 18.9 18.2
170	23	045	070	096	121	147	172	198	223	249	274	8 21.6 20.8
171		300	325	350	376	401	426	452	477	502	528	9 24.3 23.4
172	23	553	578	603	629	654	679	704	729	754	779	
173	24	805	830	855	880	905	930	955	980	*005	*030	25
174		055	080	105	130	155	180	204	229	254	279	1 2.5
175		304	329	353	378	403	428	452	477	502	527	2 5.0
176		551	576	601	625	650	674	699	724	748	773	3 7.5
177	24	797	822	846	871	895	920	944	969	993	*018	4 10.0
178	25	042	066	091	115	139	164	188	212	237	261	5 12.5
179		285	310	334	358	382	406	431	455	479	503	6 15.0
180		527	551	575	600	624	648	672	696	720	744	7 17.5
181		768	792	816	840	864	888	912	935	959	983	8 20.0
182	26	007	031	055	079	102	126	150	174	198	221	9 22.5
183		245	269	293	316	340	364	387	411	435	458	
184		482	505	529	553	576	600	623	647	670	694	24 23
185	26	717	741	764	788	811	834	858	881	905	928	1 2.4 2.3
186	27	951	975	998	*021	*045	*068	*091	*114	*138	*161	2 4.8 4.6
187		184	207	231	254	277	300	323	346	370	393	3 7.2 6.9
188		416	439	462	485	508	531	554	577	600	623	4 9.6 9.2
189		646	669	692	715	738	761	784	807	830	852	5 12.0 11.5
190	27	875	898	921	944	967	989	*012	*035	*058	*081	6 14.4 13.8
191	28	103	126	149	171	194	217	240	262	285	307	7 16.8 16.1
192		330	353	375	398	421	443	466	488	511	533	8 19.2 18.4
193		556	578	601	623	646	668	691	713	735	758	9 21.6 20.7
194		780	803	825	847	870	892	914	937	959	981	
195	29	003	026	048	070	092	115	137	159	181	203	22 21
196		226	248	270	292	314	336	358	380	403	425	1 2.2 2.1
197		447	469	491	513	535	557	579	601	623	645	2 4.4 4.2
198		667	688	710	732	754	776	798	820	842	863	3 6.6 6.3
199	29	885	907	929	951	973	994	*016	*038	*060	*081	4 8.8 8.4
200	30	103	125	146	168	190	211	233	255	276	298	5 11.0 10.5
												6 13.2 12.6
												7 15.4 14.7
												8 17.6 16.8
												9 19.8 18.9
N.	L.	0	1	2	3	4	5	6	7	8	9	P.P.

N.	L.	0	1	2	3	4	5	6	7	8	9	P.P.
200	30	103	125	146	168	190	211	233	255	276	298	
201		320	341	363	384	406	428	449	471	492	514	
202		535	557	578	600	621	643	664	685	707	728	
203		750	771	792	814	835	856	878	899	920	942	
204	30	31	963	984	*006	*027	*048	*069	*091	*112	*133	*154
205			175	197	218	239	260	281	302	323	345	366
206			387	408	429	450	471	492	513	534	555	576
207			597	618	639	660	681	702	723	744	765	785
208			806	827	848	869	890	911	931	952	973	994
209	32	015	035	056	077	098	118	139	160	181	201	
210		222	243	263	284	305	325	346	366	387	408	
211		428	449	469	490	510	531	552	572	593	613	
212	32	634	654	675	695	715	736	756	777	797	818	
213	33	838	858	879	899	919	940	960	980	*001	*021	
214		041	062	082	102	122	143	163	183	203	224	
215		244	264	284	304	325	345	365	385	405	425	
216		445	465	486	506	526	546	566	586	606	626	
217		646	666	686	706	726	746	766	786	806	826	
218	33	846	866	885	905	925	945	965	985	*005	*025	
219	34	044	064	084	104	124	143	163	183	203	223	
220		242	262	282	301	321	341	361	380	400	420	
221		439	459	479	498	518	537	557	577	596	616	
222		635	655	674	694	713	733	753	772	792	811	
223	34	830	850	869	889	908	928	947	967	986	*005	
224	35	025	044	064	083	102	122	141	160	180	199	
225		218	238	257	276	295	315	334	353	372	392	
226		411	430	449	468	488	507	526	545	564	583	
227		603	622	641	660	679	698	717	736	755	774	
228		793	813	832	851	870	889	908	927	946	965	
229	35	984	*003	*021	*040	*059	*078	*097	*116	*135	*154	
230	36	173	192	211	229	248	267	286	305	324	342	
231		361	380	399	418	436	455	474	493	511	530	
232		549	568	586	605	624	642	661	680	698	717	
233		736	754	773	791	810	829	847	866	884	903	
234	36	922	940	959	977	996	*014	*033	*051	*070	*088	
235	37	107	125	144	162	181	199	218	236	254	273	
236		291	310	328	346	365	383	401	420	438	457	
237		475	493	511	530	548	566	585	603	621	639	
238		658	676	694	712	731	749	767	785	803	822	
239	37	840	858	876	894	912	931	949	967	985	*003	
240	38	021	039	057	075	093	112	130	148	166	184	
241		202	220	238	256	274	292	310	328	346	364	
242		382	399	417	435	453	471	489	507	525	543	
243		561	578	596	614	632	650	668	686	703	721	
244		739	757	775	792	810	828	846	863	881	899	
245	38	917	934	952	970	987	*005	*023	*041	*058	*076	
246	39	094	111	129	146	164	182	199	217	235	252	
247		270	287	305	322	340	358	375	393	410	428	
248		445	463	480	498	515	533	550	568	585	602	
249		620	637	655	672	690	707	724	742	759	777	
250		794	811	829	846	863	881	898	915	933	950	
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252	40	140	157	175	192	209	226	243	261	278	295	1 1.8
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256		824	841	858	875	892	909	926	943	960	976	5 9.0
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260		497	514	531	547	564	581	597	614	631	647	9 16.2
261		664	681	697	714	731	747	764	780	797	814	
262	41	830	847	863	880	896	913	929	946	963	979	17
263	42	996	*012	*029	*045	*062	*078	*095	*111	*127	*144	1 1.7
264		160	177	193	210	226	243	259	275	292	308	2 3.4
265		325	341	357	374	390	406	423	439	455	472	3 5.1
266		488	504	521	537	553	570	586	602	619	635	4 6.8
267		651	667	684	700	716	732	749	765	781	797	5 8.5
268		813	830	846	862	878	894	911	927	943	959	6 10.2
269	42	975	991	*008	*024	*040	*056	*072	*088	*104	*120	7 11.9
270	43	136	152	169	185	201	217	233	249	265	281	8 13.6
271		297	313	329	345	361	377	393	409	425	441	9 15.3
272		457	473	489	505	521	537	553	569	584	600	
273		616	632	648	664	680	696	712	727	743	759	16
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290		240	255	270	285	300	315	330	345	359	374	6 9.0
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295	47	982	997	*012	*026	*041	*056	*070	*085	*100	*114	14
296		129	144	159	173	188	202	217	232	246	261	1 1.4
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300		712	727	741	756	770	784	799	813	828	842	5 7.0
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302	48	001	015	029	044	058	073	087	101	116	130	
303		144	159	173	187	202	216	230	244	259	273	15
304		287	302	316	330	344	359	373	387	401	416	1 1.5
305		430	444	458	473	487	501	515	530	544	558	2 3.0
306		572	586	601	615	629	643	657	671	686	700	3 4.5
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308		855	869	883	897	911	926	940	954	968	982	5 7.5
309	48 } 49 }	996	*010	*024	*038	*052	*066	*080	*094	*108	*122	6 9.0
310		136	150	164	178	192	206	220	234	248	262	7 10.5
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313		554	568	582	596	610	624	638	651	665	679	
314		693	707	721	734	748	762	776	790	803	817	
315		831	845	859	872	886	900	914	927	941	955	14
316	49 } 50 }	969	982	996	*010	*024	*037	*051	*065	*079	*092	1 1.4
317		106	120	133	147	161	174	188	202	215	229	2 2.8
318		243	256	270	284	297	311	325	338	352	365	3 4.2
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320		515	529	542	556	569	583	596	610	623	637	5 7.0
321		651	664	678	691	705	718	732	745	759	772	6 8.4
322		786	799	813	826	840	853	866	880	893	907	7 9.8
323	50 } 51 }	920	934	947	961	974	987	*001	*014	*028	*041	8 11.2
324		055	068	081	095	108	121	135	148	162	175	9 12.6
325		188	202	215	228	242	255	268	282	295	308	
326		322	335	348	362	375	388	402	415	428	441	
327		455	468	481	495	508	521	534	548	561	574	
328		587	601	614	627	640	654	667	680	693	706	
329		720	733	746	759	772	786	799	812	825	838	13
330		851	865	878	891	904	917	930	943	957	970	1 1.3
331	51 } 52 }	983	996	*009	*022	*035	*048	*061	*075	*088	*101	2 2.6
332		114	127	140	153	166	179	192	205	218	231	3 3.9
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334		375	388	401	414	427	440	453	466	479	492	5 6.5
335		504	517	530	543	556	569	582	595	608	621	6 7.8
336		634	647	660	673	686	699	711	724	737	750	7 9.1
337		763	776	789	802	815	827	840	853	866	879	8 10.4
338	52 } 53 }	892	905	917	930	943	956	969	982	994	*007	9 11.7
339		020	033	046	058	071	084	097	110	122	135	
340		148	161	173	186	199	212	224	237	250	263	
341		275	288	301	314	326	339	352	364	377	390	12
342		403	415	428	441	453	466	479	491	504	517	1 1.2
343		529	542	555	567	580	593	605	618	631	643	2 2.4
344		656	668	681	694	706	719	732	744	757	769	3 3.6
345		782	794	807	820	832	845	857	870	882	895	4 4.8
346	53 } 54 }	908	920	933	945	958	970	983	995	*008	*020	5 6.0
347		033	045	058	070	083	095	108	120	133	145	6 7.2
348		158	170	183	195	208	220	233	245	258	270	7 8.4
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350	54	407	419	432	444	456	469	481	494	506	518	
351		531	543	555	568	580	593	605	617	630	642	
352		654	667	679	691	704	716	728	741	753	765	
353		777	790	802	814	827	839	851	864	876	888	
354	54	55	900	913	925	937	949	962	974	986	*011	13
355			023	035	047	060	072	084	096	108	121	1.3
356			145	157	169	182	194	206	218	230	242	2.6
357			267	279	291	303	315	328	340	352	364	3.9
358			388	400	413	425	437	449	461	473	485	5.2
359			509	522	534	546	558	570	582	594	606	6.5
360			630	642	654	666	678	691	703	715	727	7.8
361			751	763	775	787	799	811	823	835	847	9.1
362			871	883	895	907	919	931	943	955	967	10.4
363	55	56	*003	*015	*027	*038	*050	*062	*074	*086	*098	11.7
364			110	122	134	146	158	170	182	194	205	
365			229	241	253	265	277	289	301	312	324	
366			348	360	372	384	396	407	419	431	443	12
367			467	478	490	502	514	526	538	549	561	1.2
368			585	597	608	620	632	644	656	667	679	2.4
369			703	714	726	738	750	761	773	785	797	3.6
370			820	832	844	855	867	879	891	902	914	4.8
371	56	57	937	949	961	972	984	996	*008	*019	*031	6.0
372			054	066	078	089	101	113	124	136	148	7.2
373			171	183	194	206	217	229	241	252	264	8.4
374			287	299	310	322	334	345	357	368	380	9.6
375			403	415	426	438	449	461	473	484	496	10.8
376			519	530	542	553	565	576	588	600	611	
377			634	646	657	669	680	692	703	715	726	
378			749	761	772	784	795	807	818	830	841	
379			864	875	887	898	910	921	933	944	955	11
380	57	58	*001	*013	*024	*035	*047	*058	*070	*081		1.1
381			092	104	115	127	138	149	161	172	184	2.2
382			206	218	229	240	252	263	274	286	297	3.3
383			320	331	343	354	365	377	388	399	410	4.4
384			433	444	456	467	478	490	501	512	524	5.5
385			546	557	569	580	591	602	614	625	636	6.6
386			659	670	681	692	704	715	726	737	749	7.7
387			771	782	794	805	816	827	838	850	861	8.8
388			883	894	906	917	928	939	950	961	973	9.9
389	58	59	*006	*017	*028	*040	*051	*062	*073	*084	*095	
390			106	118	129	140	151	162	173	184	195	10
391			218	229	240	251	262	273	284	295	306	1.0
392			329	340	351	362	373	384	395	406	417	2.0
393			439	450	461	472	483	494	506	517	528	3.0
394			550	561	572	583	594	605	616	627	638	4.0
395			660	671	682	693	704	715	726	737	748	5.0
396			770	780	791	802	813	824	835	846	857	6.0
397			879	890	901	912	923	934	945	956	966	7.0
398	59	60	*010	*021	*032	*043	*054	*065	*076	*086		8.0
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401		314	325	336	347	358	369	379	390	401	412	
402		423	433	444	455	466	477	487	498	509	520	
403		531	541	552	563	574	584	595	606	617	627	
404		638	649	660	670	681	692	703	713	724	735	
405		746	756	767	778	788	799	810	821	831	842	
406		853	863	874	885	895	906	917	927	938	949	
407	60	61	959	970	981	991	*002	*013	*023	*034	*045	*055
408			066	077	087	098	109	119	130	140	151	162
409			172	183	194	204	215	225	236	247	257	268
410			278	289	300	310	321	331	342	352	363	374
411			384	395	405	416	426	437	448	458	469	479
412			490	500	511	521	532	542	553	563	574	584
413			595	606	616	627	637	648	658	669	679	690
414			700	711	721	731	742	752	763	773	784	794
415		61	805	815	826	836	847	857	868	878	888	899
416	62	63	909	920	930	941	951	962	972	982	993	*003
417			014	024	034	045	055	066	076	086	097	107
418			118	128	138	149	159	170	180	190	201	211
419			221	232	242	252	263	273	284	294	304	315
420			325	335	346	356	366	377	387	397	408	418
421			428	439	449	459	469	480	490	500	511	521
422			531	542	552	562	572	583	593	603	613	624
423			634	644	655	665	675	685	696	706	716	726
424			737	747	757	767	778	788	798	808	818	829
425		62	839	849	859	870	880	890	900	910	921	931
426	63	64	941	951	961	972	982	992	*002	*012	*022	*033
427			043	053	063	073	083	094	104	114	124	134
428			144	155	165	175	185	195	205	215	225	236
429			246	256	266	276	286	296	306	317	327	337
430			347	357	367	377	387	397	407	417	428	438
431			448	458	468	478	488	498	508	518	528	538
432			548	558	568	579	589	599	609	619	629	639
433			649	659	669	679	689	699	709	719	729	739
434			749	759	769	779	789	799	809	819	829	839
435		63	849	859	869	879	889	899	909	919	929	939
436	64	65	949	959	969	979	988	998	*008	*018	*028	*038
437			048	058	068	078	088	098	108	118	128	137
438			147	157	167	177	187	197	207	217	227	237
439			246	256	266	276	286	296	306	316	326	335
440			345	355	365	375	385	395	404	414	424	434
441			444	454	464	473	483	493	503	513	523	532
442			542	552	562	572	582	591	601	611	621	631
443			640	650	660	670	680	689	699	709	719	729
444			738	748	758	768	777	787	797	807	816	826
445		64	836	846	856	865	875	885	895	904	914	924
446	65	66	933	943	953	963	972	982	992	*002	*011	*021
447			031	040	050	060	070	079	089	099	108	118
448			128	137	147	157	167	176	186	196	205	215
449			225	234	244	254	263	273	283	292	302	312
450			321	331	341	350	360	369	379	389	398	408
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452		514	523	533	543	552	562	571	581	591	600	
453		610	619	629	639	648	658	667	677	686	696	
454		706	715	725	734	744	753	763	772	782	792	
455		801	811	820	830	839	849	858	868	877	887	
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457	65	992	*001	*011	*020	*030	*039	*049	*058	*068	*077	10
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461		370	380	389	398	408	417	427	436	445	455	4 4.0
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465		745	755	764	773	783	792	801	811	820	829	8 8.0
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488		842	851	860	869	878	886	895	904	913	922	
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495		461	469	478	487	496	504	513	522	531	539	5 4.0
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503		157	165	174	183	191	200	209	217	226	234	
504		243	252	260	269	278	286	295	303	312	321	
505		329	338	346	355	364	372	381	389	398	406	
506		415	424	432	441	449	458	467	475	484	492	
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516		265	273	282	290	299	307	315	324	332	341	
517		349	357	366	374	383	391	399	408	416	425	
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522		767	775	784	792	800	809	817	825	834	842	
523		850	858	867	875	883	892	900	908	917	925	
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526		099	107	115	123	132	140	148	156	165	173	
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536		916	925	933	941	949	957	965	973	981	989	
537	72	997	*006	*014	*022	*030	*038	*046	*054	*062	*070	
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544		560	568	576	584	592	600	608	616	624	632	
545		640	648	656	664	672	679	687	695	703	711	
546		719	727	735	743	751	759	767	775	783	791	
547		799	807	815	823	830	838	846	854	862	870	
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615	78 } 79 }	888	895	902	909	916	923	930	937	944	951	
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631	80 }	003	010	017	024	030	037	044	051	058	065	
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644	80 } 81 }	889	895	902	909	916	922	929	936	943	949	
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661	82 }	020	027	033	040	046	053	060	066	073	079	1 0.7
662		086	092	099	105	112	119	125	132	138	145	2 1.4
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676	83 }	995	*001	*008	*014	*020	*027	*033	*040	*046	*052	
677		059	065	072	078	085	091	097	104	110	117	
678		123	129	136	142	149	155	161	168	174	181	
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687		696	702	708	715	721	727	734	740	746	753	
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700		510	516	522	528	535	541	547	553	559	566	
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715		431	437	443	449	455	461	467	473	479	485	
716		491	497	503	509	516	522	528	534	540	546	
717		552	558	564	570	576	582	588	594	600	606	
718		612	618	625	631	637	643	649	655	661	667	
719		673	679	685	691	697	703	709	715	721	727	
720		733	739	745	751	757	763	769	775	781	788	6 1 0.6 2 1.2 3 1.8 4 2.4 5 3.0 6 3.6 7 4.2 8 4.8 9 5.4
721		794	800	806	812	818	824	830	836	842	848	
722		854	860	866	872	878	884	890	896	902	908	
723		914	920	926	932	938	944	950	956	962	968	
724	85	974	980	986	992	998	*004	*010	*016	*022	*028	
725	86	034	040	046	052	058	064	070	076	082	088	
726		094	100	106	112	118	124	130	136	141	147	
727		153	159	165	171	177	183	189	195	201	207	
728		213	219	225	231	237	243	249	255	261	267	
729		273	279	285	291	297	303	308	314	320	326	
730		332	338	344	350	356	362	368	374	380	386	5 1 0.5 2 1.0 3 1.5 4 2.0 5 2.5 6 3.0 7 3.5 8 4.0 9 4.5
731		392	398	404	410	415	421	427	433	439	445	
732		451	457	463	469	475	481	487	493	499	504	
733		510	516	522	528	534	540	546	552	558	564	
734		570	576	581	587	593	599	605	611	617	623	
735		629	635	641	646	652	658	664	670	676	682	
736		688	694	700	705	711	717	723	729	735	741	
737		747	753	759	764	770	776	782	788	794	800	
738		806	812	817	823	829	835	841	847	853	859	
739		864	870	876	882	888	894	900	906	911	917	
740		923	929	935	941	947	953	958	964	970	976	
741	86	982	988	994	999	*005	*011	*017	*023	*029	*035	
742	87	040	046	052	058	064	070	075	081	087	093	
743		099	105	111	116	122	128	134	140	146	151	
744		157	163	169	175	181	186	192	198	204	210	
745		216	221	227	233	239	245	251	256	262	268	
746		274	280	286	291	297	303	309	315	320	326	
747		332	338	344	349	355	361	367	373	379	384	
748		390	396	402	408	413	419	425	431	437	442	
749		448	454	460	466	471	477	483	489	495	500	
750		506	512	518	523	529	535	541	547	552	558	P.P.
N.	L.	0	1	2	3	4	5	6	7	8	9	

N.	L.	0	1	2	3	4	5	6	7	8	9	P.P.
750	87	506	512	518	523	529	535	541	547	552	558	6 1 0.6 2 1.2 3 1.8 4 2.4 5 3.0 6 3.6 7 4.2 8 4.8 9 5.4

N.	L.	0	1	2	3	4	5	6	7	8	9	P.P.
800	90	309	314	320	325	331	336	342	347	352	358	6 1 0.6 2 1.2 3 1.8 4 2.4 5 3.0 6 3.6 7 4.2 8 4.8 9 5.4
801		363	369	374	380	385	390	396	401	407	412	
802		417	423	428	434	439	445	450	455	461	466	
803		472	477	482	488	493	499	504	509	515	520	
804		526	531	536	542	547	553	558	563	569	574	
805		580	585	590	596	601	607	612	617	623	628	
806		634	639	644	650	655	660	666	671	677	682	
807		687	693	698	703	709	714	720	725	730	736	
808		741	747	752	757	763	768	773	779	784	789	
809		795	800	806	811	816	822	827	832	838	843	
810		849	854	859	865	870	875	881	886	891	897	6 1 0.6 2 1.2 3 1.8 4 2.4 5 3.0 6 3.6 7 4.2 8 4.8 9 5.4
811	90	902	907	913	918	924	929	934	940	945	950	
812		956	961	966	972	977	982	988	993	998	*004	
813	91	009	014	020	025	030	036	041	046	052	057	
814		062	068	073	078	084	089	094	100	105	110	
815		116	121	126	132	137	142	148	153	158	164	
816		169	174	180	185	190	196	201	206	212	217	
817		222	228	233	238	243	249	254	259	265	270	
818		275	281	286	291	297	302	307	312	318	323	
819		328	334	339	344	350	355	360	365	371	376	
820		381	387	392	397	403	408	413	418	424	429	5 1 0.5 2 1.0 3 1.5 4 2.0 5 2.5 6 3.0 7 3.5 8 4.0 9 4.5
821		434	440	445	450	455	461	466	471	477	482	
822		487	492	498	503	508	514	519	524	529	535	
823		540	545	551	556	561	566	572	577	582	587	
824		593	598	603	609	614	619	624	630	635	640	
825		645	651	656	661	666	672	677	682	687	693	
826		698	703	709	714	719	724	730	735	740	745	
827		751	756	761	766	772	777	782	787	793	798	
828		803	808	814	819	824	829	834	840	845	850	
829		855	861	866	871	876	882	887	892	897	903	
830		908	913	918	924	929	934	939	944	950	955	5 1 0.5 2 1.0 3 1.5 4 2.0 5 2.5 6 3.0 7 3.5 8 4.0 9 4.5
831	91	960	965	971	976	981	986	991	997	*002	*007	
832		012	018	023	028	033	038	044	049	054	059	
833	92	065	070	075	080	085	091	096	101	106	111	
834		117	122	127	132	137	143	148	153	158	163	
835		169	174	179	184	189	195	200	205	210	215	
836		221	226	231	236	241	247	252	257	262	267	
837		273	278	283	288	293	298	304	309	314	319	
838		324	330	335	340	345	350	355	361	366	371	
839		376	381	387	392	397	402	407	412	418	423	
840		428	433	438	443	449	454	459	464	469	474	5 1 0.5 2 1.0 3 1.5 4 2.0 5 2.5 6 3.0 7 3.5 8 4.0 9 4.5
841		480	485	490	495	500	505	511	516	521	526	
842		531	536	542	547	552	557	562	567	572	578	
843		583	588	593	598	603	609	614	619	624	629	
844		634	639	645	650	655	660	665	670	675	681	
845		686	691	696	701	706	711	716	722	727	732	
846		737	742	747	752	758	763	768	773	778	783	
847		788	793	799	804	809	814	819	824	829	834	
848		840	845	850	855	860	865	870	875	881	886	
849		891	896	901	906	911	916	921	927	932	937	
850		942	947	952	957	962	967	973	978	983	988	
N.	L.	0	1	2	3	4	5	6	7	8	9	P.P.

N.	L.	0	1	2	3	4	5	6	7	8	9	P.P.
850	92	942	947	952	957	962	967	973	978	983	988	
851	92	993	998	*003	*008	*013	*018	*024	*029	*034	*039	
852	93	044	049	054	059	064	069	075	080	085	090	
853		095	100	105	110	115	120	125	131	136	141	
854		146	151	156	161	166	171	176	181	186	192	
855		197	202	207	212	217	222	227	232	237	242	
856		247	252	258	263	268	273	278	283	288	293	
857		298	303	308	313	318	323	328	334	339	344	
858		349	354	359	364	369	374	379	384	389	394	
859		399	404	409	414	420	425	430	435	440	445	
860		450	455	460	465	470	475	480	485	490	495	
861		500	505	510	515	520	526	531	536	541	546	
862		551	556	561	566	571	576	581	586	591	596	
863		601	606	611	616	621	626	631	636	641	646	
864		651	656	661	666	671	676	682	687	692	697	
865		702	707	712	717	722	727	732	737	742	747	
866		752	757	762	767	772	777	782	787	792	797	
867		802	807	812	817	822	827	832	837	842	847	
868		852	857	862	867	872	877	882	887	892	897	
869		902	907	912	917	922	927	932	937	942	947	
870		952	957	962	967	972	977	982	987	992	997	
871	94	002	007	012	017	022	027	032	037	042	047	
872		052	057	062	067	072	077	082	086	091	096	
873		101	106	111	116	121	126	131	136	141	146	
874		151	156	161	166	171	176	181	186	191	196	
875		201	206	211	216	221	226	231	236	240	245	
876		250	255	260	265	270	275	280	285	290	295	
877		300	305	310	315	320	325	330	335	340	345	
878		349	354	359	364	369	374	379	384	389	394	
879		399	404	409	414	419	424	429	433	438	443	
880		448	453	458	463	468	473	478	483	488	493	
881		498	503	507	512	517	522	527	532	537	542	
882		547	552	557	562	567	571	576	581	586	591	
883		596	601	606	611	616	621	626	630	635	640	
884		645	650	655	660	665	670	675	680	685	689	
885		694	699	704	709	714	719	724	729	734	738	
886		743	748	753	758	763	768	773	778	783	787	
887		792	797	802	807	812	817	822	827	832	836	
888		841	846	851	856	861	866	871	876	880	885	
889		890	895	900	905	910	915	919	924	929	934	
890		939	944	949	954	959	963	968	973	978	983	
891	94	988	993	998	*002	*007	*012	*017	*022	*027	*032	
892	95	036	041	046	051	056	061	066	071	075	080	
893		085	090	095	100	105	109	114	119	124	129	
894		134	139	143	148	153	158	163	168	173	177	
895		182	187	192	197	202	207	211	216	221	226	
896		231	236	240	245	250	255	260	265	270	274	
897		279	284	289	294	299	303	308	313	318	323	
898		328	332	337	342	347	352	357	361	366	371	
899		376	381	386	390	395	400	405	410	415	419	
900		424	429	434	439	444	448	453	458	463	468	
N.	L.	0	1	2	3	4	5	6	7	8	9	P.P.

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1 0.6
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6 2.4
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8 3.2
9 3.6

N.	L.	0	1	2	3	4	5	6	7	8	9	P.P.
900	95	424	429	434	439	444	448	453	458	463	468	
901		472	477	482	487	492	497	501	506	511	516	
902		521	525	530	535	540	545	550	554	559	564	
903		569	574	578	583	588	593	598	602	607	612	
904		617	622	626	631	636	641	646	650	655	660	
905		665	670	674	679	684	689	694	698	703	708	
906		713	718	722	727	732	737	742	746	751	756	
907		761	766	770	775	780	785	789	794	799	804	
908		809	813	818	823	828	832	837	842	847	852	
909		856	861	866	871	875	880	885	890	895	899	
910		904	909	914	918	923	928	933	938	942	947	
911	95 96 }	952	957	961	966	971	976	980	985	990	995	5
912		999	*004	*009	*014	*019	*023	*028	*033	*038	*042	1 0.5
913		047	052	057	061	066	071	076	080	085	090	2 1.0
914		095	099	104	109	114	118	123	128	133	137	3 1.5
915		142	147	152	156	161	166	171	175	180	185	4 2.0
916		190	194	199	204	209	213	218	223	227	232	5 2.5
917		237	242	246	251	256	261	265	270	275	280	6 3.0
918		284	289	294	298	303	308	313	317	322	327	7 3.5
919		332	336	341	346	350	355	360	365	369	374	8 4.0
920		379	384	388	393	398	402	407	412	417	421	9 4.5
921		426	431	435	440	445	450	454	459	464	468	
922		473	478	483	487	492	497	501	506	511	515	
923		520	525	530	534	539	544	548	553	558	562	
924		567	572	577	581	586	591	595	600	605	609	
925		614	619	624	628	633	638	642	647	652	656	
926		661	666	670	675	680	685	689	694	699	703	
927		708	713	717	722	727	731	736	741	745	750	
928		755	759	764	769	774	778	783	788	792	797	
929		802	806	811	816	820	825	830	834	839	844	
930		848	853	858	862	867	872	876	881	886	890	
931	96 97 }	895	900	904	909	914	918	923	928	932	937	4
932		942	946	951	956	960	965	970	974	979	984	1 0.4
933		988	993	997	*002	*007	*011	*016	*021	*025	*030	2 0.8
934		035	039	044	049	053	058	063	067	072	077	3 1.2
935		081	086	090	095	100	104	109	114	118	123	4 1.6
936		128	132	137	142	146	151	155	160	165	169	5 2.0
937		174	179	183	188	192	197	202	206	211	216	6 2.4
938		220	225	230	234	239	243	248	253	257	262	7 2.8
939		267	271	276	280	285	290	294	299	304	308	8 3.2
940		313	317	322	327	331	336	340	345	350	354	9 3.6
941		359	364	368	373	377	382	387	391	396	400	
942		405	410	414	419	424	428	433	437	442	447	
943		451	456	460	465	470	474	479	483	488	493	
944		497	502	506	511	516	520	525	529	534	539	
945		543	548	552	557	562	566	571	575	580	585	
946		589	594	598	603	607	612	617	621	626	630	
947		635	640	644	649	653	658	663	667	672	676	
948		681	685	690	695	699	704	708	713	717	722	
949		727	731	736	740	745	749	754	759	763	768	
950		772	777	782	786	791	795	800	804	809	813	
N.	L.	0	1	2	3	4	5	6	7	8	9	P.P.

N.	L.	0	1	2	3	4	5	6	7	8	9	P.P.
950	97	772	777	782	786	791	795	800	804	809	813	
951		818	823	827	832	836	841	845	850	855	859	
952		864	868	873	877	882	886	891	896	900	905	
953		909	914	918	923	928	932	937	941	946	950	
954		955	959	964	968	973	978	982	987	991	996	
955	98	000	005	009	014	019	023	028	032	037	041	
956		046	050	055	059	064	068	073	078	082	087	
957		091	096	100	105	109	114	118	123	127	132	
958		137	141	146	150	155	159	164	168	173	177	
959		182	186	191	195	200	204	209	214	218	223	5
960		227	232	236	241	245	250	254	259	263	268	1 0.5
961		272	277	281	286	290	295	299	304	308	313	2 1.0
962		318	322	327	331	336	340	345	349	354	358	3 1.5
963		363	367	372	376	381	385	390	394	399	403	4 2.0
964		408	412	417	421	426	430	435	439	444	448	5 2.5
965		453	457	462	466	471	475	480	484	489	493	6 3.0
966		498	502	507	511	516	520	525	529	534	538	7 3.5
967		543	547	552	556	561	565	570	574	579	583	8 4.0
968		588	592	597	601	605	610	614	619	623	628	9 4.5
969		632	637	641	646	650	655	659	664	668	673	
970		677	682	686	691	695	700	704	709	713	717	
971		722	726	731	735	740	744	749	753	758	762	
972		767	771	776	780	784	789	793	798	802	807	
973		811	816	820	825	829	834	838	843	847	851	
974		856	860	865	869	874	878	883	887	892	896	
975		900	905	909	914	918	923	927	932	936	941	
976		945	949	954	958	963	967	972	976	981	985	
977	98	989	994	998	*003	*007	*012	*016	*021	*025	*029	
978	99	034	038	043	047	052	056	061	065	069	074	
979		078	083	087	092	096	100	105	109	114	118	
980		123	127	131	136	140	145	149	154	158	162	
981		167	171	176	180	185	189	193	198	202	207	
982		211	216	220	224	229	233	238	242	247	251	
983		255	260	264	269	273	277	282	286	291	295	
984		300	304	308	313	317	322	326	330	335	339	
985		344	348	352	357	361	366	370	374	379	383	4
986		388	392	396	401	405	410	414	419	423	427	1 0.4
987		432	436	441	445	449	454	458	463	467	471	2 0.8
988		476	480	484	489	493	498	502	506	511	515	3 1.2
989		520	524	528	533	537	542	546	550	555	559	4 1.6
990		564	568	572	577	581	585	590	594	599	603	5 2.0
991		607	612	616	621	625	629	634	638	642	647	6 2.4
992		651	656	660	664	669	673	677	682	686	691	7 2.8
993		695	699	704	708	712	717	721	726	730	734	8 3.2
994		739	743	747	752	756	760	765	769	774	778	9 3.6
995		782	787	791	795	800	804	808	813	817	822	
996		826	830	835	839	843	848	852	856	861	865	
997		870	874	878	883	887	891	896	900	904	909	
998		913	917	922	926	930	935	939	944	948	952	
999		957	961	965	970	974	978	983	987	991	996	
1000	00	000	004	009	013	017	022	026	030	035	039	
N.	L.	0	1	2	3	4	5	6	7	8	9	P.P.

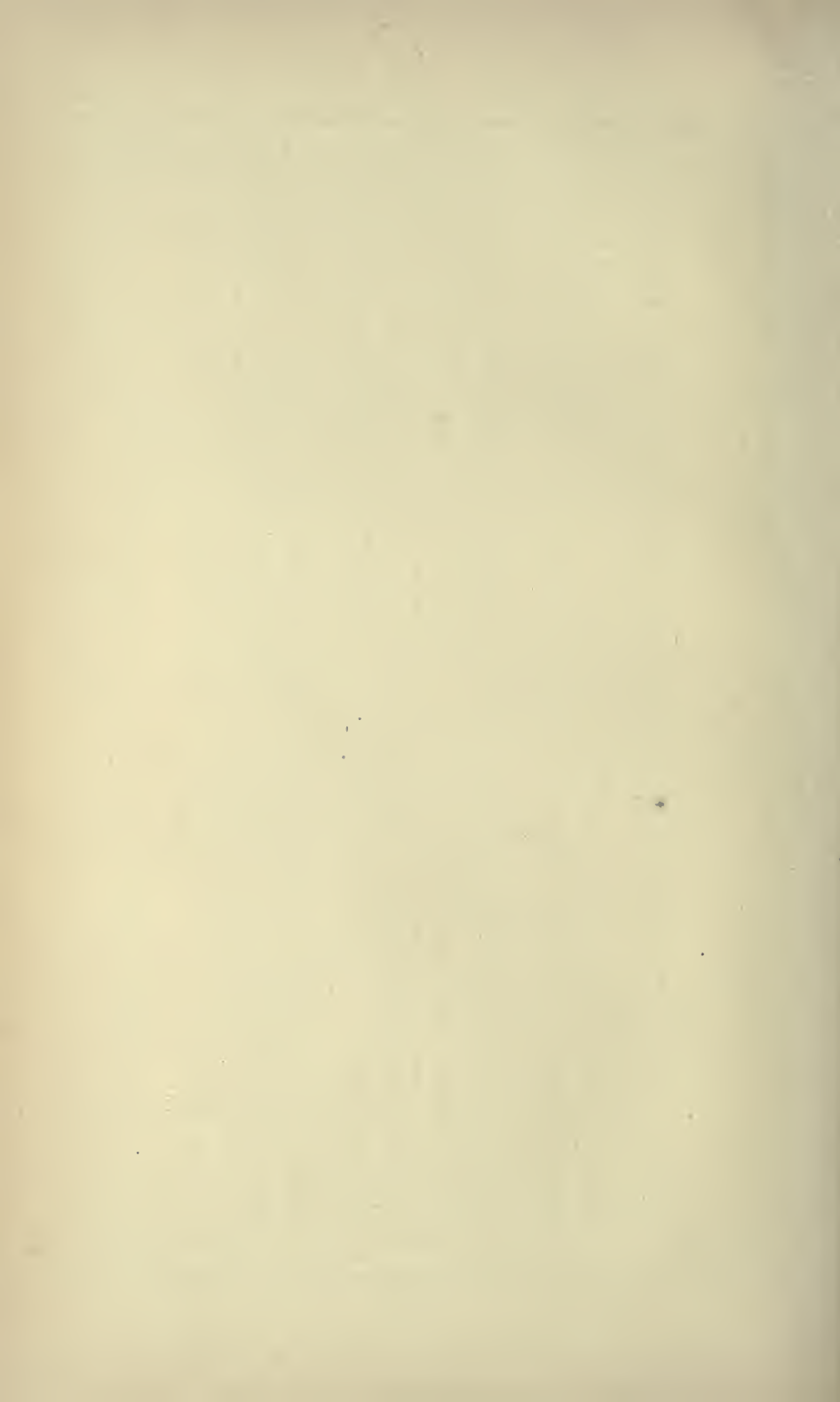


TABLE II.
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LOGARITHMS
OF THE
TRIGONOMETRIC FUNCTIONS.

S. T. 4.68		"	'	L. Sin.	L. Tan.	L. Cot.	L. Cos.	
557	557	o	0				0.00000	60
557	557	60	1	6.46373	6.46373	3.53627	0.00000	59
557	557	120	2	6.76476	6.76476	3.23524	0.00000	58
557	557	180	3	6.94085	6.94085	3.05915	0.00000	57
557	558	240	4	7.06579	7.06579	2.93421	0.00000	56
557	558	300	5	7.16270	7.16270	2.83730	0.00000	55
557	558	360	6	7.24188	7.24188	2.75812	0.00000	54
557	558	420	7	7.30882	7.30882	2.69118	0.00000	53
557	558	480	8	7.36682	7.36682	2.63318	0.00000	52
557	558	540	9	7.41797	7.41797	2.58203	0.00000	51
557	558	600	10	7.46373	7.46373	2.53627	0.00000	50
557	558	660	11	7.50512	7.50512	2.49488	0.00000	49
557	558	720	12	7.54291	7.54291	2.45709	0.00000	48
557	558	780	13	7.57767	7.57767	2.42233	0.00000	47
557	558	840	14	7.60985	7.60986	2.39014	0.00000	46
557	558	900	15	7.63982	7.63982	2.36018	0.00000	45
557	558	960	16	7.66784	7.66785	2.33215	0.00000	44
557	558	1020	17	7.69417	7.69418	2.30582	9.99999	43
557	558	1080	18	7.71900	7.71900	2.28100	9.99999	42
557	558	1140	19	7.74248	7.74248	2.25752	9.99999	41
557	558	1200	20	7.76475	7.76476	2.23524	9.99999	40
557	558	1260	21	7.78594	7.78595	2.21405	9.99999	39
557	558	1320	22	7.80615	7.80615	2.19385	9.99999	38
557	558	1380	23	7.82545	7.82546	2.17454	9.99999	37
557	558	1440	24	7.84393	7.84394	2.15606	9.99999	36
557	558	1500	25	7.86166	7.86167	2.13833	9.99999	35
557	558	1560	26	7.87870	7.87871	2.12129	9.99999	34
557	558	1620	27	7.89509	7.89510	2.10490	9.99999	33
557	558	1680	28	7.91088	7.91089	2.08911	9.99999	32
557	559	1740	29	7.92612	7.92613	2.07387	9.99998	31
557	559	1800	30	7.94084	7.94086	2.05914	9.99998	30
557	559	1860	31	7.95508	7.95510	2.04490	9.99998	29
557	559	1920	32	7.96887	7.96889	2.03111	9.99998	28
557	559	1980	33	7.98223	7.98225	2.01775	9.99998	27
557	559	2040	34	7.99520	7.99522	2.00478	9.99998	26
557	559	2100	35	8.00779	8.00781	1.99219	9.99998	25
557	559	2160	36	8.02002	8.02004	1.97996	9.99998	24
557	559	2220	37	8.03192	8.03194	1.96806	9.99997	23
557	559	2280	38	8.04350	8.04353	1.95647	9.99997	22
557	559	2340	39	8.05478	8.05481	1.94519	9.99997	21
557	559	2400	40	8.06578	8.06581	1.93419	9.99997	20
556	560	2460	41	8.07650	8.07653	1.92347	9.99997	19
556	560	2520	42	8.08696	8.08700	1.91300	9.99997	18
556	560	2580	43	8.09718	8.09722	1.90278	9.99997	17
556	560	2640	44	8.10717	8.10720	1.89280	9.99996	16
556	560	2700	45	8.11693	8.11696	1.88304	9.99996	15
556	560	2760	46	8.12647	8.12651	1.87349	9.99996	14
556	560	2820	47	8.13581	8.13585	1.86415	9.99996	13
556	560	2880	48	8.14495	8.14500	1.85500	9.99996	12
556	560	2940	49	8.15391	8.15395	1.84605	9.99996	11
556	561	3000	50	8.16268	8.16273	1.83727	9.99995	10
556	561	3060	51	8.17128	8.17133	1.82867	9.99995	9
556	561	3120	52	8.17971	8.17976	1.82024	9.99995	8
556	561	3180	53	8.18798	8.18804	1.81196	9.99995	7
556	561	3240	54	8.19610	8.19616	1.80384	9.99995	6
556	561	3300	55	8.20407	8.20413	1.79587	9.99994	5
556	561	3360	56	8.21189	8.21195	1.78805	9.99994	4
555	561	3420	57	8.21958	8.21964	1.78036	9.99994	3
555	562	3480	58	8.22713	8.22720	1.77280	9.99994	2
555	562	3540	59	8.23456	8.23462	1.76538	9.99994	1
555	562	3600	60	8.24186	8.24192	1.75808	9.99993	0
				L. Cos.	L. Cot.	L. Tan.	L. Sin.	'

S.	T.	"	'	L. Sin.	L. Tan.	L. Cot.	L. Cos.	
4.68								
555	562	3600	0	8.24186	8.24192	1.75808	9.99993	60
555	562	3660	1	8.24903	8.24910	1.75090	9.99993	59
555	562	3720	2	8.25609	8.25616	1.74384	9.99993	58
555	562	3780	3	8.26304	8.26312	1.73688	9.99993	57
555	563	3840	4	8.26988	8.26996	1.73004	9.99992	56
555	563	3900	5	8.27661	8.27669	1.72331	9.99992	55
555	563	3960	6	8.28324	8.28332	1.71668	9.99992	54
555	563	4020	7	8.28977	8.28986	1.71014	9.99992	53
555	563	4080	8	8.29621	8.29629	1.70371	9.99992	52
555	563	4140	9	8.30255	8.30263	1.69737	9.99991	51
554	563	4200	10	8.30879	8.30888	1.69112	9.99991	50
554	564	4260	11	8.31495	8.31505	1.68495	9.99991	49
554	564	4320	12	8.32103	8.32112	1.67888	9.99990	48
554	564	4380	13	8.32702	8.32711	1.67289	9.99990	47
554	564	4440	14	8.33292	8.33302	1.66698	9.99990	46
554	564	4500	15	8.33875	8.33886	1.66114	9.99990	45
554	565	4560	16	8.34450	8.34461	1.65539	9.99989	44
554	565	4620	17	8.35018	8.35029	1.64971	9.99989	43
554	565	4680	18	8.35578	8.35590	1.64410	9.99989	42
554	565	4740	19	8.36131	8.36143	1.63857	9.99989	41
554	565	4800	20	8.36678	8.36689	1.63311	9.99988	40
553	566	4860	21	8.37217	8.37229	1.62771	9.99988	39
553	566	4920	22	8.37750	8.37762	1.62238	9.99988	38
553	566	4980	23	8.38276	8.38289	1.61711	9.99987	37
553	566	5040	24	8.38796	8.38809	1.61191	9.99987	36
553	566	5100	25	8.39310	8.39323	1.60677	9.99987	35
553	567	5160	26	8.39818	8.39832	1.60168	9.99986	34
553	567	5220	27	8.40320	8.40334	1.59666	9.99986	33
553	567	5280	28	8.40816	8.40830	1.59170	9.99986	32
553	567	5340	29	8.41307	8.41321	1.58679	9.99985	31
553	567	5400	30	8.41792	8.41807	1.58193	9.99985	30
552	568	5460	31	8.42272	8.42287	1.57713	9.99985	29
552	568	5520	32	8.42746	8.42762	1.57238	9.99984	28
552	568	5580	33	8.43216	8.43232	1.56768	9.99984	27
552	568	5640	34	8.43680	8.43696	1.56304	9.99984	26
552	569	5700	35	8.44139	8.44156	1.55844	9.99983	25
552	569	5760	36	8.44594	8.44611	1.55389	9.99983	24
552	569	5820	37	8.45044	8.45061	1.54939	9.99983	23
552	569	5880	38	8.45489	8.45507	1.54493	9.99982	22
551	569	5940	39	8.45930	8.45948	1.54052	9.99982	21
551	570	6000	40	8.46366	8.46385	1.53615	9.99982	20
551	570	6060	41	8.46799	8.46817	1.53183	9.99981	19
551	570	6120	42	8.47226	8.47245	1.52755	9.99981	18
551	570	6180	43	8.47650	8.47669	1.52331	9.99981	17
551	571	6240	44	8.48069	8.48089	1.51911	9.99980	16
551	571	6300	45	8.48485	8.48505	1.51495	9.99980	15
551	571	6360	46	8.48896	8.48917	1.51083	9.99979	14
550	572	6420	47	8.49304	8.49325	1.50675	9.99979	13
550	572	6480	48	8.49708	8.49729	1.50271	9.99979	12
550	572	6540	49	8.50108	8.50130	1.49870	9.99978	11
550	572	6600	50	8.50504	8.50527	1.49473	9.99978	10
550	573	6660	51	8.50897	8.50920	1.49080	9.99977	9
550	573	6720	52	8.51287	8.51310	1.48690	9.99977	8
550	573	6780	53	8.51673	8.51696	1.48304	9.99977	7
550	573	6840	54	8.52055	8.52079	1.47921	9.99976	6
549	574	6900	55	8.52434	8.52459	1.47541	9.99976	5
549	574	6960	56	8.52810	8.52835	1.47165	9.99975	4
549	574	7020	57	8.53183	8.53208	1.46792	9.99975	3
549	575	7080	58	8.53552	8.53578	1.46422	9.99974	2
549	575	7140	59	8.53919	8.53945	1.46055	9.99974	1
549	575	7200	60	8.54282	8.54308	1.45692	9.99974	0
				L. Cos.	L. Cot.	L. Tan.	L. Sin.	'

S.	T.	"	'	L. Sin.	L. Tan.	L. Cot.	L. Cos.	
4.68								
549	575	7200	0	8.54282	8.54308	1.45692	9.99974	60
549	575	7260	1	8.54642	8.54669	1.45331	9.99973	59
548	576	7320	2	8.54999	8.55027	1.44973	9.99973	58
548	576	7380	3	8.55354	8.55382	1.44618	9.99972	57
548	576	7440	4	8.55705	8.55734	1.44266	9.99972	56
548	577	7500	5	8.56054	8.56083	1.43917	9.99971	55
548	577	7560	6	8.56400	8.56429	1.43571	9.99971	54
548	577	7620	7	8.56743	8.56773	1.43227	9.99970	53
547	578	7680	8	8.57084	8.57114	1.42886	9.99970	52
547	578	7740	9	8.57421	8.57452	1.42548	9.99969	51
547	578	7800	10	8.57757	8.57788	1.42212	9.99969	50
547	579	7860	11	8.58089	8.58121	1.41879	9.99968	49
547	579	7920	12	8.58419	8.58451	1.41549	9.99968	48
547	579	7980	13	8.58747	8.58779	1.41221	9.99967	47
546	579	8040	14	8.59072	8.59105	1.40895	9.99967	46
546	580	8100	15	8.59395	8.59428	1.40572	9.99967	45
546	580	8160	16	8.59715	8.59749	1.40251	9.99966	44
546	580	8220	17	8.60033	8.60068	1.39932	9.99966	43
546	581	8280	18	8.60349	8.60384	1.39616	9.99965	42
546	581	8340	19	8.60662	8.60698	1.39302	9.99964	41
545	582	8400	20	8.60973	8.61009	1.38991	9.99964	40
545	582	8460	21	8.61282	8.61319	1.38681	9.99963	39
545	582	8520	22	8.61589	8.61626	1.38374	9.99963	38
545	583	8580	23	8.61894	8.61931	1.38069	9.99962	37
545	583	8640	24	8.62196	8.62234	1.37766	9.99962	36
545	583	8700	25	8.62497	8.62535	1.37465	9.99961	35
544	584	8760	26	8.62795	8.62834	1.37166	9.99961	34
544	584	8820	27	8.63091	8.63131	1.36869	9.99960	33
544	584	8880	28	8.63385	8.63426	1.36574	9.99960	32
544	585	8940	29	8.63678	8.63718	1.36282	9.99959	31
544	585	9000	30	8.63968	8.64009	1.35991	9.99959	30
544	585	9060	31	8.64256	8.64298	1.35702	9.99958	29
543	586	9120	32	8.64543	8.64585	1.35415	9.99958	28
543	586	9180	33	8.64827	8.64870	1.35130	9.99957	27
543	587	9240	34	8.65110	8.65154	1.34846	9.99956	26
543	587	9300	35	8.65391	8.65435	1.34565	9.99956	25
543	587	9360	36	8.65670	8.65715	1.34285	9.99955	24
542	588	9420	37	8.65947	8.65993	1.34007	9.99955	23
542	588	9480	38	8.66223	8.66269	1.33731	9.99954	22
542	588	9540	39	8.66497	8.66543	1.33457	9.99954	21
542	589	9600	40	8.66769	8.66816	1.33184	9.99953	20
542	589	9660	41	8.67039	8.67087	1.32913	9.99952	19
541	590	9720	42	8.67308	8.67356	1.32644	9.99952	18
541	590	9780	43	8.67575	8.67624	1.32376	9.99951	17
541	590	9840	44	8.67841	8.67890	1.32110	9.99951	16
541	591	9900	45	8.68104	8.68154	1.31846	9.99950	15
541	591	9960	46	8.68367	8.68417	1.31583	9.99949	14
540	592	10020	47	8.68627	8.68678	1.31322	9.99949	13
540	592	10080	48	8.68886	8.68938	1.31062	9.99948	12
540	592	10140	49	8.69144	8.69196	1.30804	9.99948	11
540	593	10200	50	8.69400	8.69453	1.30547	9.99947	10
540	593	10260	51	8.69654	8.69708	1.30292	9.99946	9
539	594	10320	52	8.69907	8.69962	1.30038	9.99946	8
539	594	10380	53	8.70159	8.70214	1.29786	9.99945	7
539	595	10440	54	8.70409	8.70465	1.29535	9.99944	6
539	595	10500	55	8.70658	8.70714	1.29286	9.99944	5
539	595	10560	56	8.70905	8.70962	1.29038	9.99943	4
538	596	10620	57	8.71151	8.71208	1.28792	9.99942	3
538	596	10680	58	8.71395	8.71453	1.28547	9.99942	2
538	597	10740	59	8.71638	8.71697	1.28303	9.99941	1
538	597	10800	60	8.71880	8.71940	1.28060	9.99940	0
				L. Cos.	L. Cot.	L. Tan.	L. Sin.	'

S.	T.	"	'	L. Sin.	L. Tan.	L. Cot.	L. Cos.	
4.68								
538	597	10800	0	8.71880	8.71940	1.28060	9.99940	60
537	598	10860	1	8.72120	8.72181	1.27819	9.99940	59
537	598	10920	2	8.72359	8.72420	1.27580	9.99939	58
537	599	10980	3	8.72597	8.72659	1.27341	9.99938	57
537	599	11040	4	8.72834	8.72896	1.27104	9.99938	56
537	599	11100	5	8.73069	8.73132	1.26868	9.99937	55
536	600	11160	6	8.73303	8.73366	1.26634	9.99936	54
536	600	11220	7	8.73535	8.73600	1.26400	9.99936	53
536	601	11280	8	8.73767	8.73832	1.26168	9.99935	52
536	601	11340	9	8.73997	8.74063	1.25937	9.99934	51
535	602	11400	10	8.74226	8.74292	1.25708	9.99934	50
535	602	11460	11	8.74454	8.74521	1.25479	9.99933	49
535	603	11520	12	8.74680	8.74748	1.25252	9.99932	48
535	603	11580	13	8.74906	8.74974	1.25026	9.99932	47
534	604	11640	14	8.75130	8.75199	1.24801	9.99931	46
534	604	11700	15	8.75353	8.75423	1.24577	9.99930	45
534	605	11760	16	8.75575	8.75645	1.24355	9.99929	44
534	605	11820	17	8.75795	8.75867	1.24133	9.99929	43
533	606	11880	18	8.76015	8.76087	1.23913	9.99928	42
533	606	11940	19	8.76234	8.76306	1.23694	9.99927	41
533	607	12000	20	8.76451	8.76525	1.23475	9.99926	40
533	607	12060	21	8.76667	8.76742	1.23258	9.99926	39
532	608	12120	22	8.76883	8.76958	1.23042	9.99925	38
532	608	12180	23	8.77097	8.77173	1.22827	9.99924	37
532	609	12240	24	8.77310	8.77387	1.22613	9.99923	36
532	609	12300	25	8.77522	8.77600	1.22400	9.99923	35
531	610	12360	26	8.77733	8.77811	1.22189	9.99922	34
531	610	12420	27	8.77943	8.78022	1.21978	9.99921	33
531	611	12480	28	8.78152	8.78232	1.21768	9.99920	32
531	611	12540	29	8.78360	8.78441	1.21559	9.99920	31
530	612	12600	30	8.78568	8.78649	1.21351	9.99919	30
530	612	12660	31	8.78774	8.78855	1.21145	9.99918	29
530	613	12720	32	8.78979	8.79061	1.20939	9.99917	28
530	613	12780	33	8.79183	8.79266	1.20734	9.99917	27
529	614	12840	34	8.79386	8.79470	1.20530	9.99916	26
529	614	12900	35	8.79588	8.79673	1.20327	9.99915	25
529	615	12960	36	8.79789	8.79875	1.20125	9.99914	24
529	615	13020	37	8.79990	8.80076	1.19924	9.99913	23
528	616	13080	38	8.80189	8.80277	1.19723	9.99913	22
528	616	13140	39	8.80388	8.80476	1.19524	9.99912	21
528	617	13200	40	8.80585	8.80674	1.19326	9.99911	20
528	617	13260	41	8.80782	8.80872	1.19128	9.99910	19
527	618	13320	42	8.80978	8.81068	1.18932	9.99909	18
527	618	13380	43	8.81173	8.81264	1.18736	9.99909	17
527	619	13440	44	8.81367	8.81459	1.18541	9.99908	16
526	620	13500	45	8.81560	8.81653	1.18347	9.99907	15
526	620	13560	46	8.81752	8.81846	1.18154	9.99906	14
526	621	13620	47	8.81944	8.82038	1.17962	9.99905	13
526	621	13680	48	8.82134	8.82230	1.17770	9.99904	12
525	622	13740	49	8.82324	8.82420	1.17580	9.99904	11
525	622	13800	50	8.82513	8.82610	1.17390	9.99903	10
525	623	13860	51	8.82701	8.82799	1.17201	9.99902	9
525	623	13920	52	8.82888	8.82987	1.17013	9.99901	8
524	624	13980	53	8.83075	8.83175	1.16825	9.99900	7
524	625	14040	54	8.83261	8.83361	1.16639	9.99899	6
524	625	14100	55	8.83446	8.83547	1.16453	9.99898	5
523	626	14160	56	8.83630	8.83732	1.16268	9.99898	4
523	626	14220	57	8.83813	8.83916	1.16084	9.99897	3
523	627	14280	58	8.83996	8.84100	1.15900	9.99896	2
522	628	14340	59	8.84177	8.84282	1.15718	9.99895	1
522	628	14400	60	8.84358	8.84464	1.15536	9.99894	0
				L. Cos.	L. Cot.	L. Tan.	L. Sin.	'

S.	T.	"	'	L. Sin.	L. Tan.	L. Cot.	L. Cos.	
4.68								
522	628	14400	0	8.84358	8.84464	1.15536	9.99894	60
522	629	14460	1	8.84539	8.84646	1.15354	9.99893	59
522	629	14520	2	8.84718	8.84826	1.15174	9.99892	58
521	630	14580	3	8.84897	8.85006	1.14994	9.99891	57
521	631	14640	4	8.85075	8.85185	1.14815	9.99891	56
521	631	14700	5	8.85252	8.85363	1.14637	9.99890	55
520	632	14760	6	8.85429	8.85540	1.14460	9.99889	54
520	632	14820	7	8.85605	8.85717	1.14283	9.99888	53
520	633	14880	8	8.85780	8.85893	1.14107	9.99887	52
520	634	14940	9	8.85955	8.86069	1.13931	9.99886	51
519	634	15000	10	8.86128	8.86243	1.13757	9.99885	50
519	635	15060	11	8.86301	8.86417	1.13583	9.99884	49
519	635	15120	12	8.86474	8.86591	1.13409	9.99883	48
518	636	15180	13	8.86645	8.86763	1.13237	9.99882	47
518	637	15240	14	8.86816	8.86935	1.13065	9.99881	46
518	637	15300	15	8.86987	8.87106	1.12894	9.99880	45
517	638	15360	16	8.87156	8.87277	1.12723	9.99879	44
517	638	15420	17	8.87325	8.87447	1.12553	9.99879	43
517	639	15480	18	8.87494	8.87616	1.12384	9.99878	42
516	640	15540	19	8.87661	8.87785	1.12215	9.99877	41
516	640	15600	20	8.87829	8.87953	1.12047	9.99876	40
516	641	15660	21	8.87995	8.88120	1.11880	9.99875	39
515	642	15720	22	8.88161	8.88287	1.11713	9.99874	38
515	642	15780	23	8.88326	8.88453	1.11547	9.99873	37
515	643	15840	24	8.88490	8.88618	1.11382	9.99872	36
514	644	15900	25	8.88654	8.88783	1.11217	9.99871	35
514	644	15960	26	8.88817	8.88948	1.11052	9.99870	34
514	645	16020	27	8.88980	8.89111	1.10889	9.99869	33
513	646	16080	28	8.89142	8.89274	1.10726	9.99868	32
513	646	16140	29	8.89304	8.89437	1.10563	9.99867	31
513	647	16200	30	8.89464	8.89598	1.10402	9.99866	30
512	648	16260	31	8.89625	8.89760	1.10240	9.99865	29
512	648	16320	32	8.89784	8.89920	1.10080	9.99864	28
512	649	16380	33	8.89943	8.90080	1.09920	9.99863	27
511	650	16440	34	8.90102	8.90240	1.09760	9.99862	26
511	650	16500	35	8.90260	8.90399	1.09601	9.99861	25
511	651	16560	36	8.90417	8.90557	1.09443	9.99860	24
510	652	16620	37	8.90574	8.90715	1.09285	9.99859	23
510	652	16680	38	8.90730	8.90872	1.09128	9.99858	22
510	653	16740	39	8.90885	8.91029	1.08971	9.99857	21
509	654	16800	40	8.91040	8.91185	1.08815	9.99856	20
509	654	16860	41	8.91195	8.91340	1.08660	9.99855	19
509	655	16920	42	8.91349	8.91495	1.08505	9.99854	18
508	656	16980	43	8.91502	8.91650	1.08350	9.99853	17
508	656	17040	44	8.91655	8.91803	1.08197	9.99852	16
508	657	17100	45	8.91807	8.91957	1.08043	9.99851	15
507	658	17160	46	8.91959	8.92110	1.07890	9.99850	14
507	659	17220	47	8.92110	8.92262	1.07738	9.99848	13
507	659	17280	48	8.92261	8.92414	1.07586	9.99847	12
506	660	17340	49	8.92411	8.92565	1.07435	9.99846	11
506	661	17400	50	8.92561	8.92716	1.07284	9.99845	10
506	661	17460	51	8.92710	8.92866	1.07134	9.99844	9
505	662	17520	52	8.92859	8.93016	1.06984	9.99843	8
505	663	17580	53	8.93007	8.93165	1.06835	9.99842	7
505	664	17640	54	8.93154	8.93313	1.06687	9.99841	6
504	664	17700	55	8.93301	8.93462	1.06538	9.99840	5
504	665	17760	56	8.93448	8.93609	1.06391	9.99839	4
503	666	17820	57	8.93594	8.93756	1.06244	9.99838	3
503	666	17880	58	8.93740	8.93903	1.06097	9.99837	2
503	667	17940	59	8.93885	8.94049	1.05951	9.99836	1
502	668	18000	60	8.94030	8.94195	1.05805	9.99834	0
				L. Cos.	L. Cot.	L. Tan.	L. Sin.	'

	L. Sin.	d.	L. Tan.	c.d.	L. Cot.	L. Cos.		P.P.				
0	8.94 030		8.94 195	145	1.05 805	9.99 834	60					
1	8.94 174	144	8.94 340	145	1.05 660	9.99 833	59	"	145	144	143	142
2	8.94 317	144	8.94 485	145	1.05 515	9.99 832	58	5	12.1	12.0	11.9	11.8
3	8.94 461	142	8.94 630	143	1.05 370	9.99 831	57	10	24.2	24.0	23.8	23.7
4	8.94 603	143	8.94 773	144	1.05 227	9.99 830	56	15	36.2	36.0	35.8	35.5
5	8.94 746	141	8.94 917	143	1.05 083	9.99 829	55	20	48.3	48.0	47.7	47.3
6	8.94 887	142	8.95 060	142	1.04 940	9.99 828	54	25	60.4	60.0	59.6	59.2
7	8.95 029	141	8.95 202	142	1.04 798	9.99 827	53	30	72.5	72.0	71.5	71.0
8	8.95 170	140	8.95 344	142	1.04 656	9.99 825	52	35	84.6	84.0	83.4	82.8
9	8.95 310	140	8.95 486	141	1.04 514	9.99 824	51	40	96.7	96.0	95.3	94.7
10	8.95 450	139	8.95 627	140	1.04 373	9.99 823	50	45	108.8	108.0	107.2	106.5
11	8.95 589	139	8.95 767	141	1.04 233	9.99 822	49	50	120.8	120.0	119.2	118.3
12	8.95 728	139	8.95 908	139	1.04 092	9.99 821	48	55	132.9	132.0	131.1	130.2
13	8.95 867	138	8.96 047	140	1.03 953	9.99 820	47					
14	8.96 005	138	8.96 187	138	1.03 813	9.99 819	46	"	141	140	139	138
15	8.96 143	137	8.96 325	139	1.03 675	9.99 817	45	5	11.8	11.7	11.6	11.5
16	8.96 280	137	8.96 464	138	1.03 536	9.99 816	44	10	23.5	23.3	23.2	23.0
17	8.96 417	136	8.96 602	137	1.03 398	9.99 815	43	15	35.2	35.0	34.8	34.5
18	8.96 553	136	8.96 739	138	1.03 261	9.99 814	42	20	47.0	46.7	46.3	46.0
19	8.96 689	136	8.96 877	136	1.03 123	9.99 813	41	25	58.8	58.3	57.9	57.5
20	8.96 825	135	8.97 013	137	1.02 987	9.99 812	40	30	70.5	70.0	69.5	69.0
21	8.96 960	135	8.97 150	135	1.02 850	9.99 810	39	35	82.2	81.7	81.1	80.5
22	8.97 095	134	8.97 285	136	1.02 715	9.99 809	38	40	94.0	93.3	92.7	92.0
23	8.97 229	134	8.97 421	135	1.02 579	9.99 808	37	45	105.8	105.0	104.2	103.5
24	8.97 363	133	8.97 556	135	1.02 444	9.99 807	36	50	117.5	116.7	115.8	115.0
25	8.97 496	133	8.97 691	134	1.02 309	9.99 806	35	55	129.2	128.3	127.4	126.5
26	8.97 629	133	8.97 825	134	1.02 175	9.99 804	34					
27	8.97 762	132	8.97 959	133	1.02 041	9.99 803	33	"	137	136	135	134
28	8.97 894	132	8.98 092	133	1.01 908	9.99 802	32	5	11.4	11.3	11.2	11.2
29	8.98 026	131	8.98 225	133	1.01 775	9.99 801	31	10	22.8	22.7	22.5	22.3
30	8.98 157	131	8.98 358	132	1.01 642	9.99 800	30	15	34.2	34.0	33.8	33.5
31	8.98 288	131	8.98 490	132	1.01 510	9.99 798	29	20	45.7	45.3	45.0	44.7
32	8.98 419	130	8.98 622	131	1.01 378	9.99 797	28	25	57.1	56.7	56.2	55.8
33	8.98 549	130	8.98 753	131	1.01 247	9.99 796	27	30	68.5	68.0	67.5	67.0
34	8.98 679	129	8.98 884	131	1.01 116	9.99 795	26	35	79.9	79.3	78.8	78.2
35	8.98 808	129	8.99 015	130	1.00 985	9.99 793	25	40	91.3	90.7	90.0	89.3
36	8.98 937	129	8.99 145	130	1.00 855	9.99 792	24	45	102.8	102.0	101.2	100.5
37	8.99 066	128	8.99 275	130	1.00 725	9.99 791	23	50	114.2	113.3	112.5	111.7
38	8.99 194	128	8.99 405	129	1.00 595	9.99 790	22	55	125.6	124.7	123.8	122.8
39	8.99 322	128	8.99 534	128	1.00 466	9.99 788	21					
40	8.99 450	127	8.99 662	129	1.00 338	9.99 787	20	"	133	132	131	130
41	8.99 577	127	8.99 791	128	1.00 209	9.99 786	19	5	11.1	11.0	10.9	10.8
42	8.99 704	126	8.99 919	127	1.00 081	9.99 785	18	10	22.2	22.0	21.8	21.7
43	8.99 830	126	9.00 046	128	0.99 954	9.99 783	17	15	33.2	33.0	32.8	32.5
44	8.99 956	126	9.00 174	127	0.99 826	9.99 782	16	20	44.3	44.0	43.7	43.3
45	9.00 082	125	9.00 301	126	0.99 699	9.99 781	15	25	55.4	55.0	54.6	54.2
46	9.00 207	125	9.00 427	126	0.99 573	9.99 780	14	30	66.5	66.0	65.5	65.0
47	9.00 332	124	9.00 553	126	0.99 447	9.99 778	13	35	77.6	77.0	76.4	75.8
48	9.00 456	125	9.00 679	126	0.99 321	9.99 777	12	40	88.7	88.0	87.3	86.7
49	9.00 581	123	9.00 805	125	0.99 195	9.99 776	11	45	99.8	99.0	98.2	97.5
50	9.00 704	124	9.00 930	125	0.99 070	9.99 775	10	50	110.8	110.0	109.2	108.3
51	9.00 828	123	9.01 055	124	0.98 945	9.99 773	9	55	121.9	121.0	120.1	119.2
52	9.00 951	123	9.01 179	124	0.98 821	9.99 772	8					
53	9.01 074	122	9.01 303	124	0.98 697	9.99 771	7	"	129	128	127	126
54	9.01 196	122	9.01 427	123	0.98 573	9.99 769	6	5	10.8	10.7	10.6	10.5
55	9.01 318	122	9.01 550	123	0.98 450	9.99 768	5	10	21.5	21.3	21.2	21.0
56	9.01 440	121	9.01 673	123	0.98 327	9.99 767	4	15	32.2	32.0	31.8	31.5
57	9.01 561	121	9.01 796	122	0.98 204	9.99 765	3	20	43.0	42.7	42.3	42.0
58	9.01 682	121	9.01 918	122	0.98 082	9.99 764	2	25	53.8	53.3	52.9	52.5
59	9.01 803	120	9.02 040	122	0.97 960	9.99 763	1	30	64.5	64.0	63.5	63.0
60	9.01 923		9.02 162		0.97 838	9.99 761	0	35	75.2	74.7	74.1	73.5
	L. Cos.	d.	L. Cot.	c.d.	L. Tan.	L. Sin.	'	40	86.0	85.3	84.7	84.0
								45	96.8	96.0	95.2	94.5
								50	107.5	106.7	105.8	105.0
								55	118.2	117.3	116.4	115.5
								"	125	124	123	122
								5	10.4	10.3	10.2	10.2
								10	20.8	20.7	20.5	20.3
								15	31.2	31.0	30.8	30.5
								20	41.7	41.3	41.0	40.7
								25	52.1	51.7	51.2	50.8
								30	62.5	62.0	61.5	61.0
								35	72.9	72.3	71.8	71.2
								40	83.3	82.7	82.0	81.3
								45	93.8	93.0	92.2	91.5
								50	104.2	103.3	102.5	101.7
								55	114.6	113.7	112.8	111.8

'	L. Sin.	d.	L. Tan.	c.d.	L. Cot.	L. Cos.		P.P.
0	9.01 923		9.02 162		0.97 838	9.99 761	60	
1	9.02 043	120	9.02 283	121	0.97 717	9.99 760	59	" 121 120 119
2	9.02 163	120	9.02 404	121	0.97 596	9.99 759	58	5 10.1 10.0 9.9
3	9.02 283	119	9.02 525	120	0.97 475	9.99 757	57	10 20.2 20.0 19.8
4	9.02 402	118	9.02 645	121	0.97 355	9.99 756	56	15 30.2 30.0 29.8
5	9.02 520	119	9.02 766	119	0.97 234	9.99 755	55	20 40.3 40.0 39.7
6	9.02 639	118	9.02 885	120	0.97 115	9.99 753	54	25 50.4 50.0 49.6
7	9.02 757	117	9.03 005	119	0.96 995	9.99 752	53	30 60.5 60.0 59.5
8	9.02 874	118	9.03 124	118	0.96 876	9.99 751	52	35 70.6 70.0 69.4
9	9.02 992	117	9.03 242	119	0.96 758	9.99 749	51	40 80.7 80.0 79.3
10	9.03 109	117	9.03 361	118	0.96 639	9.99 748	50	45 90.8 90.0 89.2
11	9.03 226	116	9.03 479	118	0.96 521	9.99 747	49	50 100.8 100.0 99.2
12	9.03 342	116	9.03 597	117	0.96 403	9.99 745	48	55 110.9 110.0 109.1
13	9.03 458	116	9.03 714	118	0.96 286	9.99 744	47	
14	9.03 574	116	9.03 832	116	0.96 168	9.99 742	46	" 118 117 116
15	9.03 690	115	9.03 948	117	0.96 052	9.99 741	45	5 9.8 9.8 9.7
16	9.03 805	115	9.04 065	116	0.95 935	9.99 740	44	10 19.7 19.5 19.3
17	9.03 920	114	9.04 181	116	0.95 819	9.99 738	43	15 29.5 29.2 29.0
18	9.04 034	115	9.04 297	116	0.95 703	9.99 737	42	20 39.3 39.0 38.7
19	9.04 149	113	9.04 413	115	0.95 587	9.99 736	41	25 49.2 48.8 48.3
20	9.04 262	114	9.04 528	115	0.95 472	9.99 734	40	30 59.0 58.5 58.0
21	9.04 376	114	9.04 643	115	0.95 357	9.99 733	39	35 68.8 68.2 67.7
22	9.04 490	113	9.04 758	115	0.95 242	9.99 731	38	40 78.7 78.0 77.3
23	9.04 603	112	9.04 873	114	0.95 127	9.99 730	37	45 88.5 87.8 87.0
24	9.04 715	113	9.04 987	114	0.95 013	9.99 728	36	50 98.3 97.5 96.7
25	9.04 828	112	9.05 101	113	0.94 899	9.99 727	35	55 108.2 107.2 106.3
26	9.04 940	112	9.05 214	114	0.94 786	9.99 726	34	
27	9.05 052	112	9.05 328	114	0.94 672	9.99 724	33	" 115 114 113
28	9.05 164	111	9.05 441	112	0.94 559	9.99 723	32	5 9.6 9.5 9.4
29	9.05 275	111	9.05 553	113	0.94 447	9.99 721	31	10 19.2 19.0 18.8
30	9.05 386	111	9.05 666	112	0.94 334	9.99 720	30	15 28.8 28.5 28.2
31	9.05 497	110	9.05 778	112	0.94 222	9.99 718	29	20 38.3 38.0 37.7
32	9.05 607	110	9.05 890	112	0.94 110	9.99 717	28	25 47.9 47.5 47.1
33	9.05 717	110	9.06 002	111	0.93 998	9.99 716	27	30 57.5 57.0 56.5
34	9.05 827	110	9.06 113	111	0.93 887	9.99 714	26	35 67.1 66.5 65.9
35	9.05 937	109	9.06 224	111	0.93 776	9.99 713	25	40 76.7 76.0 75.3
36	9.06 046	109	9.06 335	110	0.93 665	9.99 711	24	45 86.2 85.5 84.8
37	9.06 155	109	9.06 445	111	0.93 555	9.99 710	23	50 95.8 95.0 94.2
38	9.06 264	108	9.06 556	110	0.93 444	9.99 708	22	55 105.4 104.5 103.6
39	9.06 372	109	9.06 666	109	0.93 334	9.99 707	21	
40	9.06 481	108	9.06 775	110	0.93 225	9.99 705	20	" 112 111 110
41	9.06 589	107	9.06 885	109	0.93 115	9.99 704	19	5 9.3 9.2 9.2
42	9.06 696	108	9.06 994	109	0.93 006	9.99 702	18	10 18.7 18.5 18.3
43	9.06 804	107	9.07 103	108	0.92 897	9.99 701	17	15 28.0 27.8 27.5
44	9.06 911	107	9.07 211	109	0.92 789	9.99 699	16	20 37.3 37.0 36.7
45	9.07 018	106	9.07 320	108	0.92 680	9.99 698	15	25 46.7 46.2 45.8
46	9.07 124	107	9.07 428	108	0.92 572	9.99 696	14	30 56.0 55.5 55.0
47	9.07 231	106	9.07 536	107	0.92 464	9.99 695	13	35 65.3 64.8 64.2
48	9.07 337	105	9.07 643	108	0.92 357	9.99 693	12	40 74.7 74.0 73.3
49	9.07 442	106	9.07 751	107	0.92 249	9.99 692	11	45 84.0 83.2 82.5
50	9.07 548	105	9.07 858	106	0.92 142	9.99 690	10	50 93.3 92.5 91.7
51	9.07 653	105	9.07 964	107	0.92 036	9.99 689	9	55 102.7 101.8 100.8
52	9.07 758	105	9.08 071	106	0.91 929	9.99 687	8	
53	9.07 863	105	9.08 177	106	0.91 823	9.99 686	7	" 109 108 107
54	9.07 968	105	9.08 283	106	0.91 717	9.99 684	6	5 9.1 9.0 8.9
55	9.08 072	104	9.08 389	106	0.91 611	9.99 683	5	10 18.2 18.0 17.8
56	9.08 176	104	9.08 495	105	0.91 505	9.99 681	4	15 27.2 27.0 26.8
57	9.08 280	103	9.08 600	105	0.91 400	9.99 680	3	20 36.3 36.0 35.7
58	9.08 383	103	9.08 705	105	0.91 295	9.99 678	2	25 45.4 45.0 44.6
59	9.08 486	103	9.08 810	104	0.91 190	9.99 677	1	30 54.5 54.0 53.5
60	9.08 589		9.08 914		0.91 086	9.99 675	0	35 63.6 63.0 62.4
	L. Cos.	d.	L. Cot.	c.d.	L. Tan.	L. Sin.	'	40 72.7 72.0 71.3
								45 81.8 81.0 80.2
								50 90.8 90.0 89.2
								55 99.9 99.0 98.1
								" 106 105 104
								5 8.8 8.8 8.7
								10 17.7 17.5 17.3
								15 26.5 26.2 26.0
								20 35.3 35.0 34.7
								25 44.2 43.8 43.3
								30 53.0 52.5 52.0
								35 61.8 61.2 60.7
								40 70.7 70.0 69.3
								45 79.5 78.8 78.0
								50 88.3 87.5 86.7
								55 97.2 96.2 95.3
								P.P.

	L. Sin.	d.	L. Tan.	c.d.	L. Cot.	L. Cos.		P.P.
0	9.08 589		9.08 914		0.91 086	9.99 675	60	
1	9.08 692	103	9.09 019	105	0.90 981	9.99 674	59	
2	9.08 795	103	9.09 123	104	0.90 877	9.99 672	58	
3	9.08 897	102	9.09 227	104	0.90 773	9.99 670	57	" 104 103 102
4	9.08 999	102	9.09 330	103	0.90 670	9.99 669	56	5 8.7 8.6 8.5
5	9.09 101	101	9.09 434	104	0.90 566	9.99 667	55	10 17.3 17.2 17.0
6	9.09 202	102	9.09 537	103	0.90 463	9.99 666	54	15 26.0 25.8 25.5
7	9.09 304	101	9.09 640	102	0.90 360	9.99 664	53	20 34.7 34.3 34.0
8	9.09 405	101	9.09 742	103	0.90 258	9.99 663	52	25 43.3 42.9 42.5
9	9.09 506	100	9.09 845	102	0.90 155	9.99 661	51	30 52.0 51.5 51.0
10	9.09 606	101	9.09 947	102	0.90 053	9.99 659	50	35 60.7 60.1 59.5
11	9.09 707	100	9.10 049	101	0.89 951	9.99 658	49	40 69.3 68.7 68.0
12	9.09 807	100	9.10 150	102	0.89 850	9.99 656	48	45 78.0 77.2 76.5
13	9.09 907	99	9.10 252	101	0.89 748	9.99 655	47	50 86.7 85.8 85.0
14	9.10 006	100	9.10 353	101	0.89 647	9.99 653	46	55 95.3 94.4 93.5
15	9.10 106	99	9.10 454	101	0.89 546	9.99 651	45	
16	9.10 205	99	9.10 555	101	0.89 445	9.99 650	44	" 101 100 99
17	9.10 304	98	9.10 656	100	0.89 344	9.99 648	43	5 8.4 8.3 8.2
18	9.10 402	99	9.10 756	100	0.89 244	9.99 647	42	10 16.8 16.7 16.5
19	9.10 501	98	9.10 856	100	0.89 144	9.99 645	41	15 25.2 25.0 24.8
20	9.10 599	98	9.10 956	100	0.89 044	9.99 643	40	20 33.7 33.3 33.0
21	9.10 697	98	9.11 056	99	0.88 944	9.99 642	39	25 42.1 41.7 41.2
22	9.10 795	98	9.11 155	99	0.88 845	9.99 640	38	30 50.5 50.0 49.5
23	9.10 893	97	9.11 254	99	0.88 746	9.99 638	37	35 58.9 58.3 57.8
24	9.10 990	97	9.11 353	99	0.88 647	9.99 637	36	40 67.3 66.7 66.0
25	9.11 087	97	9.11 452	99	0.88 548	9.99 635	35	45 75.8 75.0 74.2
26	9.11 184	97	9.11 551	99	0.88 449	9.99 633	34	50 84.2 83.3 82.5
27	9.11 281	96	9.11 649	98	0.88 351	9.99 632	33	55 92.6 91.7 90.8
28	9.11 377	97	9.11 747	98	0.88 253	9.99 630	32	
29	9.11 474	96	9.11 845	98	0.88 155	9.99 629	31	" 98 97 96
30	9.11 570	96	9.11 943	97	0.88 057	9.99 627	30	5 8.2 8.1 8.0
31	9.11 666	95	9.12 040	98	0.87 960	9.99 625	29	10 16.3 16.2 16.0
32	9.11 761	96	9.12 138	97	0.87 862	9.99 624	28	15 24.5 24.2 24.0
33	9.11 857	95	9.12 235	97	0.87 765	9.99 622	27	20 32.7 32.3 32.0
34	9.11 952	95	9.12 332	96	0.87 668	9.99 620	26	25 40.8 40.4 40.0
35	9.12 047	95	9.12 428	97	0.87 572	9.99 618	25	30 49.0 48.5 48.0
36	9.12 142	94	9.12 525	96	0.87 475	9.99 617	24	35 57.2 56.6 56.0
37	9.12 236	95	9.12 621	96	0.87 379	9.99 615	23	40 65.3 64.7 64.0
38	9.12 331	94	9.12 717	96	0.87 283	9.99 613	22	45 73.5 72.8 72.0
39	9.12 425	94	9.12 813	96	0.87 187	9.99 612	21	50 81.7 80.8 80.0
40	9.12 519	93	9.12 909	95	0.87 091	9.99 610	20	55 89.8 88.9 88.0
41	9.12 612	94	9.13 004	95	0.86 996	9.99 608	19	
42	9.12 706	93	9.13 099	95	0.86 901	9.99 607	18	" 95 94 93
43	9.12 799	93	9.13 194	95	0.86 806	9.99 605	17	5 7.9 7.8 7.8
44	9.12 892	93	9.13 289	95	0.86 711	9.99 603	16	10 15.8 15.7 15.5
45	9.12 985	93	9.13 384	94	0.86 616	9.99 601	15	15 23.8 23.5 23.2
46	9.13 078	93	9.13 478	95	0.86 522	9.99 600	14	20 31.7 31.3 31.0
47	9.13 171	92	9.13 573	94	0.86 427	9.99 598	13	25 39.6 39.2 38.8
48	9.13 263	92	9.13 667	94	0.86 333	9.99 596	12	30 47.5 47.0 46.5
49	9.13 355	92	9.13 761	93	0.86 239	9.99 595	11	35 55.4 54.8 54.2
50	9.13 447	92	9.13 854	94	0.86 146	9.99 593	10	40 63.3 62.7 62.0
51	9.13 539	91	9.13 948	93	0.86 052	9.99 591	9	45 71.2 70.5 69.8
52	9.13 630	92	9.14 041	93	0.85 959	9.99 589	8	50 79.2 78.3 77.5
53	9.13 722	91	9.14 134	93	0.85 866	9.99 588	7	55 87.1 86.2 85.2
54	9.13 813	91	9.14 227	93	0.85 773	9.99 586	6	
55	9.13 904	90	9.14 320	92	0.85 680	9.99 584	5	" 92 91 90
56	9.13 994	91	9.14 412	92	0.85 588	9.99 582	4	5 7.7 7.6 7.5
57	9.14 085	90	9.14 504	93	0.85 496	9.99 581	3	10 15.3 15.2 15.0
58	9.14 175	91	9.14 597	91	0.85 403	9.99 579	2	15 23.0 22.8 22.5
59	9.14 266	90	9.14 688	92	0.85 312	9.99 577	1	20 30.7 30.3 30.0
60	9.14 356		9.14 780		0.85 220	9.99 575	0	25 38.3 37.9 37.5
	L. Cos.	d.	L. Cot.	c.d.	L. Tan.	L. Sin.		P.P.

	L. Sin.	d.	L. Tan.	c.d.	L. Cot.	L. Cos.		P.P.
0	9.14 356		9.14 780		0.85 220	9.99 575	60	
1	9.14 445	89	9.14 872	92	0.85 128	9.99 574	59	
2	9.14 535	90	9.14 963	91	0.85 037	9.99 572	58	
3	9.14 624	89	9.15 054	91	0.84 946	9.99 570	57	
4	9.14 714	90	9.15 145	91	0.84 855	9.99 568	56	" 91 90 89
5	9.14 803	89	9.15 236	91	0.84 764	9.99 566	55	5 7.6 7.5 7.4
6	9.14 891	88	9.15 327	91	0.84 673	9.99 565	54	10 15.2 15.0 14.8
7	9.14 980	89	9.15 417	90	0.84 583	9.99 563	53	15 22.8 22.5 22.2
8	9.15 069	88	9.15 508	90	0.84 492	9.99 561	52	20 30.3 30.0 29.7
9	9.15 157	88	9.15 598	90	0.84 402	9.99 559	51	25 37.9 37.5 37.1
10	9.15 245	88	9.15 688	90	0.84 312	9.99 557	50	30 45.5 45.0 44.5
11	9.15 333	88	9.15 777	89	0.84 223	9.99 556	49	35 53.1 52.5 51.9
12	9.15 421	87	9.15 867	89	0.84 133	9.99 554	48	40 60.7 60.0 59.3
13	9.15 508	88	9.15 956	90	0.84 044	9.99 552	47	45 68.2 67.5 66.8
14	9.15 596	87	9.16 046	89	0.83 954	9.99 550	46	50 75.8 75.0 74.2
15	9.15 683	87	9.16 135	89	0.83 865	9.99 548	45	55 83.4 82.5 81.6
16	9.15 770	87	9.16 224	88	0.83 776	9.99 546	44	
17	9.15 857	87	9.16 312	89	0.83 688	9.99 545	43	
18	9.15 944	86	9.16 401	88	0.83 599	9.99 543	42	
19	9.16 030	86	9.16 489	88	0.83 511	9.99 541	41	" 88 87 86
20	9.16 116	87	9.16 577	88	0.83 423	9.99 539	40	5 7.3 7.2 7.2
21	9.16 203	86	9.16 665	88	0.83 335	9.99 537	39	10 14.7 14.5 14.3
22	9.16 289	85	9.16 753	88	0.83 247	9.99 535	38	15 22.0 21.8 21.5
23	9.16 374	86	9.16 841	87	0.83 159	9.99 533	37	20 29.3 29.0 28.7
24	9.16 460	85	9.16 928	88	0.83 072	9.99 532	36	25 36.7 36.2 35.8
25	9.16 545	86	9.17 016	87	0.82 984	9.99 530	35	30 44.0 43.5 43.0
26	9.16 631	85	9.17 103	87	0.82 897	9.99 528	34	35 51.3 50.8 50.2
27	9.16 716	85	9.17 190	87	0.82 810	9.99 526	33	40 58.7 58.0 57.3
28	9.16 801	85	9.17 277	86	0.82 723	9.99 524	32	45 66.0 65.2 64.5
29	9.16 886	84	9.17 363	87	0.82 637	9.99 522	31	50 73.3 72.5 71.7
30	9.16 970	85	9.17 450	86	0.82 550	9.99 520	30	55 80.7 79.8 78.8
31	9.17 055	84	9.17 536	86	0.82 464	9.99 518	29	
32	9.17 139	84	9.17 622	86	0.82 378	9.99 517	28	
33	9.17 223	84	9.17 708	86	0.82 292	9.99 515	27	" 85 84 83
34	9.17 307	84	9.17 794	86	0.82 206	9.99 513	26	5 7.1 7.0 6.9
35	9.17 391	83	9.17 880	85	0.82 120	9.99 511	25	10 14.2 14.0 13.8
36	9.17 474	84	9.17 965	86	0.82 035	9.99 509	24	15 21.2 21.0 20.8
37	9.17 558	83	9.18 051	85	0.81 949	9.99 507	23	20 28.3 28.0 27.7
38	9.17 641	83	9.18 136	85	0.81 864	9.99 505	22	25 35.4 35.0 34.6
39	9.17 724	83	9.18 221	85	0.81 779	9.99 503	21	30 42.5 42.0 41.5
40	9.17 807	83	9.18 306	85	0.81 694	9.99 501	20	35 49.6 49.0 48.4
41	9.17 890	83	9.18 391	84	0.81 609	9.99 499	19	40 56.7 56.0 55.3
42	9.17 973	82	9.18 475	85	0.81 525	9.99 497	18	45 63.8 63.0 62.2
43	9.18 055	82	9.18 560	84	0.81 440	9.99 495	17	50 70.8 70.0 69.2
44	9.18 137	83	9.18 644	84	0.81 356	9.99 494	16	55 77.9 77.0 76.1
45	9.18 220	82	9.18 728	84	0.81 272	9.99 492	15	
46	9.18 302	81	9.18 812	84	0.81 188	9.99 490	14	
47	9.18 383	82	9.18 896	83	0.81 104	9.99 488	13	
48	9.18 465	82	9.18 979	84	0.81 021	9.99 486	12	
49	9.18 547	81	9.19 063	83	0.80 937	9.99 484	11	" 82 81 80
50	9.18 628	81	9.19 146	83	0.80 854	9.99 482	10	5 6.8 6.8 6.7
51	9.18 709	81	9.19 229	83	0.80 771	9.99 480	9	10 13.7 13.5 13.3
52	9.18 790	81	9.19 312	83	0.80 688	9.99 478	8	15 20.5 20.2 20.0
53	9.18 871	81	9.19 395	83	0.80 605	9.99 476	7	20 27.3 27.0 26.7
54	9.18 952	81	9.19 478	83	0.80 522	9.99 474	6	25 34.2 33.8 33.3
55	9.19 033	80	9.19 561	82	0.80 439	9.99 472	5	30 41.0 40.5 40.0
56	9.19 113	80	9.19 643	82	0.80 357	9.99 470	4	35 47.8 47.2 46.7
57	9.19 193	80	9.19 725	82	0.80 275	9.99 468	3	40 54.7 54.0 53.3
58	9.19 273	80	9.19 807	82	0.80 193	9.99 466	2	45 61.5 60.8 60.0
59	9.19 353	80	9.19 889	82	0.80 111	9.99 464	1	50 68.3 67.5 66.7
60	9.19 433		9.19 971		0.80 029	9.99 462	0	55 75.2 74.2 73.3
	L. Cos.	d.	L. Cot.	c.d.	L. Tan.	L. Sin.		P.P.

'	L. Sin.	d.	L. Tan.	c.d.	L. Cot.	L. Cos.		P.P.
0	9.19 433		9.19 971		0.80 029	9.99 462	60	
1	9.19 513	80	9.20 053	82	0.79 947	9.99 460	59	
2	9.19 592	79	9.20 134	81	0.79 866	9.99 458	58	
3	9.19 672	80	9.20 216	82	0.79 784	9.99 456	57	
4	9.19 751	79	9.20 297	81	0.79 703	9.99 454	56	" 82 81 80
5	9.19 830	79	9.20 378	81	0.79 622	9.99 452	55	5 6.8 6.8 6.7
6	9.19 909	79	9.20 459	81	0.79 541	9.99 450	54	10 13.7 13.5 13.3
7	9.19 988	79	9.20 540	81	0.79 460	9.99 448	53	15 20.5 20.2 20.0
8	9.20 067	78	9.20 621	80	0.79 379	9.99 446	52	20 27.3 27.0 26.7
9	9.20 145	78	9.20 701	80	0.79 299	9.99 444	51	25 34.2 33.8 33.3
10	9.20 223	78	9.20 782	81	0.79 218	9.99 442	50	30 41.0 40.5 40.0
11	9.20 302	79	9.20 862	80	0.79 138	9.99 440	49	35 47.8 47.2 46.7
12	9.20 380	78	9.20 942	80	0.79 058	9.99 438	48	40 54.7 54.0 53.3
13	9.20 458	77	9.21 022	80	0.78 978	9.99 436	47	45 61.5 60.8 60.0
14	9.20 535	78	9.21 102	80	0.78 898	9.99 434	46	50 68.3 67.5 66.7
15	9.20 613	78	9.21 182	79	0.78 818	9.99 432	45	55 75.2 74.2 73.3
16	9.20 691	77	9.21 261	80	0.78 739	9.99 429	44	
17	9.20 768	77	9.21 341	79	0.78 659	9.99 427	43	
18	9.20 845	77	9.21 420	79	0.78 580	9.99 425	42	
19	9.20 922	77	9.21 499	79	0.78 501	9.99 423	41	" 79 78 77
20	9.20 999	77	9.21 578	79	0.78 422	9.99 421	40	5 6.6 6.5 6.4
21	9.21 076	77	9.21 657	79	0.78 343	9.99 419	39	10 13.2 13.0 12.8
22	9.21 153	76	9.21 736	78	0.78 264	9.99 417	38	15 19.8 19.5 19.2
23	9.21 229	77	9.21 814	79	0.78 186	9.99 415	37	20 26.3 26.0 25.7
24	9.21 306	76	9.21 893	78	0.78 107	9.99 413	36	25 32.9 32.5 32.1
25	9.21 382	76	9.21 971	78	0.78 029	9.99 411	35	30 39.5 39.0 38.5
26	9.21 458	76	9.22 049	78	0.77 951	9.99 409	34	35 46.1 45.5 44.9
27	9.21 534	76	9.22 127	78	0.77 873	9.99 407	33	40 52.7 52.0 51.3
28	9.21 610	75	9.22 205	78	0.77 795	9.99 404	32	45 59.2 58.5 57.8
29	9.21 685	76	9.22 283	78	0.77 717	9.99 402	31	50 65.8 65.0 64.2
30	9.21 761	75	9.22 361	77	0.77 639	9.99 400	30	55 72.4 71.5 70.6
31	9.21 836	76	9.22 438	78	0.77 562	9.99 398	29	
32	9.21 912	75	9.22 516	77	0.77 484	9.99 396	28	
33	9.21 987	75	9.22 593	77	0.77 407	9.99 394	27	
34	9.22 062	75	9.22 670	77	0.77 330	9.99 392	26	" 76 75 74
35	9.22 137	74	9.22 747	77	0.77 253	9.99 390	25	5 6.3 6.2 6.2
36	9.22 211	75	9.22 824	77	0.77 176	9.99 388	24	10 12.7 12.5 12.3
37	9.22 286	75	9.22 901	76	0.77 099	9.99 385	23	15 19.0 18.8 18.5
38	9.22 361	74	9.22 977	77	0.77 023	9.99 383	22	20 25.3 25.0 24.7
39	9.22 435	74	9.23 054	76	0.76 946	9.99 381	21	25 31.7 31.2 30.8
40	9.22 509	74	9.23 130	76	0.76 870	9.99 379	20	30 38.0 37.5 37.0
41	9.22 583	74	9.23 206	77	0.76 794	9.99 377	19	35 44.3 43.8 43.2
42	9.22 657	74	9.23 283	76	0.76 717	9.99 375	18	40 50.7 50.0 49.3
43	9.22 731	74	9.23 359	76	0.76 641	9.99 372	17	45 57.0 56.2 55.5
44	9.22 805	73	9.23 435	75	0.76 565	9.99 370	16	50 63.3 62.5 61.7
45	9.22 878	74	9.23 510	76	0.76 490	9.99 368	15	55 69.7 68.8 67.8
46	9.22 952	73	9.23 586	75	0.76 414	9.99 366	14	
47	9.23 025	73	9.23 661	76	0.76 339	9.99 364	13	
48	9.23 098	73	9.23 737	75	0.76 263	9.99 362	12	
49	9.23 171	73	9.23 812	75	0.76 188	9.99 359	11	" 73 72 71
50	9.23 244	73	9.23 887	75	0.76 113	9.99 357	10	5 6.1 6.0 5.9
51	9.23 317	73	9.23 962	75	0.76 038	9.99 355	9	10 12.2 12.0 11.8
52	9.23 390	72	9.24 037	75	0.75 963	9.99 353	8	15 18.2 18.0 17.8
53	9.23 462	73	9.24 112	74	0.75 888	9.99 351	7	20 24.3 24.0 23.7
54	9.23 535	72	9.24 186	75	0.75 814	9.99 348	6	25 30.4 30.0 29.6
55	9.23 607	72	9.24 261	74	0.75 739	9.99 346	5	30 36.5 36.0 35.5
56	9.23 679	73	9.24 335	75	0.75 665	9.99 344	4	35 42.6 42.0 41.4
57	9.23 752	71	9.24 410	74	0.75 590	9.99 342	3	40 48.7 48.0 47.3
58	9.23 823	72	9.24 484	74	0.75 516	9.99 340	2	45 54.8 54.0 53.2
59	9.23 895	72	9.24 558	74	0.75 442	9.99 337	1	50 60.8 60.0 59.2
60	9.23 967		9.24 632		0.75 368	9.99 335	0	55 66.9 66.0 65.1
'	L. Cos.	d.	L. Cot.	c.d.	L. Tan.	L. Sin.		P.P.

	L. Sin.	d.	L. Tan.	c.d.	L. Cot.	L. Cos.	d.		P.P.
0	9.23 967		9.24 632		0.75 368	9.99 335		60	
1	9.24 039	72	9.24 706	74	0.75 294	9.99 333	2	59	
2	9.24 110	71	9.24 779	73	0.75 221	9.99 331	2	58	
3	9.24 181	71	9.24 853	74	0.75 147	9.99 328	3	57	" 74 73 72
4	9.24 253	72	9.24 926	73	0.75 074	9.99 326	2	56	5 6.2 6.1 6.0
5	9.24 324	71	9.25 000	74	0.75 000	9.99 324	2	55	10 12.3 12.2 12.0
6	9.24 395	71	9.25 073	73	0.74 927	9.99 322	2	54	15 18.5 18.2 18.0
7	9.24 466	70	9.25 146	73	0.74 854	9.99 319	3	53	20 24.7 24.3 24.0
8	9.24 536	71	9.25 219	73	0.74 781	9.99 317	2	52	25 30.8 30.4 30.0
9	9.24 607	71	9.25 292	73	0.74 708	9.99 315	2	51	30 37.0 36.5 36.0
10	9.24 677	70	9.25 365	73	0.74 635	9.99 313	2	50	35 43.2 42.6 42.0
11	9.24 748	71	9.25 437	72	0.74 563	9.99 310	3	49	40 49.3 48.7 48.0
12	9.24 818	70	9.25 510	73	0.74 490	9.99 308	2	48	45 55.5 54.8 54.0
13	9.24 888	70	9.25 582	72	0.74 418	9.99 306	2	47	50 61.7 60.8 60.0
14	9.24 958	70	9.25 655	73	0.74 345	9.99 304	3	46	55 67.8 66.9 66.0
15	9.25 028	70	9.25 727	72	0.74 273	9.99 301	2	45	
16	9.25 098	70	9.25 799	72	0.74 201	9.99 299	2	44	
17	9.25 168	69	9.25 871	72	0.74 129	9.99 297	3	43	
18	9.25 237	70	9.25 943	72	0.74 057	9.99 294	2	42	" 71 70 69
19	9.25 307	69	9.26 015	71	0.73 985	9.99 292	2	41	5 5.9 5.8 5.8
20	9.25 376	69	9.26 086	72	0.73 914	9.99 290	2	40	10 11.8 11.7 11.5
21	9.25 445	69	9.26 158	71	0.73 842	9.99 288	3	39	15 17.8 17.5 17.2
22	9.25 514	69	9.26 229	72	0.73 771	9.99 285	2	38	20 23.7 23.3 23.0
23	9.25 583	69	9.26 301	71	0.73 699	9.99 283	2	37	25 29.6 29.2 28.8
24	9.25 652	69	9.26 372	71	0.73 628	9.99 281	3	36	30 35.5 35.0 34.5
25	9.25 721	69	9.26 443	71	0.73 557	9.99 278	2	35	35 41.4 40.8 40.2
26	9.25 790	68	9.26 514	71	0.73 486	9.99 276	2	34	40 47.3 46.7 46.0
27	9.25 858	69	9.26 585	70	0.73 415	9.99 274	3	33	45 53.2 52.5 51.8
28	9.25 927	68	9.26 655	71	0.73 345	9.99 271	2	32	50 59.2 58.3 57.5
29	9.25 995	68	9.26 726	71	0.73 274	9.99 269	2	31	55 65.1 64.2 63.2
30	9.26 063	68	9.26 797	70	0.73 203	9.99 267	3	30	
31	9.26 131	68	9.26 867	70	0.73 133	9.99 264	2	29	
32	9.26 199	68	9.26 937	71	0.73 063	9.99 262	2	28	" 68 67 66
33	9.26 267	68	9.27 008	70	0.72 992	9.99 260	3	27	5 5.7 5.6 5.5
34	9.26 335	68	9.27 078	70	0.72 922	9.99 257	2	26	10 11.3 11.2 11.0
35	9.26 403	67	9.27 148	70	0.72 852	9.99 255	3	25	15 17.0 16.8 16.5
36	9.26 470	68	9.27 218	70	0.72 782	9.99 252	2	24	20 22.7 22.3 22.0
37	9.26 538	67	9.27 288	69	0.72 712	9.99 250	2	23	25 28.3 27.9 27.5
38	9.26 605	67	9.27 357	70	0.72 643	9.99 248	3	22	30 34.0 33.5 33.0
39	9.26 672	67	9.27 427	69	0.72 573	9.99 245	2	21	35 39.7 39.1 38.5
40	9.26 739	67	9.27 496	70	0.72 504	9.99 243	2	20	40 45.3 44.7 44.0
41	9.26 806	67	9.27 566	69	0.72 434	9.99 241	3	19	45 51.0 50.2 49.5
42	9.26 873	67	9.27 635	69	0.72 365	9.99 238	2	18	50 56.7 55.8 55.0
43	9.26 940	67	9.27 704	69	0.72 296	9.99 236	3	17	55 62.3 61.4 60.5
44	9.27 007	66	9.27 773	69	0.72 227	9.99 233	2	16	
45	9.27 073	67	9.27 842	69	0.72 158	9.99 231	2	15	" 65 3 2
46	9.27 140	66	9.27 911	69	0.72 089	9.99 229	3	14	5 5.4 5.3 5.2
47	9.27 206	67	9.27 980	69	0.72 020	9.99 226	2	13	10 10.8 10.5 10.3
48	9.27 273	66	9.28 049	68	0.71 951	9.99 224	3	12	15 16.2 16.0 15.8
49	9.27 339	66	9.28 117	69	0.71 883	9.99 221	2	11	20 21.7 21.4 21.0
50	9.27 405	66	9.28 186	68	0.71 814	9.99 219	3	10	25 27.1 26.7 26.3
51	9.27 471	66	9.28 254	69	0.71 746	9.99 217	2	9	30 32.5 32.1 31.7
52	9.27 537	65	9.28 323	68	0.71 677	9.99 214	3	8	35 37.9 37.4 37.0
53	9.27 602	66	9.28 391	68	0.71 609	9.99 212	2	7	40 43.3 42.8 42.3
54	9.27 668	66	9.28 459	68	0.71 541	9.99 209	3	6	45 48.8 48.2 47.7
55	9.27 734	65	9.28 527	68	0.71 473	9.99 207	2	5	50 54.2 53.5 52.8
56	9.27 799	65	9.28 595	67	0.71 405	9.99 204	2	4	55 59.6 58.8 58.0
57	9.27 864	66	9.28 662	68	0.71 338	9.99 202	3	3	
58	9.27 930	65	9.28 730	68	0.71 270	9.99 200	2	2	
59	9.27 995	65	9.28 798	67	0.71 202	9.99 197	2	1	
60	9.28 060		9.28 865		0.71 135	9.99 195		0	
	L. Cos.	d.	L. Cot.	c.d.	L. Tan.	L. Sin.	d.		P.P.

'	L. Sin.	d.	L. Tan.	c.d.	L. Cot.	L. Cos.	d.		P.P.
0	9.28 060		9.28 865		0.71 135	9.99 195		60	
1	9.28 125	65	9.28 933	68	0.71 067	9.99 192	3	59	
2	9.28 190	64	9.29 000	67	0.71 000	9.99 190	2	58	
3	9.28 254	65	9.29 067	67	0.70 933	9.99 187	3	57	
4	9.28 319	65	9.29 134	67	0.70 866	9.99 185	2	56	" 68 67 66
5	9.28 384	64	9.29 201	67	0.70 799	9.99 182	3	55	5 5.7 5.6 5.5
6	9.28 448	64	9.29 268	67	0.70 732	9.99 180	2	54	10 11.3 11.2 11.0
7	9.28 512	65	9.29 335	67	0.70 665	9.99 177	3	53	15 17.0 16.8 16.5
8	9.28 577	64	9.29 402	66	0.70 598	9.99 175	2	52	20 22.7 22.3 22.0
9	9.28 641	64	9.29 468	67	0.70 532	9.99 172	3	51	25 28.3 27.9 27.5
10	9.28 705	64	9.29 535	66	0.70 465	9.99 170	2	50	30 34.0 33.5 33.0
11	9.28 769	64	9.29 601	67	0.70 399	9.99 167	3	49	35 39.7 39.1 38.5
12	9.28 833	63	9.29 668	66	0.70 332	9.99 165	2	48	40 45.3 44.7 44.0
13	9.28 896	64	9.29 734	66	0.70 266	9.99 162	3	47	45 51.0 50.2 49.5
14	9.28 960	64	9.29 800	66	0.70 200	9.99 160	2	46	50 56.7 55.8 55.0
15	9.29 024	63	9.29 866	66	0.70 134	9.99 157	3	45	55 62.3 61.4 60.5
16	9.29 087	63	9.29 932	66	0.70 068	9.99 155	2	44	
17	9.29 150	64	9.29 998	66	0.70 002	9.99 152	3	43	
18	9.29 214	63	9.30 064	66	0.69 936	9.99 150	2	42	
19	9.29 277	63	9.30 130	65	0.69 870	9.99 147	3	41	" 65 64 63
20	9.29 340	63	9.30 195	66	0.69 805	9.99 145	2	40	5 5.4 5.3 5.2
21	9.29 403	63	9.30 261	65	0.69 739	9.99 142	3	39	10 10.8 10.7 10.5
22	9.29 466	63	9.30 326	65	0.69 674	9.99 140	2	38	15 16.2 16.0 15.8
23	9.29 529	62	9.30 391	66	0.69 609	9.99 137	3	37	20 21.7 21.3 21.0
24	9.29 591	63	9.30 457	65	0.69 543	9.99 135	2	36	25 27.1 26.7 26.2
25	9.29 654	62	9.30 522	65	0.69 478	9.99 132	3	35	30 32.5 32.0 31.5
26	9.29 716	63	9.30 587	65	0.69 413	9.99 130	2	34	35 37.9 37.3 36.8
27	9.29 779	62	9.30 652	65	0.69 348	9.99 127	3	33	40 43.3 42.7 42.0
28	9.29 841	62	9.30 717	65	0.69 283	9.99 124	2	32	45 48.8 48.0 47.2
29	9.29 903	63	9.30 782	64	0.69 218	9.99 122	3	31	50 54.2 53.3 52.5
30	9.29 966	62	9.30 846	65	0.69 154	9.99 119	2	30	55 59.6 58.7 57.8
31	9.30 028	62	9.30 911	64	0.69 089	9.99 117	3	29	
32	9.30 090	61	9.30 975	65	0.69 025	9.99 114	2	28	" 62 61 60
33	9.30 151	62	9.31 040	64	0.68 960	9.99 112	3	27	5 5.2 5.1 5.0
34	9.30 213	62	9.31 104	64	0.68 896	9.99 109	2	26	10 10.3 10.2 10.0
35	9.30 275	61	9.31 168	65	0.68 832	9.99 106	3	25	15 15.5 15.2 15.0
36	9.30 336	62	9.31 233	64	0.68 767	9.99 104	2	24	20 20.7 20.3 20.0
37	9.30 398	61	9.31 297	64	0.68 703	9.99 101	3	23	25 25.8 25.4 25.0
38	9.30 459	62	9.31 361	64	0.68 639	9.99 099	2	22	30 31.0 30.5 30.0
39	9.30 521	61	9.31 425	64	0.68 575	9.99 096	3	21	35 36.2 35.6 35.0
40	9.30 582	61	9.31 489	63	0.68 511	9.99 093	2	20	40 41.3 40.7 40.0
41	9.30 643	61	9.31 552	64	0.68 448	9.99 091	3	19	45 46.5 45.8 45.0
42	9.30 704	61	9.31 616	63	0.68 384	9.99 088	2	18	50 51.7 50.8 50.0
43	9.30 765	61	9.31 679	64	0.68 321	9.99 086	3	17	55 56.8 55.9 55.0
44	9.30 826	61	9.31 743	63	0.68 257	9.99 083	2	16	
45	9.30 887	60	9.31 806	64	0.68 194	9.99 080	3	15	
46	9.30 947	61	9.31 870	63	0.68 130	9.99 078	2	14	
47	9.31 008	60	9.31 933	63	0.68 067	9.99 075	3	13	
48	9.31 068	61	9.31 996	63	0.68 004	9.99 072	2	12	
49	9.31 129	60	9.32 059	63	0.67 941	9.99 070	3	11	" 59 3 2
50	9.31 189	61	9.32 122	63	0.67 878	9.99 067	2	10	5 4.9 0.2 0.2
51	9.31 250	60	9.32 185	63	0.67 815	9.99 064	3	9	10 9.8 0.5 0.3
52	9.31 310	60	9.32 248	63	0.67 752	9.99 062	2	8	15 14.8 0.8 0.5
53	9.31 370	60	9.32 311	62	0.67 689	9.99 059	3	7	20 19.7 1.0 0.7
54	9.31 430	60	9.32 373	63	0.67 627	9.99 056	2	6	25 24.6 1.2 0.8
55	9.31 490	59	9.32 436	62	0.67 564	9.99 054	3	5	30 29.5 1.5 1.0
56	9.31 549	60	9.32 498	63	0.67 502	9.99 051	2	4	35 34.4 1.8 1.2
57	9.31 609	60	9.32 561	62	0.67 439	9.99 048	3	3	40 39.3 2.0 1.3
58	9.31 669	59	9.32 623	62	0.67 377	9.99 046	2	2	45 44.2 2.2 1.5
59	9.31 728	60	9.32 685	62	0.67 315	9.99 043	3	1	50 49.2 2.5 1.7
60	9.31 788		9.32 747		0.67 253	9.99 040		0	55 54.1 2.8 1.8
	L. Cos.	d.	L. Cot.	c.d.	L. Tan.	L. Sin.	d.	'	P.P.

'	L. Sin.	d.	L. Tan.	c.d.	L. Cot.	L. Cos.	d.		P.P.
0	9.31 788	59	9.32 747	63	0.67 253	9.99 040	2	60	
1	9.31 847	60	9.32 810	62	0.67 190	9.99 038	3	59	
2	9.31 907	59	9.32 872	61	0.67 128	9.99 035	3	58	
3	9.31 966	59	9.32 933	62	0.67 067	9.99 032	2	57	
4	9.32 025	59	9.32 995	62	0.67 005	9.99 030	3	56	" 63 62 61
5	9.32 084	59	9.33 057	62	0.66 943	9.99 027	3	55	5 5.2 5.2 5.1
6	9.32 143	59	9.33 119	61	0.66 881	9.99 024	2	54	10 10.5 10.3 10.2
7	9.32 202	59	9.33 180	62	0.66 820	9.99 022	3	53	15 15.8 15.5 15.2
8	9.32 261	58	9.33 242	61	0.66 758	9.99 019	3	52	20 21.0 20.7 20.3
9	9.32 319	59	9.33 303	62	0.66 697	9.99 016	3	51	25 26.2 25.8 25.4
10	9.32 378	59	9.33 365	61	0.66 635	9.99 013	2	50	30 31.5 31.0 30.5
11	9.32 437	58	9.33 426	61	0.66 574	9.99 011	3	49	35 36.8 36.2 35.6
12	9.32 495	58	9.33 487	61	0.66 513	9.99 008	3	48	40 42.0 41.3 40.7
13	9.32 553	59	9.33 548	61	0.66 452	9.99 005	3	47	45 47.2 46.5 45.8
14	9.32 612	58	9.33 609	61	0.66 391	9.99 002	2	46	50 52.5 51.7 50.8
15	9.32 670	58	9.33 670	61	0.66 330	9.99 000	3	45	55 57.8 56.8 55.9
16	9.32 728	58	9.33 731	61	0.66 269	9.98 997	3	44	
17	9.32 786	58	9.33 792	61	0.66 208	9.98 994	3	43	
18	9.32 844	58	9.33 853	60	0.66 147	9.98 991	2	42	
19	9.32 902	58	9.33 913	61	0.66 087	9.98 989	3	41	" 60 59 58
20	9.32 960	58	9.33 974	60	0.66 026	9.98 986	3	40	5 5.0 4.9 4.8
21	9.33 018	57	9.34 034	61	0.65 966	9.98 983	3	39	10 10.0 9.8 9.7
22	9.33 075	58	9.34 095	60	0.65 905	9.98 980	2	38	15 15.0 14.8 14.5
23	9.33 133	57	9.34 155	60	0.65 845	9.98 978	3	37	20 20.0 19.7 19.3
24	9.33 190	58	9.34 215	61	0.65 785	9.98 975	3	36	25 25.0 24.6 24.2
25	9.33 248	57	9.34 276	60	0.65 724	9.98 972	3	35	30 30.0 29.5 29.0
26	9.33 305	57	9.34 336	60	0.65 664	9.98 969	2	34	35 35.0 34.4 33.8
27	9.33 362	58	9.34 396	60	0.65 604	9.98 967	3	33	40 40.0 39.3 38.7
28	9.33 420	57	9.34 456	60	0.65 544	9.98 964	3	32	45 45.0 44.2 43.5
29	9.33 477	57	9.34 516	60	0.65 484	9.98 961	3	31	50 50.0 49.2 48.3
30	9.33 534	57	9.34 576	59	0.65 424	9.98 958	3	30	55 55.0 54.1 53.2
31	9.33 591	56	9.34 635	60	0.65 365	9.98 955	2	29	
32	9.33 647	57	9.34 695	60	0.65 305	9.98 953	3	28	
33	9.33 704	57	9.34 755	59	0.65 245	9.98 950	3	27	" 57 56 55
34	9.33 761	57	9.34 814	60	0.65 186	9.98 947	3	26	5 4.8 4.7 4.6
35	9.33 818	56	9.34 874	59	0.65 126	9.98 944	3	25	10 9.5 9.3 9.2
36	9.33 874	57	9.34 933	59	0.65 067	9.98 941	3	24	15 14.2 14.0 13.8
37	9.33 931	56	9.34 992	59	0.65 008	9.98 938	2	23	20 19.0 18.7 18.3
38	9.33 987	56	9.35 051	60	0.64 949	9.98 936	3	22	25 23.8 23.3 22.9
39	9.34 043	57	9.35 111	59	0.64 889	9.98 933	3	21	30 28.5 28.0 27.5
40	9.34 100	56	9.35 170	59	0.64 830	9.98 930	3	20	35 33.2 32.7 32.1
41	9.34 156	56	9.35 229	59	0.64 771	9.98 927	3	19	40 38.0 37.3 36.7
42	9.34 212	56	9.35 288	59	0.64 712	9.98 924	3	18	45 42.8 42.0 41.2
43	9.34 268	56	9.35 347	58	0.64 653	9.98 921	2	17	50 47.5 46.7 45.8
44	9.34 324	56	9.35 405	59	0.64 595	9.98 919	3	16	55 52.2 51.3 50.4
45	9.34 380	56	9.35 464	59	0.64 536	9.98 916	3	15	
46	9.34 436	55	9.35 523	58	0.64 477	9.98 913	3	14	
47	9.34 491	56	9.35 581	59	0.64 419	9.98 910	3	13	
48	9.34 547	55	9.35 640	58	0.64 360	9.98 907	3	12	" 3 2
49	9.34 602	56	9.35 698	59	0.64 302	9.98 904	3	11	5 0.2 0.2
50	9.34 658	55	9.35 757	58	0.64 243	9.98 901	3	10	10 0.5 0.3
51	9.34 713	56	9.35 815	58	0.64 185	9.98 898	2	9	15 0.8 0.5
52	9.34 769	55	9.35 873	58	0.64 127	9.98 896	3	8	20 1.0 0.7
53	9.34 824	55	9.35 931	58	0.64 069	9.98 893	3	7	25 1.2 0.8
54	9.34 879	55	9.35 989	58	0.64 011	9.98 890	3	6	30 1.5 1.0
55	9.34 934	55	9.36 047	58	0.63 953	9.98 887	3	5	35 1.8 1.2
56	9.34 989	55	9.36 105	58	0.63 895	9.98 884	3	4	40 2.0 1.3
57	9.35 044	55	9.36 163	58	0.63 837	9.98 881	3	3	45 2.2 1.5
58	9.35 099	55	9.36 221	58	0.63 779	9.98 878	3	2	50 2.5 1.7
59	9.35 154	55	9.36 279	57	0.63 721	9.98 875	3	1	55 2.8 1.8
60	9.35 209		9.36 336		0.63 664	9.98 872		0	
	L. Cos.	d.	L. Cot.	c.d.	L. Tan.	L. Sin.	d.	'	P.P.

	L. Sin.	d.	L. Tan.	c.d.	L. Cot.	L. Cos.	d.		P.P.
0	9.35 209		9.36 336		0.63 664	9.98 872		60	
1	9.35 263	54	9.36 394	58	0.63 606	9.98 869	3	59	
2	9.35 318	55	9.36 452	58	0.63 548	9.98 867	2	58	
3	9.35 373	55	9.36 509	57	0.63 491	9.98 864	3	57	
4	9.35 427	54	9.36 566	57	0.63 434	9.98 861	3	56	" 58 57 56
5	9.35 481	54	9.36 624	58	0.63 376	9.98 858	3	55	5 4.8 4.8 4.7
6	9.35 536	55	9.36 681	57	0.63 319	9.98 855	3	54	10 9.7 9.5 9.3
7	9.35 590	54	9.36 738	57	0.63 262	9.98 852	3	53	15 14.5 14.2 14.0
8	9.35 644	54	9.36 795	57	0.63 205	9.98 849	3	52	20 19.3 19.0 18.7
9	9.35 698	54	9.36 852	57	0.63 148	9.98 846	3	51	25 24.2 23.8 23.3
10	9.35 752	54	9.36 909	57	0.63 091	9.98 843	3	50	30 29.0 28.5 28.0
11	9.35 806	54	9.36 966	57	0.63 034	9.98 840	3	49	35 33.8 33.2 32.7
12	9.35 860	54	9.37 023	57	0.62 977	9.98 837	3	48	40 38.7 38.0 37.3
13	9.35 914	54	9.37 080	57	0.62 920	9.98 834	3	47	45 43.5 42.8 42.0
14	9.35 968	54	9.37 137	56	0.62 863	9.98 831	3	46	50 48.3 47.5 46.7
15	9.36 022	53	9.37 193	57	0.62 807	9.98 828	3	45	55 53.2 52.2 51.3
16	9.36 075	54	9.37 250	56	0.62 750	9.98 825	3	44	
17	9.36 129	53	9.37 306	57	0.62 694	9.98 822	3	43	
18	9.36 182	54	9.37 363	56	0.62 637	9.98 819	3	42	
19	9.36 236	53	9.37 419	57	0.62 581	9.98 816	3	41	" 55 54 53
20	9.36 289	53	9.37 476	56	0.62 524	9.98 813	3	40	5 4.6 4.5 4.4
21	9.36 342	53	9.37 532	56	0.62 468	9.98 810	3	39	10 9.2 9.0 8.8
22	9.36 395	54	9.37 588	56	0.62 412	9.98 807	3	38	15 13.8 13.5 13.2
23	9.36 449	53	9.37 644	56	0.62 356	9.98 804	3	37	20 18.3 18.0 17.7
24	9.36 502	53	9.37 700	56	0.62 300	9.98 801	3	36	25 22.9 22.5 22.1
25	9.36 555	53	9.37 756	56	0.62 244	9.98 798	3	35	30 27.5 27.0 26.5
26	9.36 608	52	9.37 812	56	0.62 188	9.98 795	3	34	35 32.1 31.5 30.9
27	9.36 660	53	9.37 868	56	0.62 132	9.98 792	3	33	40 36.7 36.0 35.3
28	9.36 713	53	9.37 924	56	0.62 076	9.98 789	3	32	45 41.2 40.5 39.8
29	9.36 766	53	9.37 980	55	0.62 020	9.98 786	3	31	50 45.8 45.0 44.2
30	9.36 819	52	9.38 035	56	0.61 965	9.98 783	3	30	55 50.4 49.5 48.6
31	9.36 871	53	9.38 091	56	0.61 909	9.98 780	3	29	
32	9.36 924	52	9.38 147	55	0.61 853	9.98 777	3	28	
33	9.36 976	52	9.38 202	55	0.61 798	9.98 774	3	27	" 52 51
34	9.37 028	53	9.38 257	56	0.61 743	9.98 771	3	26	5 4.3 4.2
35	9.37 081	52	9.38 313	55	0.61 687	9.98 768	3	25	10 8.7 8.5
36	9.37 133	52	9.38 368	55	0.61 632	9.98 765	3	24	15 13.0 12.8
37	9.37 185	52	9.38 423	56	0.61 577	9.98 762	3	23	20 17.3 17.0
38	9.37 237	52	9.38 479	55	0.61 521	9.98 759	3	22	25 21.7 21.2
39	9.37 289	52	9.38 534	55	0.61 466	9.98 756	3	21	30 26.0 25.5
40	9.37 341	52	9.38 589	55	0.61 411	9.98 753	3	20	35 30.3 29.8
41	9.37 393	52	9.38 644	55	0.61 356	9.98 750	3	19	40 34.7 34.0
42	9.37 445	52	9.38 699	55	0.61 301	9.98 746	4	18	45 39.0 38.2
43	9.37 497	52	9.38 754	54	0.61 246	9.98 743	3	17	50 43.3 42.5
44	9.37 549	51	9.38 808	55	0.61 192	9.98 740	3	16	55 47.7 46.8
45	9.37 600	52	9.38 863	55	0.61 137	9.98 737	3	15	
46	9.37 652	51	9.38 918	54	0.61 082	9.98 734	3	14	
47	9.37 703	52	9.38 972	55	0.61 028	9.98 731	3	13	
48	9.37 755	51	9.39 027	55	0.60 973	9.98 728	3	12	" 4 3 2
49	9.37 806	52	9.39 082	54	0.60 918	9.98 725	3	11	5 0.3 0.2 0.2
50	9.37 858	51	9.39 136	54	0.60 864	9.98 722	3	10	10 0.7 0.5 0.3
51	9.37 909	51	9.39 190	55	0.60 810	9.98 719	4	9	15 1.0 0.8 0.5
52	9.37 960	51	9.39 245	54	0.60 755	9.98 715	3	8	20 1.3 1.0 0.7
53	9.38 011	51	9.39 299	54	0.60 701	9.98 712	3	7	25 1.7 1.2 0.8
54	9.38 062	51	9.39 353	54	0.60 647	9.98 709	3	6	30 2.0 1.5 1.0
55	9.38 113	51	9.39 407	54	0.60 593	9.98 706	3	5	35 2.3 1.8 1.2
56	9.38 164	51	9.39 461	54	0.60 539	9.98 703	3	4	40 2.7 2.0 1.3
57	9.38 215	51	9.39 515	54	0.60 485	9.98 700	3	3	45 3.0 2.2 1.5
58	9.38 266	51	9.39 569	54	0.60 431	9.98 697	3	2	50 3.3 2.5 1.7
59	9.38 317	51	9.39 623	54	0.60 377	9.98 694	4	1	55 3.7 2.8 1.8
60	9.38 368		9.39 677		0.60 323	9.98 690		0	
	L. Cos.	d.	L. Cot.	c.d.	L. Tan.	L. Sin.	d.		P.P.

'	L. Sin.	d.	L. Tan.	c.d.	L. Cot.	L. Cos.	d.		P.P.
0	9.38 368	50	9.39 677	54	0.60 323	9.98 690	60		
1	9.38 418	51	9.39 731	54	0.60 269	9.98 687	3	59	
2	9.38 469	50	9.39 785	53	0.60 215	9.98 684	3	58	
3	9.38 519	51	9.39 838	54	0.60 162	9.98 681	3	57	" 54 53 52
4	9.38 570	50	9.39 892	53	0.60 108	9.98 678	3	56	5 4.5 4.4 4.3
5	9.38 620	50	9.39 945	54	0.60 055	9.98 675	4	55	10 9.0 8.8 8.7
6	9.38 670	51	9.39 999	53	0.60 001	9.98 671	3	54	15 13.5 13.2 13.0
7	9.38 721	50	9.40 052	54	0.59 948	9.98 668	3	53	20 18.0 17.7 17.3
8	9.38 771	50	9.40 106	53	0.59 894	9.98 665	3	52	25 22.5 22.1 21.7
9	9.38 821	50	9.40 159	53	0.59 841	9.98 662	3	51	30 27.0 26.5 26.0
10	9.38 871	50	9.40 212	54	0.59 788	9.98 659	3	50	35 31.5 30.9 30.3
11	9.38 921	50	9.40 266	53	0.59 734	9.98 656	4	49	40 36.0 35.3 34.7
12	9.38 971	50	9.40 319	53	0.59 681	9.98 652	3	48	45 40.5 39.8 39.0
13	9.39 021	50	9.40 372	53	0.59 628	9.98 649	3	47	50 45.0 44.2 43.3
14	9.39 071	50	9.40 425	53	0.59 575	9.98 646	3	46	55 49.5 48.6 47.7
15	9.39 121	49	9.40 478	53	0.59 522	9.98 643	3	45	
16	9.39 170	50	9.40 531	53	0.59 469	9.98 640	4	44	
17	9.39 220	50	9.40 584	52	0.59 416	9.98 636	3	43	
18	9.39 270	49	9.40 636	53	0.59 364	9.98 633	3	42	
19	9.39 319	50	9.40 689	53	0.59 311	9.98 630	3	41	" 51 50 49
20	9.39 369	49	9.40 742	53	0.59 258	9.98 627	4	40	5 4.2 4.2 4.1
21	9.39 418	49	9.40 795	52	0.59 205	9.98 623	3	39	10 8.5 8.3 8.2
22	9.39 467	50	9.40 847	53	0.59 153	9.98 620	3	38	15 12.8 12.5 12.2
23	9.39 517	49	9.40 900	52	0.59 100	9.98 617	3	37	20 17.0 16.7 16.3
24	9.39 566	49	9.40 952	53	0.59 048	9.98 614	4	36	25 21.2 20.8 20.4
25	9.39 615	49	9.41 005	52	0.58 995	9.98 610	3	35	30 25.5 25.0 24.5
26	9.39 664	49	9.41 057	52	0.58 943	9.98 607	3	34	35 29.8 29.2 28.6
27	9.39 713	49	9.41 109	52	0.58 891	9.98 604	3	33	40 34.0 33.3 32.7
28	9.39 762	49	9.41 161	53	0.58 839	9.98 601	4	32	45 38.2 37.5 36.8
29	9.39 811	49	9.41 214	52	0.58 786	9.98 597	3	31	50 42.5 41.7 40.8
30	9.39 860	49	9.41 266	52	0.58 734	9.98 594	3	30	55 46.8 45.8 44.9
31	9.39 909	49	9.41 318	52	0.58 682	9.98 591	3	29	
32	9.39 958	48	9.41 370	52	0.58 630	9.98 588	4	28	
33	9.40 006	49	9.41 422	52	0.58 578	9.98 584	3	27	
34	9.40 055	48	9.41 474	52	0.58 526	9.98 581	3	26	" 48 47
35	9.40 103	49	9.41 526	52	0.58 474	9.98 578	4	25	5 4.0 3.9
36	9.40 152	48	9.41 578	51	0.58 422	9.98 574	3	24	10 8.0 7.8
37	9.40 200	49	9.41 629	52	0.58 371	9.98 571	3	23	15 12.0 11.8
38	9.40 249	48	9.41 681	52	0.58 319	9.98 568	3	22	20 16.0 15.7
39	9.40 297	49	9.41 733	51	0.58 267	9.98 565	4	21	25 20.0 19.6
40	9.40 346	48	9.41 784	52	0.58 216	9.98 561	3	20	30 24.0 23.5
41	9.40 394	48	9.41 836	51	0.58 164	9.98 558	3	19	35 28.0 27.4
42	9.40 442	48	9.41 887	52	0.58 113	9.98 555	4	18	40 32.0 31.3
43	9.40 490	48	9.41 939	51	0.58 061	9.98 551	3	17	45 36.0 35.2
44	9.40 538	48	9.41 990	51	0.58 010	9.98 548	3	16	50 40.0 39.2
45	9.40 586	48	9.42 041	52	0.57 959	9.98 545	4	15	55 44.0 43.1
46	9.40 634	48	9.42 093	51	0.57 907	9.98 541	3	14	
47	9.40 682	48	9.42 144	51	0.57 856	9.98 538	3	13	
48	9.40 730	48	9.42 195	51	0.57 805	9.98 535	4	12	
49	9.40 778	47	9.42 246	51	0.57 754	9.98 531	3	11	" 4 3
50	9.40 825	48	9.42 297	51	0.57 703	9.98 528	3	10	5 0.3 0.2
51	9.40 873	48	9.42 348	51	0.57 652	9.98 525	4	9	10 0.7 0.5
52	9.40 921	47	9.42 399	51	0.57 601	9.98 521	3	8	15 1.0 0.8
53	9.40 968	48	9.42 450	51	0.57 550	9.98 518	3	7	20 1.3 1.0
54	9.41 016	47	9.42 501	51	0.57 499	9.98 515	4	6	25 1.7 1.2
55	9.41 063	48	9.42 552	51	0.57 448	9.98 511	3	5	30 2.0 1.5
56	9.41 111	47	9.42 603	50	0.57 397	9.98 508	3	4	35 2.3 1.8
57	9.41 158	47	9.42 653	51	0.57 347	9.98 505	4	3	40 2.7 2.0
58	9.41 205	47	9.42 704	51	0.57 296	9.98 501	3	2	45 3.0 2.2
59	9.41 252	48	9.42 755	50	0.57 245	9.98 498	4	1	50 3.3 2.5
60	9.41 300		9.42 805		0.57 195	9.98 494	0		55 3.7 2.8
	L. Cos.	d.	L. Cot.	c.d.	L. Tan.	L. Sin.	d.	'	P.P.

'	L. Sin.	d.	L. Tan.	c.d.	L. Cot.	L. Cos.	d.		P.P.
0	9.41 300		9.42 805		0.57 195	9.98 494		60	
1	9.41 347	47	9.42 856	51	0.57 144	9.98 491	3	59	
2	9.41 394	47	9.42 906	50	0.57 094	9.98 488	3	58	
3	9.41 441	47	9.42 957	51	0.57 043	9.98 484	4	57	
4	9.41 488	47	9.43 007	50	0.56 993	9.98 481	3	56	" 51 50 49
5	9.41 535	47	9.43 057	51	0.56 943	9.98 477	4	55	5 4.2 4.2 4.1
6	9.41 582	46	9.43 108	50	0.56 892	9.98 474	3	54	10 8.5 8.3 8.2
7	9.41 628	47	9.43 158	50	0.56 842	9.98 471	4	53	15 12.8 12.5 12.2
8	9.41 675	47	9.43 208	50	0.56 792	9.98 467	3	52	20 17.0 16.7 16.3
9	9.41 722	46	9.43 258	50	0.56 742	9.98 464	4	51	25 21.2 20.8 20.4
10	9.41 768	47	9.43 308	50	0.56 692	9.98 460	3	50	30 25.5 25.0 24.5
11	9.41 815	46	9.43 358	50	0.56 642	9.98 457	4	49	35 29.8 29.2 28.6
12	9.41 861	47	9.43 408	50	0.56 592	9.98 453	3	48	40 34.0 33.3 32.7
13	9.41 908	46	9.43 458	50	0.56 542	9.98 450	4	47	45 38.2 37.5 36.8
14	9.41 954	47	9.43 508	50	0.56 492	9.98 447	3	46	50 42.5 41.7 40.8
15	9.42 001	46	9.43 558	49	0.56 442	9.98 443	4	45	55 46.8 45.8 44.9
16	9.42 047	46	9.43 607	50	0.56 393	9.98 440	3	44	
17	9.42 093	47	9.43 657	50	0.56 343	9.98 436	4	43	
18	9.42 140	46	9.43 707	49	0.56 293	9.98 433	3	42	
19	9.42 186	46	9.43 756	50	0.56 244	9.98 429	4	41	" 48 47 46
20	9.42 232	46	9.43 806	49	0.56 194	9.98 426	3	40	5 4.0 3.9 3.8
21	9.42 278	46	9.43 855	50	0.56 145	9.98 422	4	39	10 8.0 7.8 7.7
22	9.42 324	46	9.43 905	49	0.56 095	9.98 419	3	38	15 12.0 11.8 11.5
23	9.42 370	46	9.43 954	50	0.56 046	9.98 415	4	37	20 16.0 15.7 15.3
24	9.42 416	45	9.44 004	49	0.55 996	9.98 412	3	36	25 20.0 19.6 19.2
25	9.42 461	46	9.44 053	49	0.55 947	9.98 409	4	35	30 24.0 23.5 23.0
26	9.42 507	46	9.44 102	49	0.55 898	9.98 405	3	34	35 28.0 27.4 26.8
27	9.42 553	46	9.44 151	50	0.55 849	9.98 402	4	33	40 32.0 31.3 30.7
28	9.42 599	45	9.44 201	49	0.55 799	9.98 398	3	32	45 36.0 35.2 34.5
29	9.42 644	46	9.44 250	49	0.55 750	9.98 395	4	31	50 40.0 39.2 38.3
30	9.42 690	45	9.44 299	49	0.55 701	9.98 391	3	30	55 44.0 43.1 42.2
31	9.42 735	46	9.44 348	49	0.55 652	9.98 388	4	29	
32	9.42 781	45	9.44 397	49	0.55 603	9.98 384	3	28	
33	9.42 826	46	9.44 446	49	0.55 554	9.98 381	4	27	" 45 44
34	9.42 872	45	9.44 495	49	0.55 505	9.98 377	3	26	5 3.8 3.7
35	9.42 917	45	9.44 544	48	0.55 456	9.98 373	4	25	10 7.5 7.3
36	9.42 962	46	9.44 592	49	0.55 408	9.98 370	3	24	15 11.2 11.0
37	9.43 008	45	9.44 641	49	0.55 359	9.98 366	4	23	20 15.0 14.7
38	9.43 053	45	9.44 690	48	0.55 310	9.98 363	3	22	25 18.8 18.3
39	9.43 098	45	9.44 738	49	0.55 262	9.98 359	4	21	30 22.5 22.0
40	9.43 143	45	9.44 787	59	0.55 213	9.98 356	3	20	35 26.2 25.7
41	9.43 188	45	9.44 836	48	0.55 164	9.98 352	4	19	40 30.0 29.3
42	9.43 233	45	9.44 884	49	0.55 116	9.98 349	3	18	45 33.8 33.0
43	9.43 278	45	9.44 933	48	0.55 067	9.98 345	4	17	50 37.5 36.7
44	9.43 323	44	9.44 981	48	0.55 019	9.98 342	3	16	55 41.2 40.3
45	9.43 367	45	9.45 029	49	0.54 971	9.98 338	4	15	
46	9.43 412	45	9.45 078	48	0.54 922	9.98 334	3	14	
47	9.43 457	45	9.45 126	48	0.54 874	9.98 331	4	13	
48	9.43 502	44	9.45 174	48	0.54 826	9.98 327	3	12	
49	9.43 546	45	9.45 222	49	0.54 778	9.98 324	4	11	" 4 3
50	9.43 591	44	9.45 271	48	0.54 729	9.98 320	3	10	5 0.3 0.2
51	9.43 635	45	9.45 319	48	0.54 681	9.98 317	4	9	10 0.7 0.5
52	9.43 680	44	9.45 367	48	0.54 633	9.98 313	3	8	15 1.0 0.8
53	9.43 724	45	9.45 415	48	0.54 585	9.98 309	4	7	20 1.3 1.0
54	9.43 769	44	9.45 463	48	0.54 537	9.98 306	3	6	25 1.7 1.2
55	9.43 813	44	9.45 511	48	0.54 489	9.98 302	4	5	30 2.0 1.5
56	9.43 857	44	9.45 559	47	0.54 441	9.98 299	3	4	35 2.3 1.8
57	9.43 901	45	9.45 606	48	0.54 394	9.98 295	4	3	40 2.7 2.0
58	9.43 946	44	9.45 654	48	0.54 346	9.98 291	3	2	45 3.0 2.2
59	9.43 990	44	9.45 702	48	0.54 298	9.98 288	4	1	50 3.3 2.5
60	9.44 034		9.45 750		0.54 250	9.98 284		0	55 3.7 2.8
	L. Cos.	d.	L. Cot.	c.d.	L. Tan.	L. Sin.	d.	'	P.P.

'	L. Sin.	d.	L. Tan.	c.d.	L. Cot.	L. Cos.	d.		P.P.
0	9.44 034		9.45 750		0.54 250	9.98 284		60	
1	9.44 078	44	9.45 797	47	0.54 203	9.98 281	3	59	
2	9.44 122	44	9.45 845	48	0.54 155	9.98 277	4	58	
3	9.44 166	44	9.45 892	47	0.54 108	9.98 273	4	57	
4	9.44 210	44	9.45 940	48	0.54 060	9.98 270	3	56	" 48 47 46
5	9.44 253	43	9.45 987	47	0.54 013	9.98 266	4	55	5 4.0 3.9 3.8
6	9.44 297	44	9.46 035	48	0.53 965	9.98 262	4	54	10 8.0 7.8 7.7
7	9.44 341	44	9.46 082	47	0.53 918	9.98 259	3	53	15 12.0 11.8 11.5
8	9.44 385	44	9.46 130	48	0.53 870	9.98 255	4	52	20 16.0 15.7 15.3
9	9.44 428	43	9.46 177	47	0.53 823	9.98 251	4	51	25 20.0 19.6 19.2
10	9.44 472	44	9.46 224	48	0.53 776	9.98 248	3	50	30 24.0 23.5 23.0
11	9.44 516	44	9.46 271	47	0.53 729	9.98 244	4	49	35 28.0 27.4 26.8
12	9.44 559	43	9.46 319	48	0.53 681	9.98 240	3	48	40 32.0 31.3 30.7
13	9.44 602	44	9.46 366	47	0.53 634	9.98 237	4	47	45 36.0 35.2 34.5
14	9.44 646	43	9.46 413	48	0.53 587	9.98 233	3	46	50 40.0 39.2 38.3
15	9.44 689	44	9.46 460	47	0.53 540	9.98 229	4	45	55 44.0 43.1 42.2
16	9.44 733	43	9.46 507	48	0.53 493	9.98 226	3	44	
17	9.44 776	43	9.46 554	47	0.53 446	9.98 222	4	43	
18	9.44 819	43	9.46 601	48	0.53 399	9.98 218	3	42	
19	9.44 862	43	9.46 648	46	0.53 352	9.98 215	4	41	" 45 44 43
20	9.44 905	43	9.46 694	47	0.53 306	9.98 211	4	40	5 3.8 3.7 3.6
21	9.44 948	44	9.46 741	47	0.53 259	9.98 207	3	39	10 7.5 7.3 7.2
22	9.44 992	43	9.46 788	48	0.53 212	9.98 204	4	38	15 11.2 11.0 10.8
23	9.45 035	42	9.46 835	47	0.53 165	9.98 200	3	37	20 15.0 14.7 14.3
24	9.45 077	43	9.46 881	46	0.53 119	9.98 196	4	36	25 18.8 18.3 17.9
25	9.45 120	43	9.46 928	47	0.53 072	9.98 192	3	35	30 22.5 22.0 21.5
26	9.45 163	43	9.46 975	46	0.53 025	9.98 189	4	34	35 26.2 25.7 25.1
27	9.45 206	43	9.47 021	47	0.52 979	9.98 185	3	33	40 30.0 29.3 28.7
28	9.45 249	43	9.47 068	46	0.52 932	9.98 181	4	32	45 33.8 33.0 32.2
29	9.45 292	42	9.47 114	46	0.52 886	9.98 177	3	31	50 37.5 36.7 35.8
30	9.45 334	43	9.47 160	47	0.52 840	9.98 174	4	30	55 41.2 40.3 39.4
31	9.45 377	42	9.47 207	46	0.52 793	9.98 170	3	29	
32	9.45 419	43	9.47 253	46	0.52 747	9.98 166	4	28	
33	9.45 462	42	9.47 299	47	0.52 701	9.98 162	3	27	" 42 41
34	9.45 504	43	9.47 346	46	0.52 654	9.98 159	4	26	5 3.5 3.4
35	9.45 547	42	9.47 392	46	0.52 608	9.98 155	3	25	10 7.0 6.8
36	9.45 589	43	9.47 438	46	0.52 562	9.98 151	4	24	15 10.5 10.2
37	9.45 632	42	9.47 484	46	0.52 516	9.98 147	3	23	20 14.0 13.7
38	9.45 674	42	9.47 530	46	0.52 470	9.98 144	4	22	25 17.5 17.1
39	9.45 716	42	9.47 576	46	0.52 424	9.98 140	3	21	30 21.0 20.5
40	9.45 758	43	9.47 622	46	0.52 378	9.98 136	4	20	35 24.5 23.9
41	9.45 801	42	9.47 668	46	0.52 332	9.98 132	3	19	40 28.0 27.3
42	9.45 843	42	9.47 714	46	0.52 286	9.98 129	4	18	45 31.5 30.8
43	9.45 885	42	9.47 760	46	0.52 240	9.98 125	3	17	50 35.0 34.2
44	9.45 927	42	9.47 806	46	0.52 194	9.98 121	4	16	55 38.5 37.6
45	9.45 969	42	9.47 852	45	0.52 148	9.98 117	3	15	
46	9.46 011	42	9.47 897	46	0.52 103	9.98 113	4	14	
47	9.46 053	42	9.47 943	46	0.52 057	9.98 110	3	13	
48	9.46 095	41	9.47 989	46	0.52 011	9.98 106	4	12	" 4 3
49	9.46 136	42	9.48 035	45	0.51 965	9.98 102	3	11	5 0.3 0.2
50	9.46 178	42	9.48 080	46	0.51 920	9.98 098	4	10	10 0.7 0.5
51	9.46 220	42	9.48 126	45	0.51 874	9.98 094	3	9	15 1.0 0.8
52	9.46 262	41	9.48 171	46	0.51 829	9.98 090	4	8	20 1.3 1.0
53	9.46 303	42	9.48 217	45	0.51 783	9.98 087	3	7	25 1.7 1.2
54	9.46 345	41	9.48 262	45	0.51 738	9.98 083	4	6	30 2.0 1.5
55	9.46 386	42	9.48 307	46	0.51 693	9.98 079	3	5	35 2.3 1.8
56	9.46 428	41	9.48 353	45	0.51 647	9.98 075	4	4	40 2.7 2.0
57	9.46 469	42	9.48 398	45	0.51 602	9.98 071	3	3	45 3.0 2.2
58	9.46 511	41	9.48 443	46	0.51 557	9.98 067	4	2	50 3.3 2.5
59	9.46 552	42	9.48 489	45	0.51 511	9.98 063	3	1	55 3.7 2.8
60	9.46 594		9.48 534		0.51 466	9.98 060		0	
	L. Cos.	d.	L. Cot.	c.d.	L. Tan.	L. Sin.	d.	'	P.P.

'	L. Sin.	d.	L. Tan.	c.d.	L. Cot.	L. Cos.	d.		P.P.
0	9.46 594	41	9.48 534	45	0.51 466	9.98 060	4	60	
1	9.46 635	41	9.48 579	45	0.51 421	9.98 056	4	59	
2	9.46 676	41	9.48 624	45	0.51 376	9.98 052	4	58	
3	9.46 717	41	9.48 669	45	0.51 331	9.98 048	4	57	
4	9.46 758	42	9.48 714	45	0.51 286	9.98 044	4	56	" 45 44 43
5	9.46 800	41	9.48 759	45	0.51 241	9.98 040	4	55	5 3.8 3.7 3.6
6	9.46 841	41	9.48 804	45	0.51 196	9.98 036	4	54	10 7.5 7.3 7.2
7	9.46 882	41	9.48 849	45	0.51 151	9.98 032	4	53	15 11.2 11.0 10.8
8	9.46 923	41	9.48 894	45	0.51 106	9.98 029	3	52	20 15.0 14.7 14.3
9	9.46 964	41	9.48 939	45	0.51 061	9.98 025	4	51	25 18.8 18.3 17.9
10	9.47 005	40	9.48 984	45	0.51 016	9.98 021	4	50	30 22.5 22.0 21.5
11	9.47 045	41	9.49 029	44	0.50 971	9.98 017	4	49	35 26.2 25.7 25.1
12	9.47 086	41	9.49 073	45	0.50 927	9.98 013	4	48	40 30.0 29.3 28.7
13	9.47 127	41	9.49 118	45	0.50 882	9.98 009	4	47	45 33.8 33.0 32.2
14	9.47 168	41	9.49 163	44	0.50 837	9.98 005	4	46	50 37.5 36.7 35.8
15	9.47 209	40	9.49 207	45	0.50 793	9.98 001	4	45	55 41.2 40.3 39.4
16	9.47 249	41	9.49 252	44	0.50 748	9.97 997	4	44	
17	9.47 290	40	9.49 296	45	0.50 704	9.97 993	4	43	
18	9.47 330	41	9.49 341	44	0.50 659	9.97 989	3	42	
19	9.47 371	40	9.49 385	45	0.50 615	9.97 986	4	41	" 42 41
20	9.47 411	41	9.49 430	44	0.50 570	9.97 982	4	40	5 3.5 3.4
21	9.47 452	40	9.49 474	45	0.50 526	9.97 978	4	39	10 7.0 6.8
22	9.47 492	41	9.49 519	44	0.50 481	9.97 974	4	38	15 10.5 10.2
23	9.47 533	40	9.49 563	44	0.50 437	9.97 970	4	37	20 14.0 13.7
24	9.47 573	40	9.49 607	45	0.50 393	9.97 966	4	36	25 17.5 17.1
25	9.47 613	41	9.49 652	44	0.50 348	9.97 962	4	35	30 21.0 20.5
26	9.47 654	40	9.49 696	44	0.50 304	9.97 958	4	34	35 24.5 23.9
27	9.47 694	40	9.49 740	44	0.50 260	9.97 954	4	33	40 28.0 27.3
28	9.47 734	40	9.49 784	44	0.50 216	9.97 950	4	32	45 31.5 30.8
29	9.47 774	40	9.49 828	44	0.50 172	9.97 946	4	31	50 35.0 34.2
30	9.47 814	40	9.49 872	44	0.50 128	9.97 942	4	30	55 38.5 37.6
31	9.47 854	40	9.49 916	44	0.50 084	9.97 938	4	29	
32	9.47 894	40	9.49 960	44	0.50 040	9.97 934	4	28	
33	9.47 934	40	9.50 004	44	0.49 996	9.97 930	4	27	" 40 39
34	9.47 974	40	9.50 048	44	0.49 952	9.97 926	4	26	5 3.3 3.2
35	9.48 014	40	9.50 092	44	0.49 908	9.97 922	4	25	10 6.7 6.5
36	9.48 054	40	9.50 136	44	0.49 864	9.97 918	4	24	15 10.0 9.8
37	9.48 094	39	9.50 180	43	0.49 820	9.97 914	4	23	20 13.3 13.0
38	9.48 133	40	9.50 223	44	0.49 777	9.97 910	4	22	25 16.7 16.2
39	9.48 173	40	9.50 267	44	0.49 733	9.97 906	4	21	30 20.0 19.5
40	9.48 213	39	9.50 311	44	0.49 689	9.97 902	4	20	35 23.3 22.8
41	9.48 252	40	9.50 355	43	0.49 645	9.97 898	4	19	40 26.7 26.0
42	9.48 292	40	9.50 398	44	0.49 602	9.97 894	4	18	45 30.0 29.2
43	9.48 332	39	9.50 442	43	0.49 558	9.97 890	4	17	50 33.3 32.5
44	9.48 371	40	9.50 485	44	0.49 515	9.97 886	4	16	55 36.7 35.8
45	9.48 411	39	9.50 529	43	0.49 471	9.97 882	4	15	
46	9.48 450	40	9.50 572	44	0.49 428	9.97 878	4	14	
47	9.48 490	39	9.50 616	43	0.49 384	9.97 874	4	13	
48	9.48 529	39	9.50 659	44	0.49 341	9.97 870	4	12	
49	9.48 568	39	9.50 703	43	0.49 297	9.97 866	5	11	" 5 4 3
50	9.48 607	40	9.50 746	43	0.49 254	9.97 861	4	10	5 0.4 0.3 0.2
51	9.48 647	39	9.50 789	44	0.49 211	9.97 857	4	9	10 0.8 0.7 0.5
52	9.48 686	39	9.50 833	43	0.49 167	9.97 853	4	8	15 1.2 1.0 0.8
53	9.48 725	39	9.50 876	43	0.49 124	9.97 849	4	7	20 1.7 1.3 1.0
54	9.48 764	39	9.50 919	43	0.49 081	9.97 845	4	6	25 2.1 1.7 1.2
55	9.48 803	39	9.50 962	43	0.49 038	9.97 841	4	5	30 2.5 2.0 1.5
56	9.48 842	39	9.51 005	43	0.48 995	9.97 837	4	4	35 2.9 2.3 1.8
57	9.48 881	39	9.51 048	44	0.48 952	9.97 833	4	3	40 3.3 2.7 2.0
58	9.48 920	39	9.51 092	43	0.48 908	9.97 829	4	2	45 3.8 3.0 2.2
59	9.48 959	39	9.51 135	43	0.48 865	9.97 825	4	1	50 4.2 3.3 2.5
60	9.48 998	39	9.51 178	43	0.48 822	9.97 821	4	0	55 4.6 3.7 2.8
	L. Cos.	d.	L. Cot.	c.d.	L. Tan.	L. Sin.	d.	'	P.P.

'	L. Sin.	d.	L. Tan.	c.d.	L. Cot.	L. Cos.	d.		P.P.
0	9.48 998		9.51 178		0.48 822	9.97 821		60	
1	9.49 037	39	9.51 221	43	0.48 779	9.97 817	4	59	
2	9.49 076	39	9.51 264	42	0.48 736	9.97 812	5	58	
3	9.49 115	38	9.51 306	43	0.48 694	9.97 808	4	57	
4	9.49 153	39	9.51 349	43	0.48 651	9.97 804	4	56	
5	9.49 192	39	9.51 392	43	0.48 608	9.97 800	4	55	
6	9.49 231	38	9.51 435	43	0.48 565	9.97 796	4	54	
7	9.49 269	39	9.51 478	42	0.48 522	9.97 792	4	53	" 43 42 41
8	9.49 308	39	9.51 520	43	0.48 480	9.97 788	4	52	5 3.6 3.5 3.4
9	9.49 347	38	9.51 563	43	0.48 437	9.97 784	4	51	10 7.2 7.0 6.8
10	9.49 385	39	9.51 606	42	0.48 394	9.97 779	5	50	15 10.8 10.5 10.2
11	9.49 424	38	9.51 648	43	0.48 352	9.97 775	4	49	20 14.3 14.0 13.7
12	9.49 462	38	9.51 691	43	0.48 309	9.97 771	4	48	25 17.9 17.5 17.1
13	9.49 500	39	9.51 734	42	0.48 266	9.97 767	4	47	30 21.5 21.0 20.5
14	9.49 539	38	9.51 776	43	0.48 224	9.97 763	4	46	35 25.1 24.5 23.9
15	9.49 577	38	9.51 819	42	0.48 181	9.97 759	5	45	40 28.7 28.0 27.3
16	9.49 615	39	9.51 861	42	0.48 139	9.97 754	4	44	45 32.2 31.5 30.8
17	9.49 654	38	9.51 903	43	0.48 097	9.97 750	4	43	50 35.8 35.0 34.2
18	9.49 692	38	9.51 946	42	0.48 054	9.97 746	4	42	55 39.4 38.5 37.6
19	9.49 730	38	9.51 988	43	0.48 012	9.97 742	4	41	
20	9.49 768	38	9.52 031	42	0.47 969	9.97 738	4	40	
21	9.49 806	38	9.52 073	42	0.47 927	9.97 734	5	39	
22	9.49 844	38	9.52 115	42	0.47 885	9.97 729	4	38	
23	9.49 882	38	9.52 157	43	0.47 843	9.97 725	4	37	
24	9.49 920	38	9.52 200	42	0.47 800	9.97 721	4	36	
25	9.49 958	38	9.52 242	42	0.47 758	9.97 717	4	35	
26	9.49 996	38	9.52 284	42	0.47 716	9.97 713	5	34	" 39 38 37
27	9.50 034	38	9.52 326	42	0.47 674	9.97 708	4	33	5 3.2 3.2 3.1
28	9.50 072	38	9.52 368	42	0.47 632	9.97 704	4	32	10 6.5 6.3 6.2
29	9.50 110	38	9.52 410	42	0.47 590	9.97 700	4	31	15 9.8 9.5 9.2
30	9.50 148	37	9.52 452	42	0.47 548	9.97 696	4	30	20 13.0 12.7 12.3
31	9.50 185	38	9.52 494	42	0.47 506	9.97 691	5	29	25 16.2 15.8 15.4
32	9.50 223	38	9.52 536	42	0.47 464	9.97 687	4	28	30 19.5 19.0 18.5
33	9.50 261	37	9.52 578	42	0.47 422	9.97 683	4	27	35 22.8 22.2 21.6
34	9.50 298	38	9.52 620	41	0.47 380	9.97 679	5	26	40 26.0 25.3 24.7
35	9.50 336	38	9.52 661	42	0.47 339	9.97 674	4	25	45 29.2 28.5 27.8
36	9.50 374	37	9.52 703	42	0.47 297	9.97 670	4	24	50 32.5 31.7 30.8
37	9.50 411	38	9.52 745	42	0.47 255	9.97 666	4	23	55 35.8 34.8 33.9
38	9.50 449	37	9.52 787	42	0.47 213	9.97 662	5	22	
39	9.50 486	37	9.52 829	41	0.47 171	9.97 657	4	21	
40	9.50 523	38	9.52 870	42	0.47 130	9.97 653	4	20	
41	9.50 561	37	9.52 912	41	0.47 088	9.97 649	4	19	
42	9.50 598	37	9.52 953	42	0.47 047	9.97 645	5	18	
43	9.50 635	38	9.52 995	42	0.47 005	9.97 640	4	17	
44	9.50 673	37	9.53 037	41	0.46 963	9.97 636	4	16	
45	9.50 710	37	9.53 078	42	0.46 922	9.97 632	4	15	" 36 35 34
46	9.50 747	37	9.53 120	41	0.46 880	9.97 628	5	14	5 3.0 0.4 0.3
47	9.50 784	37	9.53 161	41	0.46 839	9.97 623	4	13	10 6.0 0.8 0.7
48	9.50 821	37	9.53 202	42	0.46 798	9.97 619	4	12	15 9.0 1.2 1.0
49	9.50 858	38	9.53 244	41	0.46 756	9.97 615	5	11	20 12.0 1.7 1.3
50	9.50 896	37	9.53 285	42	0.46 715	9.97 610	4	10	25 15.0 2.1 1.7
51	9.50 933	37	0.53 327	41	0.46 673	9.97 606	4	9	30 18.0 2.5 2.0
52	9.50 970	37	9.53 368	41	0.46 632	9.97 602	5	8	35 21.0 2.9 2.3
53	9.51 007	36	9.53 409	41	0.46 591	9.97 597	4	7	40 24.0 3.3 2.7
54	9.51 043	37	9.53 450	42	0.46 550	9.97 593	4	6	45 27.0 3.8 3.0
55	9.51 080	37	9.53 492	41	0.46 508	9.97 589	5	5	50 30.0 4.2 3.3
56	9.51 117	37	9.53 533	41	0.46 467	9.97 584	4	4	55 33.0 4.6 3.7
57	9.51 154	37	9.53 574	41	0.46 426	9.97 580	4	3	
58	9.51 191	36	9.53 615	41	0.46 385	9.97 576	5	2	
59	9.51 227	37	9.53 656	41	0.46 344	9.97 571	4	1	
60	9.51 264		9.53 697		0.46 303	9.97 567		0	
	L. Cos.	d.	L. Cot.	c.d.	L. Tan.	L. Sin.	d.	'	P.P.

'	L. Sin.	d.	L. Tan.	c.d.	L. Cot.	L. Cos.	d.		P.P.
0	9.51 264		9.53 697		0.46 303	9.97 567		60	
1	9.51 301	37	9.53 738	41	0.46 262	9.97 563	4	59	
2	9.51 338	37	9.53 779	41	0.46 221	9.97 558	5	58	
3	9.51 374	36	9.53 820	41	0.46 180	9.97 554	4	57	
4	9.51 411	37	9.53 861	41	9.46 139	9.97 550	4	56	
5	9.51 447	36	9.53 902	41	0.46 098	9.97 545	5	55	
6	9.51 484	37	9.53 943	41	0.46 057	9.97 541	4	54	" 41 40 39
7	9.51 520	36	9.53 984	41	0.46 016	9.97 536	5	53	5 3.4 3.3 3.2
8	9.51 557	37	9.54 025	41	0.45 975	9.97 532	4	52	10 6.8 6.7 6.5
9	9.51 593	36	9.54 065	41	0.45 935	9.97 528	4	51	15 10.2 10.0 9.8
10	9.51 629	36	9.54 106	41	0.45 894	9.97 523	5	50	20 13.7 13.3 13.0
11	9.51 666	37	9.54 147	41	0.45 853	9.97 519	4	49	25 17.1 16.7 16.2
12	9.51 702	36	9.54 187	40	0.45 813	9.97 515	4	48	30 20.5 20.0 19.5
13	9.51 738	36	9.54 228	41	0.45 772	9.97 510	5	47	35 23.9 23.3 22.8
14	9.51 774	36	9.54 269	41	0.45 731	9.97 506	4	46	40 27.3 26.7 26.0
15	9.51 811	37	9.54 309	40	0.45 691	9.97 501	5	45	45 30.8 30.0 29.2
16	9.51 847	36	9.54 350	41	0.45 650	9.97 497	4	44	50 34.2 33.3 32.5
17	9.51 883	36	9.54 390	40	0.45 610	9.97 492	5	43	55 37.6 36.7 35.8
18	9.51 919	36	9.54 431	41	0.45 569	9.97 488	4	42	
19	9.51 955	36	9.54 471	41	0.45 529	9.97 484	5	41	
20	9.51 991	36	9.54 512	40	0.45 488	9.97 479	4	40	
21	9.52 027	36	9.54 552	41	0.45 448	9.97 475	5	39	
22	9.52 063	36	9.54 593	40	0.45 407	9.97 470	4	38	
23	9.52 099	36	9.54 633	40	0.45 367	9.97 466	5	37	
24	9.52 135	36	9.54 673	41	0.45 327	9.97 461	4	36	
25	9.52 171	36	9.54 714	40	0.45 286	9.97 457	5	35	
26	9.52 207	35	9.54 754	40	0.45 246	9.97 453	4	34	" 37 36 35
27	9.52 242	36	9.54 794	41	0.45 206	9.97 448	5	33	5 3.1 3.0 2.9
28	9.52 278	36	9.54 835	40	0.45 165	9.97 444	4	32	10 6.2 6.0 5.8
29	9.52 314	36	9.54 875	40	0.45 125	9.97 439	5	31	15 9.2 9.0 8.8
30	9.52 350	35	9.54 915	40	0.45 085	9.97 435	4	30	20 12.3 12.0 11.7
31	9.52 385	36	9.54 955	40	0.45 045	9.97 430	5	29	25 15.4 15.0 14.6
32	9.52 421	35	9.54 995	40	0.45 005	9.97 426	4	28	30 18.5 18.0 17.5
33	9.52 456	36	9.55 035	40	0.44 965	9.97 421	5	27	35 21.6 21.0 20.4
34	9.52 492	35	9.55 075	40	0.44 925	9.97 417	4	26	40 24.7 24.0 23.3
35	9.52 527	36	9.55 115	40	0.44 885	9.97 412	5	25	45 27.8 27.0 26.2
36	9.52 563	35	9.55 155	40	0.44 845	9.97 408	4	24	50 30.8 30.0 29.2
37	9.52 598	36	9.55 195	40	0.44 805	9.97 403	5	23	55 33.9 33.0 32.1
38	9.52 634	35	9.55 235	40	0.44 765	9.97 399	4	22	
39	9.52 669	36	9.55 275	40	0.44 725	9.97 394	5	21	
40	9.52 705	35	9.55 315	40	0.44 685	9.97 390	4	20	
41	9.52 740	35	9.55 355	40	0.44 645	9.97 385	5	19	
42	9.52 775	36	9.55 395	39	0.44 605	9.97 381	4	18	
43	9.52 811	35	9.55 434	40	0.44 566	9.97 376	5	17	
44	9.52 846	35	9.55 474	40	0.44 526	9.97 372	4	16	
45	9.52 881	35	9.55 514	40	0.44 486	9.97 367	5	15	
46	9.52 916	35	9.55 554	39	0.44 446	9.97 363	4	14	" 34 5 4
47	9.52 951	35	9.55 593	40	0.44 407	9.97 358	5	13	5 2.8 0.4 0.3
48	9.52 986	35	9.55 633	40	0.44 367	9.97 353	4	12	10 5.7 0.8 0.7
49	9.53 021	35	9.55 673	39	0.44 327	9.97 349	5	11	15 8.5 1.2 1.0
50	9.53 056	36	9.55 712	40	0.44 288	9.97 344	4	10	20 11.3 1.7 1.3
51	9.53 092	34	9.55 752	39	0.44 248	9.97 340	5	9	25 14.2 2.1 1.7
52	9.53 126	35	9.55 791	40	0.44 209	9.97 335	4	8	30 17.0 2.5 2.0
53	9.53 161	35	9.55 831	39	0.44 169	9.97 331	5	7	35 19.8 2.9 2.3
54	9.53 196	35	9.55 870	40	0.44 130	9.97 326	4	6	40 22.7 3.3 2.7
55	9.53 231	35	9.55 910	39	0.44 090	9.97 322	5	5	45 25.5 3.8 3.0
56	9.53 266	35	9.55 949	40	0.44 051	9.97 317	4	4	50 28.3 4.2 3.3
57	9.53 301	35	9.55 989	39	0.44 011	9.97 312	5	3	55 31.2 4.6 3.7
58	9.53 336	34	9.56 028	39	0.43 972	9.97 308	4	2	
59	9.53 370	35	9.56 067	40	0.43 933	9.97 303	5	1	
60	9.53 405		9.56 107		0.43 893	9.97 299	4	0	
	L. Cos.	d.	L. Cot.	c.d.	L. Tan.	L. Sin.	d.	'	P.P.

'	L. Sin.	d.	L. Tan.	c.d.	L. Cot.	L. Cos.	d.		P.P.
0	9.53 405		9.56 107		0.43 893	9.97 299		60	
1	9.53 440	35	9.56 146	39	0.43 854	9.97 294	5	59	
2	9.53 475	35	9.56 185	39	0.43 815	9.97 289	5	58	
3	9.53 509	34	9.56 224	39	0.43 776	9.97 285	4	57	
4	9.53 544	35	9.56 264	40	0.43 736	9.97 280	5	56	
5	9.53 578	34	9.56 303	39	0.43 697	9.97 276	4	55	
6	9.53 613	35	9.56 342	39	0.43 658	9.97 271	5	54	
7	9.53 647	34	9.56 381	39	0.43 619	9.97 266	5	53	" 40 39 38
8	9.53 682	35	9.56 420	39	0.43 580	9.97 262	4	52	5 3.3 3.2 3.2
9	9.53 716	34	9.56 459	39	0.43 541	9.97 257	5	51	10 6.7 6.5 6.3
10	9.53 751	35	9.56 498	39	0.43 502	9.97 252	5	50	15 10.0 9.8 9.5
11	9.53 785	34	9.56 537	39	0.43 463	9.97 248	4	49	20 13.3 13.0 12.7
12	9.53 819	35	9.56 576	39	0.43 424	9.97 243	5	48	25 16.7 16.2 15.8
13	9.53 854	34	9.56 615	39	0.43 385	9.97 238	4	47	30 20.0 19.5 19.0
14	9.53 888	35	9.56 654	39	0.43 346	9.97 234	5	46	35 23.3 22.8 22.2
15	9.53 922	34	9.56 693	39	0.43 307	9.97 229	4	45	40 26.7 26.0 25.3
16	9.53 957	35	9.56 732	39	0.43 268	9.97 224	5	44	45 30.0 29.2 28.5
17	9.53 991	34	9.56 771	39	0.43 229	9.97 220	4	43	50 33.3 32.5 31.7
18	9.54 025	35	9.56 810	39	0.43 190	9.97 215	5	42	55 36.7 35.8 34.8
19	9.54 059	34	9.56 849	38	0.43 151	9.97 210	4	41	
20	9.54 093	35	9.56 887	39	0.43 113	9.97 206	5	40	
21	9.54 127	34	9.56 926	39	0.43 074	9.97 201	4	39	
22	9.54 161	35	9.56 965	39	0.43 035	9.97 196	5	38	
23	9.54 195	34	9.57 004	39	0.42 996	9.97 192	4	37	
24	9.54 229	35	9.57 042	38	0.42 958	9.97 187	5	36	
25	9.54 263	34	9.57 081	39	0.42 919	9.97 182	4	35	
26	9.54 297	35	9.57 120	39	0.42 880	9.97 178	5	34	" 37 35 34
27	9.54 331	34	9.57 158	38	0.42 842	9.97 173	4	33	5 3.1 2.9 2.8
28	9.54 365	35	9.57 197	39	0.42 803	9.97 168	5	32	10 6.2 5.8 5.7
29	9.54 399	34	9.57 235	38	0.42 765	9.97 163	4	31	15 9.2 8.8 8.5
30	9.54 433	35	9.57 274	39	0.42 726	9.97 159	5	30	20 12.3 11.7 11.3
31	9.54 466	34	9.57 312	38	0.42 688	9.97 154	4	29	25 15.4 14.6 14.2
32	9.54 500	35	9.57 351	39	0.42 649	9.97 149	5	28	30 18.5 17.5 17.0
33	9.54 534	34	9.57 389	38	0.42 611	9.97 145	4	27	35 21.6 20.4 19.8
34	9.54 567	35	9.57 428	39	0.42 572	9.97 140	5	26	40 24.7 23.3 22.7
35	9.54 601	34	9.57 466	38	0.42 534	9.97 135	4	25	45 27.8 26.2 25.5
36	9.54 635	35	9.57 504	39	0.42 496	9.97 130	5	24	50 30.8 29.2 28.3
37	9.54 668	34	9.57 543	38	0.42 457	9.97 126	4	23	55 33.9 32.1 31.2
38	9.54 702	35	9.57 581	39	0.42 419	9.97 121	5	22	
39	9.54 735	34	9.57 619	38	0.42 381	9.97 116	4	21	
40	9.54 769	35	9.57 658	39	0.42 342	9.97 111	5	20	
41	9.54 802	34	9.57 696	38	0.42 304	9.97 107	4	19	
42	9.54 836	35	9.57 734	39	0.42 266	9.97 102	5	18	
43	9.54 869	34	9.57 772	38	0.42 228	9.97 097	4	17	
44	9.54 903	35	9.57 810	39	0.42 190	9.97 092	5	16	
45	9.54 936	34	9.57 849	38	0.42 151	9.97 087	4	15	
46	9.54 969	35	9.57 887	39	0.42 113	9.97 083	5	14	" 33 31 30
47	9.55 003	34	9.57 925	38	0.42 075	9.97 078	4	13	5 2.8 0.4 0.3
48	9.55 036	35	9.57 963	39	0.42 037	9.97 073	5	12	10 5.5 0.8 0.7
49	9.55 069	34	9.58 001	38	0.41 999	9.97 068	4	11	15 8.2 1.2 1.0
50	9.55 102	35	9.58 039	39	0.41 961	9.97 063	5	10	20 11.0 1.7 1.3
51	9.55 136	34	9.58 077	38	0.41 923	9.97 059	4	9	25 13.8 2.1 1.7
52	9.55 169	35	9.58 115	39	0.41 885	9.97 054	5	8	30 16.5 2.5 2.0
53	9.55 202	34	9.58 153	38	0.41 847	9.97 049	4	7	35 19.2 2.9 2.3
54	9.55 235	35	9.58 191	39	0.41 809	9.97 044	5	6	40 22.0 3.3 2.7
55	9.55 268	34	9.58 229	38	0.41 771	9.97 039	4	5	45 24.8 3.8 3.0
56	9.55 301	35	9.58 267	39	0.41 733	9.97 035	5	4	50 27.5 4.2 3.3
57	9.55 334	34	9.58 304	38	0.41 696	9.97 030	4	3	55 30.2 4.6 3.7
58	9.55 367	35	9.58 342	39	0.41 658	9.97 025	5	2	
59	9.55 400	34	9.58 380	38	0.41 620	9.97 020	4	1	
60	9.55 433	33	9.58 418	37	0.41 582	9.97 015	5	0	
	L. Cos.	d.	L. Cot.	c.d.	L. Tan.	L. Sin.	d.	'	P.P.

	L. Sin.	d.	L. Tan.	c.d.	L. Cot.	L. Cos.	d.		P.P.
0	9.55 433	33	9.58 418	37	0.41 582	9.97 015	5	60	
1	9.55 466	33	9.58 455	38	0.41 545	9.97 010	5	59	
2	9.55 499	33	9.58 493	38	0.41 507	9.97 005	5	58	
3	9.55 532	32	9.58 531	38	0.41 469	9.97 001	4	57	
4	9.55 564	33	9.58 569	37	0.41 431	9.96 996	5	56	
5	9.55 597	33	9.58 606	38	0.41 394	9.96 991	5	55	
6	9.55 630	33	9.58 644	37	0.41 356	9.96 986	5	54	
7	9.55 663	32	9.58 681	38	0.41 319	9.96 981	5	53	" 38 37 36
8	9.55 695	33	9.58 719	38	0.41 281	9.96 976	5	52	5 3.2 3.1 3.0
9	9.55 728	33	9.58 757	37	0.41 243	9.96 971	5	51	10 6.3 6.2 6.0
10	9.55 761	32	9.58 794	38	0.41 206	9.96 966	5	50	15 9.5 9.2 9.0
11	9.55 793	33	9.58 832	37	0.41 168	9.96 962	4	49	20 12.7 12.3 12.0
12	9.55 826	32	9.58 869	38	0.41 131	9.96 957	5	48	25 15.8 15.4 15.0
13	9.55 858	33	9.58 907	37	0.41 093	9.96 952	5	47	30 19.0 18.5 18.0
14	9.55 891	32	9.58 944	37	0.41 056	9.96 947	5	46	35 22.2 21.6 21.0
15	9.55 923	33	9.58 981	38	0.41 019	9.96 942	5	45	40 25.3 24.7 24.0
16	9.55 956	32	9.59 019	37	0.40 981	9.96 937	5	44	45 28.5 27.8 27.0
17	9.55 988	33	9.59 056	38	0.40 944	9.96 932	5	43	50 31.7 30.8 30.0
18	9.56 021	32	9.59 094	37	0.40 906	9.96 927	5	42	55 34.8 33.9 33.0
19	9.56 053	32	9.59 131	37	0.40 869	9.96 922	5	41	
20	9.56 085	33	9.59 168	37	0.40 832	9.96 917	5	40	
21	9.56 118	32	9.59 205	38	0.40 795	9.96 912	5	39	
22	9.56 150	32	9.59 243	37	0.40 757	9.96 907	4	38	
23	9.56 182	33	9.59 280	37	0.40 720	9.96 903	5	37	
24	9.56 215	32	9.59 317	37	0.40 683	9.96 898	5	36	
25	9.56 247	32	9.59 354	37	0.40 646	9.96 893	5	35	
26	9.56 279	32	9.59 391	38	0.40 609	9.96 888	5	34	" 33 32 31
27	9.56 311	32	9.59 429	37	0.40 571	9.96 883	5	33	5 2.8 2.7 2.6
28	9.56 343	32	9.59 466	37	0.40 534	9.96 878	5	32	10 5.5 5.3 5.2
29	9.56 375	33	9.59 503	37	0.40 497	9.96 873	5	31	15 8.2 8.0 7.8
30	9.56 408	32	9.59 540	37	0.40 460	9.96 868	5	30	20 11.0 10.7 10.3
31	9.56 440	32	9.59 577	37	0.40 423	9.96 863	5	29	25 13.8 13.3 12.9
32	9.56 472	32	9.59 614	37	0.40 386	9.96 858	5	28	30 16.5 16.0 15.5
33	9.56 504	32	9.59 651	37	0.40 349	9.96 853	5	27	35 19.2 18.7 18.1
34	9.56 536	32	9.59 688	37	0.40 312	9.96 848	5	26	40 22.0 21.3 20.7
35	9.56 568	31	9.59 725	37	0.40 275	9.96 843	5	25	45 24.8 24.0 23.2
36	9.56 599	32	9.59 762	37	0.40 238	9.96 838	5	24	50 27.5 26.7 25.8
37	9.56 631	32	9.59 799	36	0.40 201	9.96 833	5	23	55 30.2 29.3 28.4
38	9.56 663	32	9.59 835	37	0.40 165	9.96 828	5	22	
39	9.56 695	32	9.59 872	37	0.40 128	9.96 823	5	21	
40	9.56 727	32	9.59 909	37	0.40 091	9.96 818	5	20	
41	9.56 759	31	9.59 946	37	0.40 054	9.96 813	5	19	
42	9.56 790	32	9.59 983	36	0.40 017	9.96 808	5	18	
43	9.56 822	32	9.60 019	37	0.39 981	9.96 803	5	17	
44	9.56 854	32	9.60 056	37	0.39 944	9.96 798	5	16	
45	9.56 886	31	9.60 093	37	0.39 907	9.96 793	5	15	" 6 5 4
46	9.56 917	32	9.60 130	37	0.39 870	9.96 788	5	14	5 0.5 0.4 0.3
47	9.56 949	31	9.60 166	36	0.39 834	9.96 783	5	13	10 1.0 0.8 0.7
48	9.56 980	32	9.60 203	37	0.39 797	9.96 778	6	12	15 1.5 1.2 1.0
49	9.57 012	32	9.60 240	36	0.39 760	9.96 772	5	11	20 2.0 1.7 1.3
50	9.57 044	31	9.60 276	37	0.39 724	9.96 767	5	10	25 2.5 2.1 1.7
51	9.57 075	32	9.60 313	36	0.39 687	9.96 762	5	9	30 3.0 2.5 2.0
52	9.57 107	31	9.60 349	37	0.39 651	9.96 757	5	8	35 3.5 2.9 2.3
53	9.57 138	31	9.60 386	36	0.39 614	9.96 752	5	7	40 4.0 3.3 2.7
54	9.57 169	32	9.60 422	37	0.39 578	9.96 747	5	6	45 4.5 3.8 3.0
55	9.57 201	31	9.60 459	36	0.39 541	9.96 742	5	5	50 5.0 4.2 3.3
56	9.57 232	32	9.60 495	37	0.39 505	9.96 737	5	4	55 5.5 4.6 3.7
57	9.57 264	31	9.60 532	36	0.39 468	9.96 732	5	3	
58	9.57 295	31	9.60 568	37	0.39 432	9.96 727	5	2	
59	9.57 326	32	9.60 605	36	0.39 395	9.96 722	5	1	
60	9.57 358	32	9.60 641	36	0.39 359	9.96 717	5	0	
	L. Cos.	d.	L. Cot.	c.d.	L. Tan.	L. Sin.	d.		P.P.

	L. Sin.	d.	L. Tan.	c.d.	L. Cot.	L. Cos.	d.		P.P.
0	9.57 358		9.60 641		0.39 359	9.96 717		60	
1	9.57 389	31	9.60 677	36	0.39 323	9.96 711	6	59	
2	9.57 420	31	9.60 714	36	0.39 286	9.96 706	5	58	
3	9.57 451	31	9.60 750	36	0.39 250	9.96 701	5	57	
4	9.57 482	31	9.60 786	36	0.39 214	9.96 696	5	56	
5	9.57 514	31	9.60 823	36	0.39 177	9.96 691	5	55	
6	9.57 545	31	9.60 859	36	0.39 141	9.96 686	5	54	" 37 36 35
7	9.57 576	31	9.60 895	36	0.39 105	9.96 681	5	53	5 3.1 3.0 2.9
8	9.57 607	31	9.60 931	36	0.39 069	9.96 676	5	52	10 6.2 6.0 5.8
9	9.57 638	31	9.60 967	36	0.39 033	9.96 670	6	51	15 9.2 9.0 8.8
10	9.57 669	31	9.61 004	37	0.38 996	9.96 665	5	50	20 12.3 12.0 11.7
11	9.57 700	31	9.61 040	36	0.38 960	9.96 660	5	49	25 15.4 15.0 14.6
12	9.57 731	31	9.61 076	36	0.38 924	9.96 655	5	48	30 18.5 18.0 17.5
13	9.57 762	31	9.61 112	36	0.38 888	9.96 650	5	47	35 21.6 21.0 20.4
14	9.57 793	31	9.61 148	36	0.38 852	9.96 645	5	46	40 24.7 24.0 23.3
15	9.57 824	31	9.61 184	36	0.38 816	9.96 640	5	45	45 27.8 27.0 26.2
16	9.57 855	30	9.61 220	36	0.38 780	9.96 634	6	44	50 30.8 30.0 29.2
17	9.57 885	31	9.61 256	36	0.38 744	9.96 629	5	43	55 33.9 33.0 32.1
18	9.57 916	31	9.61 292	36	0.38 708	9.96 624	5	42	
19	9.57 947	31	9.61 328	36	0.38 672	9.96 619	5	41	
20	9.57 978	30	9.61 364	36	0.38 636	9.96 614	6	40	
21	9.58 008	31	9.61 400	36	0.38 600	9.96 608	5	39	
22	9.58 039	31	9.61 436	36	0.38 564	9.96 603	5	38	
23	9.58 070	31	9.61 472	36	0.38 528	9.96 598	5	37	
24	9.58 101	30	9.61 508	36	0.38 492	9.96 593	5	36	
25	9.58 131	31	9.61 544	35	0.38 456	9.96 588	6	35	
26	9.58 162	30	9.61 579	36	0.38 421	9.96 582	5	34	" 32 31 30
27	9.58 192	31	9.61 615	36	0.38 385	9.96 577	5	33	5 2.7 2.6 2.5
28	9.58 223	30	9.61 651	36	0.38 349	9.96 572	5	32	10 5.3 5.2 5.0
29	9.58 253	31	9.61 687	35	0.38 313	9.96 567	5	31	15 8.0 7.8 7.5
30	9.58 284	30	9.61 722	36	0.38 278	9.96 562	6	30	20 10.7 10.3 10.0
31	9.58 314	31	9.61 758	36	0.38 242	9.96 556	5	29	25 13.3 12.9 12.5
32	9.58 345	30	9.61 794	36	0.38 206	9.96 551	5	28	30 16.0 15.5 15.0
33	9.58 375	31	9.61 830	35	0.38 170	9.96 546	5	27	35 18.7 18.1 17.5
34	9.58 406	30	9.61 865	36	0.38 135	9.96 541	6	26	40 21.3 20.7 20.0
35	9.58 436	31	9.61 901	35	0.38 099	9.96 535	5	25	45 24.0 23.2 22.5
36	9.58 467	30	9.61 936	36	0.38 064	9.96 530	5	24	50 26.7 25.8 25.0
37	9.58 497	30	9.61 972	36	0.38 028	9.96 525	5	23	55 29.3 28.4 27.5
38	9.58 527	30	9.62 008	35	0.37 992	9.96 520	6	22	
39	9.58 557	31	9.62 043	36	0.37 957	9.96 514	5	21	
40	9.58 588	30	9.62 079	35	0.37 921	9.96 509	5	20	
41	9.58 618	30	9.62 114	36	0.37 886	9.96 504	6	19	
42	9.58 648	30	9.62 150	35	0.37 850	9.96 498	5	18	
43	9.58 678	31	9.62 185	36	0.37 815	9.96 493	5	17	
44	9.58 709	30	9.62 221	35	0.37 779	9.96 488	5	16	
45	9.58 739	30	9.62 256	36	0.37 744	9.96 483	6	15	
46	9.58 769	30	9.62 292	35	0.37 708	9.96 477	5	14	" 29 6 5
47	9.58 799	30	9.62 327	35	0.37 673	9.96 472	5	13	5 2.4 0.5 0.4
48	9.58 829	30	9.62 362	36	0.37 638	9.96 467	6	12	10 4.8 1.0 0.8
49	9.58 859	30	9.62 398	35	0.37 602	9.96 461	5	11	15 7.2 1.5 1.2
50	9.58 889	30	9.62 433	35	0.37 567	9.96 456	5	10	20 9.7 2.0 1.7
51	9.58 919	30	9.62 468	36	0.37 532	9.96 451	5	9	25 12.1 2.5 2.1
52	9.58 949	30	9.62 504	35	0.37 496	9.96 445	6	8	30 14.5 2.0 2.5
53	9.58 979	30	9.62 539	35	0.37 461	9.96 440	5	7	35 16.9 3.5 2.9
54	9.59 009	30	9.62 574	35	0.37 426	9.96 435	6	6	40 19.3 4.0 3.3
55	9.59 039	30	9.62 609	36	0.37 391	9.96 429	5	5	45 21.8 4.5 3.8
56	9.59 069	29	9.62 645	35	0.37 355	9.96 424	5	4	50 24.2 5.0 4.2
57	9.59 098	30	9.62 680	35	0.37 320	9.96 419	5	3	55 26.6 5.5 4.6
58	9.59 128	30	9.62 715	35	0.37 285	9.96 413	6	2	
59	9.59 158	30	9.62 750	35	0.37 250	9.96 408	5	1	
60	9.59 188		9.62 785		0.37 215	9.96 403		0	
	L. Cos.	d.	L. Cot.	c.d.	L. Tan.	L. Sin.	d.		P.P.

	L. Sin.	d.	L. Tan.	c.d.	L. Cot.	L. Cos.	d.		P.P.
0	9.59 188	30	9.62 785	35	0.37 215	9.96 403	6	60	
1	9.59 218	29	9.62 820	35	0.37 180	9.96 397	5	59	
2	9.59 247	30	9.62 855	35	0.37 145	9.96 392	5	58	
3	9.59 277	30	9.62 890	36	0.37 110	9.96 387	6	57	
4	9.59 307	29	9.62 926	35	0.37 074	9.96 381	5	56	
5	9.59 336	30	9.62 961	35	0.37 039	9.96 376	6	55	
6	9.59 366	30	9.62 996	35	0.37 004	9.96 370	5	54	
7	9.59 396	29	9.63 031	35	0.36 969	9.96 365	5	53	" 36 35 34
8	9.59 425	30	9.63 066	35	0.36 934	9.96 360	6	52	5 3.0 2.9 2.8
9	9.59 455	29	9.63 101	34	0.36 899	9.96 354	5	51	10 6.0 5.8 5.7
10	9.59 484	30	9.63 135	35	0.36 865	9.96 349	6	50	15 9.0 8.8 8.5
11	9.59 514	29	9.63 170	35	0.36 830	9.96 343	5	49	20 12.0 11.7 11.3
12	9.59 543	30	9.63 205	35	0.36 795	9.96 338	5	48	25 15.0 14.6 14.2
13	9.59 573	29	9.63 240	35	0.36 760	9.96 333	6	47	30 18.0 17.5 17.0
14	9.59 602	30	9.63 275	35	0.36 725	9.96 327	5	46	35 21.0 20.4 19.8
15	9.59 632	29	9.63 310	35	0.36 690	9.96 322	6	45	40 24.0 23.3 22.7
16	9.59 661	30	9.63 345	34	0.36 655	9.96 316	5	44	45 27.0 26.2 25.5
17	9.59 690	29	9.63 379	35	0.36 621	9.96 311	6	43	50 30.0 29.2 28.3
18	9.59 720	29	9.63 414	35	0.36 586	9.96 305	5	42	55 33.0 32.1 31.2
19	9.59 749	29	9.63 449	35	0.36 551	9.96 300	6	41	
20	9.59 778	30	9.63 484	35	0.36 516	9.96 294	5	40	
21	9.59 808	29	9.63 519	34	0.36 481	9.96 289	5	39	
22	9.59 837	29	9.63 553	35	0.36 447	9.96 284	6	38	
23	9.59 866	29	9.63 588	35	0.36 412	9.96 278	5	37	
24	9.59 895	29	9.63 623	34	0.36 377	9.96 273	6	36	
25	9.59 924	30	9.63 657	35	0.36 343	9.96 267	5	35	
26	9.59 954	29	9.63 692	34	0.36 308	9.96 262	6	34	" 30 29 28
27	9.59 983	29	9.63 726	35	0.36 274	9.96 256	5	33	5 2.5 2.4 2.3
28	9.60 012	29	9.63 761	35	0.36 239	9.96 251	6	32	10 5.0 4.8 4.7
29	9.60 041	29	9.63 796	34	0.36 204	9.96 245	5	31	15 7.5 7.2 7.0
30	9.60 070	29	9.63 830	35	0.36 170	9.96 240	6	30	20 10.0 9.7 9.3
31	9.60 099	29	9.63 865	34	0.36 135	9.96 234	5	29	25 12.5 12.1 11.7
32	9.60 128	29	9.63 899	35	0.36 101	9.96 229	6	28	30 15.0 14.5 14.0
33	9.60 157	29	9.63 934	34	0.36 066	9.96 223	5	27	35 17.5 16.9 16.3
34	9.60 186	29	9.63 968	35	0.36 032	9.96 218	6	26	40 20.0 19.3 18.7
35	9.60 215	29	9.64 003	34	0.35 997	9.96 212	5	25	45 22.5 21.8 21.0
36	9.60 244	29	9.64 037	35	0.35 963	9.96 207	6	24	50 25.0 24.2 23.3
37	9.60 273	29	9.64 072	34	0.35 928	9.96 201	5	23	55 27.5 26.6 25.7
38	9.60 302	29	9.64 106	34	0.35 894	9.96 196	6	22	
39	9.60 331	28	9.64 140	35	0.35 860	9.96 190	5	21	
40	9.60 359	29	9.64 175	34	0.35 825	9.96 185	6	20	
41	9.60 388	29	9.64 209	34	0.35 791	9.96 179	5	19	
42	9.60 417	29	9.64 243	35	0.35 757	9.96 174	6	18	
43	9.60 446	28	9.64 278	34	0.35 722	9.96 168	5	17	
44	9.60 474	29	9.64 312	34	0.35 688	9.96 162	6	16	
45	9.60 503	29	9.64 346	35	0.35 654	9.96 157	5	15	
46	9.60 532	29	9.64 381	34	0.35 619	9.96 151	6	14	" 6 5
47	9.60 561	28	9.64 415	34	0.35 585	9.96 146	5	13	5 0.5 0.4
48	9.60 589	29	9.64 449	34	0.35 551	9.96 140	6	12	10 1.0 0.8
49	9.60 618	28	9.64 483	34	0.35 517	9.96 135	5	11	15 1.5 1.2
50	9.60 646	29	9.64 517	35	0.35 483	9.96 129	6	10	20 2.0 1.7
51	9.60 675	29	9.64 552	34	0.35 448	9.96 123	5	9	25 2.5 2.1
52	9.60 704	28	9.64 586	34	0.35 414	9.96 118	6	8	30 3.0 2.5
53	9.60 732	29	9.64 620	34	0.35 380	9.96 112	5	7	35 3.5 2.9
54	9.60 761	28	9.64 654	34	0.35 346	9.96 107	6	6	40 4.0 3.3
55	9.60 789	29	9.64 688	34	0.35 312	9.96 101	5	5	45 4.5 3.8
56	9.60 818	28	9.64 722	34	0.35 278	9.96 095	6	4	50 5.0 4.2
57	9.60 846	29	9.64 756	34	0.35 244	9.96 090	5	3	55 5.5 4.6
58	9.60 875	28	9.64 790	34	0.35 210	9.96 084	6	2	
59	9.60 903	28	9.64 824	34	0.35 176	9.96 079	5	1	
60	9.60 931		9.64 858		0.35 142	9.96 073	6	0	
	L. Cos.	d.	L. Cot.	c.d.	L. Tan.	L. Sin.	d.		P.P.

'	L. Sin.	d.	L. Tan.	c.d.	L. Cot.	L. Cos.	d.		P.P.
0	9.60 931	29	9.64 858	34	0.35 142	9.96 073	6	60	
1	9.60 960	28	9.64 892	34	0.35 108	9.96 067	5	59	
2	9.60 988	28	9.64 926	34	0.35 074	9.96 062	6	58	
3	9.61 016	29	9.64 960	34	0.35 040	9.96 056	6	57	
4	9.61 045	28	9.64 994	34	0.35 006	9.96 050	5	56	
5	9.61 073	28	9.65 028	34	0.34 972	9.96 045	6	55	
6	9.61 101	28	9.65 062	34	0.34 938	9.96 039	5	54	
7	9.61 129	29	9.65 096	34	0.34 904	9.96 034	6	53	" 34 33
8	9.61 158	28	9.65 130	34	0.34 870	9.96 028	6	52	5 2.8 2.8
9	9.61 186	28	9.65 164	33	0.34 836	9.96 022	5	51	10 5.7 5.5
10	9.61 214	28	9.65 197	34	0.34 803	9.96 017	6	50	15 8.5 8.2
11	9.61 242	28	9.65 231	34	0.34 769	9.96 011	6	49	20 11.3 11.0
12	9.61 270	28	9.65 265	34	0.34 735	9.96 005	5	48	25 14.2 13.8
13	9.61 298	28	9.65 299	34	0.34 701	9.96 000	6	47	30 17.0 16.5
14	9.61 326	28	9.65 333	33	0.34 667	9.95 994	6	46	35 19.8 19.2
15	9.61 354	28	9.65 366	34	0.34 634	9.95 988	6	45	40 22.7 22.0
16	9.61 382	29	9.65 400	34	0.34 600	9.95 982	5	44	45 25.5 24.8
17	9.61 411	27	9.65 434	33	0.34 566	9.95 977	6	43	50 28.3 27.5
18	9.61 438	28	9.65 467	34	0.34 533	9.95 971	6	42	55 31.2 30.2
19	9.61 466	28	9.65 501	34	0.34 499	9.95 965	5	41	
20	9.61 494	28	9.65 535	33	0.34 465	9.95 960	6	40	
21	9.61 522	28	9.65 568	34	0.34 432	9.95 954	6	39	
22	9.61 550	28	9.65 602	34	0.34 398	9.95 948	6	38	
23	9.61 578	28	9.65 636	33	0.34 364	9.95 942	5	37	
24	9.61 606	28	9.65 669	34	0.34 331	9.95 937	6	36	
25	9.61 634	28	9.65 703	33	0.34 297	9.95 931	6	35	
26	9.61 662	27	9.65 736	34	0.34 264	9.95 925	5	34	" 29 28 27
27	9.61 689	28	9.65 770	33	0.34 230	9.95 920	6	33	5 2.4 2.3 2.2
28	9.61 717	28	9.65 803	34	0.34 197	9.95 914	6	32	10 4.8 4.7 4.5
29	9.61 745	28	9.65 837	33	0.34 163	9.95 908	6	31	15 7.2 7.0 6.8
30	9.61 773	27	9.65 870	34	0.34 130	9.95 902	5	30	20 9.7 9.3 9.0
31	9.61 800	28	9.65 904	33	0.34 096	9.95 897	6	29	25 12.1 11.7 11.2
32	9.61 828	28	9.65 937	34	0.34 063	9.95 891	6	28	30 14.5 14.0 13.5
33	9.61 856	27	9.65 971	33	0.34 029	9.95 885	6	27	35 16.9 16.3 15.8
34	9.61 883	28	9.66 004	34	0.33 996	9.95 879	6	26	40 19.3 18.7 18.0
35	9.61 911	28	9.66 038	33	0.33 962	9.95 873	5	25	45 21.8 21.0 20.2
36	9.61 939	27	9.66 071	33	0.33 929	9.95 868	6	24	50 24.2 23.3 22.5
37	9.61 966	28	9.66 104	34	0.33 896	9.95 862	6	23	55 26.6 25.7 24.8
38	9.61 994	27	9.66 138	33	0.33 862	9.95 856	6	22	
39	9.62 021	28	9.66 171	33	0.33 829	9.95 850	6	21	
40	9.62 049	27	9.66 204	34	0.33 796	9.95 844	5	20	
41	9.62 076	28	9.66 238	33	0.33 762	9.95 839	6	19	
42	9.62 104	27	9.66 271	33	0.33 729	9.95 833	6	18	
43	9.62 131	28	9.66 304	33	0.33 696	9.95 827	6	17	
44	9.62 159	27	9.66 337	34	0.33 663	9.95 821	6	16	
45	9.62 186	28	9.66 371	33	0.33 629	9.95 815	5	15	
46	9.62 214	27	9.66 404	33	0.33 596	9.95 810	6	14	" 6 5
47	9.62 241	27	9.66 437	33	0.33 563	9.95 804	6	13	5 0.5 0.4
48	9.62 268	28	9.66 470	33	0.33 530	9.95 798	6	12	10 1.0 0.8
49	9.62 296	27	9.66 503	34	0.33 497	9.95 792	6	11	15 1.5 1.2
50	9.62 323	27	9.66 537	33	0.33 463	9.95 786	6	10	20 2.0 1.7
51	9.62 350	27	9.66 570	33	0.33 430	9.95 780	5	9	25 2.5 2.1
52	9.62 377	28	9.66 603	33	0.33 397	9.95 775	6	8	30 3.0 2.5
53	9.62 405	27	9.66 636	33	0.33 364	9.95 769	6	7	35 3.5 2.9
54	9.62 432	27	9.66 669	33	0.33 331	9.95 763	6	6	40 4.0 3.3
55	9.62 459	27	9.66 702	33	0.33 298	9.95 757	6	5	45 4.5 3.8
56	9.62 486	28	9.66 735	33	0.33 265	9.95 751	6	4	50 5.0 4.2
57	9.62 513	27	9.66 768	33	0.33 232	9.95 745	6	3	55 5.5 4.6
58	9.62 541	27	9.66 801	33	0.33 199	9.95 739	6	2	
59	9.62 568	27	9.66 834	33	0.33 166	9.95 733	5	1	
60	9.62 595		9.66 867		0.33 133	9.95 728		0	
	L. Cos.	d.	L. Cot.	c.d.	L. Tan.	L. Sin.	d.	'	P.P.

'	L. Sin.	d.	L. Tan.	c.d.	L. Cot.	L. Cos.	d.		P.P.
0	9.62 595	27	9.66 867	33	0.33 133	9.95 728	6	60	
1	9.62 622	27	9.66 900	33	0.33 100	9.95 722	6	59	
2	9.62 649	27	9.66 933	33	0.33 067	9.95 716	6	58	
3	9.62 676	27	9.66 966	33	0.33 034	9.95 710	6	57	
4	9.62 703	27	9.66 999	33	0.33 001	9.95 704	6	56	
5	9.62 730	27	9.67 032	33	0.32 968	9.95 698	6	55	
6	9.62 757	27	9.67 065	33	0.32 935	9.95 692	6	54	" 33 32
7	9.62 784	27	9.67 098	33	0.32 902	9.95 686	6	53	5 2.8 2.7
8	9.62 811	27	9.67 131	32	0.32 869	9.95 680	6	52	10 5.5 5.3
9	9.62 838	27	9.67 163	33	0.32 837	9.95 674	6	51	15 8.2 8.0
10	9.62 865	27	9.67 196	33	0.32 804	9.95 668	6	50	20 11.0 10.7
11	9.62 892	26	9.67 229	33	0.32 771	9.95 663	6	49	25 13.8 13.3
12	9.62 918	27	9.67 262	33	0.32 738	9.95 657	6	48	30 16.5 16.0
13	9.62 945	27	9.67 295	32	0.32 705	9.95 651	6	47	35 19.2 18.7
14	9.62 972	27	9.67 327	33	0.32 673	9.95 645	6	46	40 22.0 21.3
15	9.62 999	27	9.67 360	33	0.32 640	9.95 639	6	45	45 24.8 24.0
16	9.63 026	26	9.67 393	33	0.32 607	9.95 633	6	44	50 27.5 26.7
17	9.63 052	27	9.67 426	32	0.32 574	9.95 627	6	43	55 30.2 29.3
18	9.63 079	27	9.67 458	33	0.32 542	9.95 621	6	42	
19	9.63 106	27	9.67 491	33	0.32 509	9.95 615	6	41	
20	9.63 133	26	9.67 524	32	0.32 476	9.95 609	6	40	
21	9.63 159	27	9.67 556	33	0.32 444	9.95 603	6	39	
22	9.63 186	27	9.67 589	33	0.32 411	9.95 597	6	38	
23	9.63 213	26	9.67 622	32	0.32 378	9.95 591	6	37	
24	9.63 239	27	9.67 654	33	0.32 346	9.95 585	6	36	
25	9.63 266	26	9.67 687	32	0.32 313	9.95 579	6	35	
26	9.63 292	27	9.67 719	33	0.32 281	9.95 573	6	34	" 27 26
27	9.63 319	26	9.67 752	33	0.32 248	9.95 567	6	33	5 2.2 2.2
28	9.63 345	27	9.67 785	32	0.32 215	9.95 561	6	32	10 4.5 4.3
29	9.63 372	26	9.67 817	33	0.32 183	9.95 555	6	31	15 6.8 6.5
30	9.63 398	27	9.67 850	32	0.32 150	9.95 549	6	30	20 9.0 8.7
31	9.63 425	26	9.67 882	33	0.32 118	9.95 543	6	29	25 11.2 10.8
32	9.63 451	27	9.67 915	32	0.32 085	9.95 537	6	28	30 13.5 13.0
33	9.63 478	26	9.67 947	33	0.32 053	9.95 531	6	27	35 15.8 15.2
34	9.63 504	27	9.67 980	32	0.32 020	9.95 525	6	26	40 18.0 17.3
35	9.63 531	26	9.68 012	32	0.31 988	9.95 519	6	25	45 20.2 19.5
36	9.63 557	26	9.68 044	33	0.31 956	9.95 513	6	24	50 22.5 21.7
37	9.63 583	27	9.68 077	32	0.31 923	9.95 507	6	23	55 24.8 23.8
38	9.63 610	26	9.68 109	33	0.31 891	9.95 500	6	22	
39	9.63 636	26	9.68 142	32	0.31 858	9.95 494	6	21	
40	9.63 662	27	9.68 174	32	0.31 826	9.95 488	6	20	
41	9.63 689	26	9.68 206	33	0.31 794	9.95 482	6	19	
42	9.63 715	26	9.68 239	32	0.31 761	9.95 476	6	18	
43	9.63 741	26	9.68 271	32	0.31 729	9.95 470	6	17	
44	9.63 767	27	9.68 303	33	0.31 697	9.95 464	6	16	
45	9.63 794	26	9.68 336	32	0.31 664	9.95 458	6	15	
46	9.63 820	26	9.68 368	32	0.31 632	9.95 452	6	14	" 7 6 5
47	9.63 846	26	9.68 400	32	0.31 600	9.95 446	6	13	5 0.6 0.5 0.4
48	9.63 872	26	9.68 432	33	0.31 568	9.95 440	6	12	10 1.2 1.0 0.8
49	9.63 898	26	9.68 465	32	0.31 535	9.95 434	6	11	15 1.8 1.5 1.2
50	9.63 924	26	9.68 497	32	0.31 503	9.95 427	7	10	20 2.3 2.0 1.7
51	9.63 950	26	9.68 529	32	0.31 471	9.95 421	6	9	25 2.9 2.5 2.1
52	9.63 976	26	9.68 561	32	0.31 439	9.95 415	6	8	30 3.5 3.0 2.5
53	9.64 002	26	9.68 593	33	0.31 407	9.95 409	6	7	35 4.1 3.5 2.9
54	9.64 028	26	9.68 626	32	0.31 374	9.95 403	6	6	40 4.7 4.0 3.3
55	9.64 054	26	9.68 658	32	0.31 342	9.95 397	6	5	45 5.2 4.5 3.8
56	9.64 080	26	9.68 690	32	0.31 310	9.95 391	7	4	50 5.8 5.0 4.2
57	9.64 106	26	9.68 722	32	0.31 278	9.95 384	6	3	55 6.4 5.5 4.6
58	9.64 132	26	9.68 754	32	0.31 246	9.95 378	6	2	
59	9.64 158	26	9.68 786	32	0.31 214	9.95 372	6	1	
60	9.64 184		9.68 818		0.31 182	9.95 366		0	
	L. Cos.	d.	L. Cot.	c.d.	L. Tan.	L. Sin.	d.	'	P.P.

'	L. Sin.	d.	L. Tan.	c.d.	L. Cot.	L. Cos.	d.		P.P.
0	9.64 184	26	9.68 818	32	0.31 182	9.95 366	6	60	
1	9.64 210	26	9.68 850	32	0.31 150	9.95 360	6	59	
2	9.64 236	26	9.68 882	32	0.31 118	9.95 354	6	58	
3	9.64 262	26	9.68 914	32	0.31 086	9.95 348	6	57	
4	9.64 288	25	9.68 946	32	0.31 054	9.95 341	6	56	
5	9.64 313	26	9.68 978	32	0.31 022	9.95 335	6	55	
6	9.64 339	26	9.69 010	32	0.30 990	9.95 329	6	54	" 32 31
7	9.64 365	26	9.69 042	32	0.30 958	9.95 323	6	53	5 2.7 2.6
8	9.64 391	26	9.69 074	32	0.30 926	9.95 317	7	52	10 5.3 5.2
9	9.64 417	25	9.69 106	32	0.30 894	9.95 310	6	51	15 8.0 7.8
10	9.64 442	26	9.69 138	32	0.30 862	9.95 304	6	50	20 10.7 10.3
11	9.64 468	26	9.69 170	32	0.30 830	9.95 298	6	49	25 13.3 12.9
12	9.64 494	25	9.69 202	32	0.30 798	9.95 292	6	48	30 16.0 15.5
13	9.64 519	26	9.69 234	32	0.30 766	9.95 286	6	47	35 18.7 18.1
14	9.64 545	26	9.69 266	32	0.30 734	9.95 279	6	46	40 21.3 20.7
15	9.64 571	25	9.69 298	31	0.30 702	9.95 273	6	45	45 24.0 23.2
16	9.64 596	26	9.69 329	32	0.30 671	9.95 267	6	44	50 26.7 25.8
17	9.64 622	25	9.69 361	32	0.30 639	9.95 261	6	43	55 29.3 28.4
18	9.64 647	26	9.69 393	32	0.30 607	9.95 254	6	42	
19	9.64 673	25	9.69 425	32	0.30 575	9.95 248	6	41	
20	9.64 698	26	9.69 457	31	0.30 543	9.95 242	6	40	
21	9.64 724	25	9.69 488	32	0.30 512	9.95 236	7	39	
22	9.64 749	26	9.69 520	32	0.30 480	9.95 229	6	38	
23	9.64 775	25	9.69 552	32	0.30 448	9.95 223	6	37	
24	9.64 800	26	9.69 584	31	0.30 416	9.95 217	6	36	
25	9.64 826	25	9.69 615	32	0.30 385	9.95 211	7	35	
26	9.64 851	26	9.69 647	32	0.30 353	9.95 204	6	34	" 26 25 24
27	9.64 877	25	9.69 679	31	0.30 321	9.95 198	6	33	5 2.2 2.1 2.0
28	9.64 902	25	9.69 710	32	0.30 290	9.95 192	7	32	10 4.3 4.2 4.0
29	9.64 927	26	9.69 742	32	0.30 258	9.95 185	6	31	15 6.5 6.2 6.0
30	9.64 953	25	9.69 774	31	0.30 226	9.95 179	6	30	20 8.7 8.3 8.0
31	9.64 978	25	9.69 805	32	0.30 195	9.95 173	6	29	25 10.8 10.4 10.0
32	9.65 003	26	9.69 837	31	0.30 163	9.95 167	6	28	30 13.0 12.5 12.0
33	9.65 029	25	9.69 868	32	0.30 132	9.95 160	7	27	35 15.2 14.6 14.0
34	9.65 054	25	9.69 900	32	0.30 100	9.95 154	6	26	40 17.3 16.7 16.0
35	9.65 079	25	9.69 932	31	0.30 068	9.95 148	6	25	45 19.5 18.8 18.0
36	9.65 104	26	9.69 963	32	0.30 037	9.95 141	7	24	50 21.7 20.8 20.0
37	9.65 130	25	9.69 995	31	0.30 005	9.95 135	6	23	55 23.8 22.9 22.0
38	9.65 155	25	9.70 026	32	0.29 974	9.95 129	6	22	
39	9.65 180	25	9.70 058	31	0.29 942	9.95 122	7	21	
40	9.65 205	25	9.70 089	32	0.29 911	9.95 116	6	20	
41	9.65 230	25	9.70 121	31	0.29 879	9.95 110	7	19	
42	9.65 255	26	9.70 152	32	0.29 848	9.95 103	6	18	
43	9.65 281	25	9.70 184	31	0.29 816	9.95 097	7	17	
44	9.65 306	25	9.70 215	32	0.29 785	9.95 090	6	16	
45	9.65 331	25	9.70 247	31	0.29 753	9.95 084	6	15	
46	9.65 356	25	9.70 278	31	0.29 722	9.95 078	7	14	" 7 6
47	9.65 381	25	9.70 309	32	0.29 691	9.95 071	6	13	5 0.6 0.5
48	9.65 406	25	9.70 341	31	0.29 659	9.95 065	6	12	10 1.2 1.0
49	9.65 431	25	9.70 372	32	0.29 628	9.95 059	7	11	15 1.8 1.5
50	9.65 456	25	9.70 404	31	0.29 596	9.95 052	6	10	20 2.3 2.0
51	9.65 481	25	9.70 435	31	0.29 565	9.95 046	6	9	25 2.9 2.5
52	9.65 506	25	9.70 466	32	0.29 534	9.95 039	7	8	30 3.5 3.0
53	9.65 531	25	9.70 498	31	0.29 502	9.95 033	6	7	35 4.1 3.5
54	9.65 556	24	9.70 529	31	0.29 471	9.95 027	7	6	40 4.7 4.0
55	9.65 580	25	9.70 560	32	0.29 440	9.95 020	6	5	45 5.2 4.5
56	9.65 605	25	9.70 592	31	0.29 408	9.95 014	7	4	50 5.8 5.0
57	9.65 630	25	9.70 623	31	0.29 377	9.95 007	6	3	55 6.4 5.5
58	9.65 655	25	9.70 654	31	0.29 346	9.95 001	6	2	
59	9.65 680	25	9.70 685	32	0.29 315	9.94 995	7	1	
60	9.65 705		9.70 717		0.29 283	9.94 988		0	
	L. Cos.	d.	L. Cot.	c.d.	L. Tan.	L. Sin.	d.	'	P.P.

'	L. Sin.	d.	L. Tan.	c.d.	L. Cot.	L. Cos.	d.		P.P.
0	9.65 705		9.70 717		0.29 283	9.94 988		60	
1	9.65 729	24	9.70 748	31	0.29 252	9.94 982	6	59	
2	9.65 754	25	9.70 779	31	0.29 221	9.94 975	7	58	
3	9.65 779	25	9.70 810	31	0.29 190	9.94 969	6	57	
4	9.65 804	25	9.70 841	31	0.29 159	9.94 962	7	56	
5	9.65 828	24	9.70 873	32	0.29 127	9.94 956	6	55	
6	9.65 853	25	9.70 904	31	0.29 096	9.94 949	7	54	
7	9.65 878	25	9.70 935	31	0.29 065	9.94 943	6	53	" 32 31 30
8	9.65 902	24	9.70 966	31	0.29 034	9.94 936	7	52	5 2.7 2.6 2.5
9	9.65 927	25	9.70 997	31	0.29 003	9.94 930	6	51	10 5.3 5.2 5.0
10	9.65 952	25	9.71 028	31	0.28 972	9.94 923	7	50	15 8.0 7.8 7.5
11	9.65 976	24	9.71 059	31	0.28 941	9.94 917	6	49	20 10.7 10.3 10.0
12	9.66 001	25	9.71 090	31	0.28 910	9.94 911	7	48	25 13.3 12.9 12.5
13	9.66 025	24	9.71 121	31	0.28 879	9.94 904	6	47	30 16.0 15.5 15.0
14	9.66 050	25	9.71 153	32	0.28 847	9.94 898	7	46	35 18.7 18.1 17.5
15	9.66 075	25	9.71 184	31	0.28 816	9.94 891	6	45	40 21.3 20.7 20.0
16	9.66 099	24	9.71 215	31	0.28 785	9.94 885	7	44	45 24.0 23.2 22.5
17	9.66 124	25	9.71 246	31	0.28 754	9.94 878	6	43	50 26.7 25.8 25.0
18	9.66 148	24	9.71 277	31	0.28 723	9.94 871	7	42	55 29.3 28.4 27.5
19	9.66 173	24	9.71 308	31	0.28 692	9.94 865	6	41	
20	9.66 197	24	9.71 339	31	0.28 661	9.94 858	7	40	
21	9.66 221	25	9.71 370	31	0.28 630	9.94 852	6	39	
22	9.66 246	24	9.71 401	31	0.28 599	9.94 845	7	38	
23	9.66 270	25	9.71 431	30	0.28 569	9.94 839	6	37	
24	9.66 295	24	9.71 462	31	0.28 538	9.94 832	7	36	
25	9.66 319	25	9.71 493	31	0.28 507	9.94 826	6	35	
26	9.66 343	24	9.71 524	31	0.28 476	9.94 819	7	34	
27	9.66 368	25	9.71 555	31	0.28 445	9.94 813	6	33	" 25 24 23
28	9.66 392	24	9.71 586	31	0.28 414	9.94 806	7	32	5 2.1 2.0 1.9
29	9.66 416	25	9.71 617	31	0.28 383	9.94 799	6	31	10 4.2 4.0 3.8
30	9.66 441	24	9.71 648	31	0.28 352	9.94 793	7	30	15 6.2 6.0 5.8
31	9.66 465	25	9.71 679	31	0.28 321	9.94 786	6	29	20 8.3 8.0 7.7
32	9.66 489	24	9.71 709	30	0.28 291	9.94 780	7	28	25 10.4 10.0 9.6
33	9.66 513	25	9.71 740	31	0.28 260	9.94 773	6	27	30 12.5 12.0 11.5
34	9.66 537	24	9.71 771	31	0.28 229	9.94 767	7	26	35 14.6 14.0 13.4
35	9.66 562	25	9.71 802	31	0.28 198	9.94 760	6	25	40 16.7 16.0 15.3
36	9.66 586	24	9.71 833	31	0.28 167	9.94 753	7	24	45 18.8 18.0 17.2
37	9.66 610	25	9.71 863	30	0.28 137	9.94 747	6	23	50 20.8 20.0 19.2
38	9.66 634	24	9.71 894	31	0.28 106	9.94 740	7	22	55 22.9 22.0 21.1
39	9.66 658	25	9.71 925	31	0.28 075	9.94 734	6	21	
40	9.66 682	24	9.71 955	30	0.28 045	9.94 727	7	20	
41	9.66 706	25	9.71 986	31	0.28 014	9.94 720	6	19	
42	9.66 731	24	9.72 017	31	0.27 983	9.94 714	7	18	
43	9.66 755	25	9.72 048	31	0.27 952	9.94 707	6	17	
44	9.66 779	24	9.72 078	30	0.27 922	9.94 700	7	16	
45	9.66 803	25	9.72 109	31	0.27 891	9.94 694	6	15	
46	9.66 827	24	9.72 140	31	0.27 860	9.94 687	7	14	
47	9.66 851	25	9.72 170	30	0.27 830	9.94 680	6	13	" 7 6
48	9.66 875	24	9.72 201	31	0.27 799	9.94 674	7	12	5 0.6 0.5
49	9.66 899	25	9.72 231	31	0.27 769	9.94 667	6	11	10 1.2 1.0
50	9.66 922	24	9.72 262	30	0.27 738	9.94 660	7	10	15 1.8 1.5
51	9.66 946	25	9.72 293	31	0.27 707	9.94 654	6	9	20 2.3 2.0
52	9.66 970	24	9.72 323	31	0.27 677	9.94 647	7	8	25 2.9 2.5
53	9.66 994	25	9.72 354	31	0.27 646	9.94 640	6	7	30 3.5 3.0
54	9.67 018	24	9.72 384	30	0.27 616	9.94 634	7	6	35 4.1 3.5
55	9.67 042	25	9.72 415	31	0.27 585	9.94 627	6	5	40 4.7 4.0
56	9.67 066	24	9.72 445	30	0.27 555	9.94 620	7	4	45 5.2 4.5
57	9.67 090	25	9.72 476	31	0.27 524	9.94 614	6	3	50 5.8 5.0
58	9.67 113	24	9.72 506	30	0.27 494	9.94 607	7	2	55 6.4 5.5
59	9.67 137	25	9.72 537	31	0.27 463	9.94 600	6	1	
60	9.67 161	24	9.72 567	30	0.27 433	9.94 593	7	0	
	L. Cos.	d.	L. Cot.	c.d.	L. Tan.	L. Sin.	d.		P.P.

'	L. Sin.	d.	L. Tan.	c.d.	L. Cot.	L. Cos.	d.		P.P.
0	9.67 161		9.72 567		0.27 433	9.94 593		60	
1	9.67 185	24	9.72 598	31	0.27 402	9.94 587	6	59	
2	9.67 208	23	9.72 628	30	0.27 372	9.94 580	7	58	
3	9.67 232	24	9.72 659	31	0.27 341	9.94 573	7	57	
4	9.67 256	24	9.72 689	30	0.27 311	9.94 567	6	56	
5	9.67 280	24	9.72 720	31	0.27 280	9.94 560	7	55	
6	9.67 303	23	9.72 750	30	0.27 250	9.94 553	7	54	
7	9.67 327	24	9.72 780	31	0.27 220	9.94 546	7	53	" 31 30 29
8	9.67 350	23	9.72 811	30	0.27 189	9.94 540	6	52	5 2.6 2.5 2.4
9	9.67 374	24	9.72 841	31	0.27 159	9.94 533	7	51	10 5.2 5.0 4.8
10	9.67 398	24	9.72 872	30	0.27 128	9.94 526	7	50	15 7.8 7.5 7.2
11	9.67 421	23	9.72 902	31	0.27 098	9.94 519	7	49	20 10.3 10.0 9.7
12	9.67 445	24	9.72 932	30	0.27 068	9.94 513	6	48	25 12.9 12.5 12.1
13	9.67 468	23	9.72 963	31	0.27 037	9.94 506	7	47	30 15.5 15.0 14.5
14	9.67 492	24	9.72 993	30	0.27 007	9.94 499	7	46	35 18.1 17.5 16.9
15	9.67 515	23	9.73 023	31	0.26 977	9.94 492	7	45	40 20.7 20.0 19.3
16	9.67 539	24	9.73 054	30	0.26 946	9.94 485	7	44	45 23.2 22.5 21.8
17	9.67 562	23	9.73 084	31	0.26 916	9.94 479	6	43	50 25.8 25.0 24.2
18	9.67 586	24	9.73 114	30	0.26 886	9.94 472	7	42	55 28.4 27.5 26.6
19	9.67 609	23	9.73 144	31	0.26 856	9.94 465	7	41	
20	9.67 633	24	9.73 175	30	0.26 825	9.94 458	7	40	
21	9.67 656	23	9.73 205	31	0.26 795	9.94 451	6	39	
22	9.67 680	24	9.73 235	30	0.26 765	9.94 445	7	38	
23	9.67 703	23	9.73 265	31	0.26 735	9.94 438	7	37	
24	9.67 726	24	9.73 295	30	0.26 705	9.94 431	7	36	
25	9.67 750	23	9.73 326	31	0.26 674	9.94 424	7	35	
26	9.67 773	24	9.73 356	30	0.26 644	9.94 417	7	34	" 24 23 22
27	9.67 796	23	9.73 386	31	0.26 614	9.94 410	6	33	5 2.0 1.9 1.8
28	9.67 820	24	9.73 416	30	0.26 584	9.94 404	7	32	10 4.0 3.8 3.7
29	9.67 843	23	9.73 446	31	0.26 554	9.94 397	7	31	15 6.0 5.8 5.5
30	9.67 866	24	9.73 476	30	0.26 524	9.94 390	7	30	20 8.0 7.7 7.3
31	9.67 890	23	9.73 507	31	0.26 493	9.94 383	7	29	25 10.0 9.6 9.2
32	9.67 913	24	9.73 537	30	0.26 463	9.94 376	6	28	30 12.0 11.5 11.0
33	9.67 936	23	9.73 567	31	0.26 433	9.94 369	7	27	35 14.0 13.4 12.8
34	9.67 959	24	9.73 597	30	0.26 403	9.94 362	7	26	40 16.0 15.3 14.7
35	9.67 982	23	9.73 627	31	0.26 373	9.94 355	7	25	45 18.0 17.2 16.5
36	9.68 006	24	9.73 657	30	0.26 343	9.94 349	6	24	50 20.0 19.2 18.3
37	9.68 029	23	9.73 687	31	0.26 313	9.94 342	7	23	55 22.0 21.1 20.2
38	9.68 052	24	9.73 717	30	0.26 283	9.94 335	7	22	
39	9.68 075	23	9.73 747	31	0.26 253	9.94 328	7	21	
40	9.68 098	24	9.73 777	30	0.26 223	9.94 321	7	20	
41	9.68 121	23	9.73 807	31	0.26 193	9.94 314	7	19	
42	9.68 144	24	9.73 837	30	0.26 163	9.94 307	7	18	
43	9.68 167	23	9.73 867	31	0.26 133	9.94 300	7	17	
44	9.68 190	24	9.73 897	30	0.26 103	9.94 293	7	16	
45	9.68 213	23	9.73 927	31	0.26 073	9.94 286	7	15	
46	9.68 237	24	9.73 957	30	0.26 043	9.94 279	7	14	
47	9.68 260	23	9.73 987	31	0.26 013	9.94 273	6	13	" 7 6
48	9.68 283	24	9.74 017	30	0.25 983	9.94 266	7	12	5 0.6 0.5
49	9.68 305	23	9.74 047	31	0.25 953	9.94 259	7	11	10 1.2 1.0
50	9.68 328	24	9.74 077	30	0.25 923	9.94 252	7	10	15 1.8 1.5
51	9.68 351	23	9.74 107	31	0.25 893	9.94 245	7	9	20 2.3 2.0
52	9.68 374	24	9.74 137	30	0.25 863	9.94 238	7	8	25 2.9 2.5
53	9.68 397	23	9.74 166	31	0.25 834	9.94 231	7	7	30 3.5 3.0
54	9.68 420	24	9.74 196	30	0.25 804	9.94 224	7	6	35 4.1 3.5
55	9.68 443	23	9.74 226	31	0.25 774	9.94 217	7	5	40 4.7 4.0
56	9.68 466	24	9.74 256	30	0.25 744	9.94 210	7	4	45 5.2 4.5
57	9.68 489	23	9.74 286	31	0.25 714	9.94 203	7	3	50 5.8 5.0
58	9.68 512	24	9.74 316	30	0.25 684	9.94 196	7	2	55 6.4 5.5
59	9.68 534	23	9.74 345	31	0.25 655	9.94 189	7	1	
60	9.68 557	24	9.74 375	30	0.25 625	9.94 182	7	0	
	L. Cos.	d.	L. Cot.	c.d.	L. Tan.	L. Sin.	d.	'	P.P.

'	L. Sin.	d.	L. Tan.	c.d.	L. Cot.	L. Cos.	d.		P.P.
0	9.68 557	23	9.74 375	30	0.25 625	9.94 182	7	60	
1	9.68 580	23	9.74 405	30	0.25 595	9.94 175	7	59	
2	9.68 603	22	9.74 435	30	0.25 565	9.94 168	7	58	
3	9.68 625	23	9.74 465	29	0.25 535	9.94 161	7	57	
4	9.68 648	23	9.74 494	30	0.25 506	9.94 154	7	56	
5	9.68 671	23	9.74 524	30	0.25 476	9.94 147	7	55	
6	9.68 694	22	9.74 554	29	0.25 446	9.94 140	7	54	
7	9.68 716	23	9.74 583	30	0.25 417	9.94 133	7	53	
8	9.68 739	23	9.74 613	30	0.25 387	9.94 126	7	52	
9	9.68 762	22	9.74 643	30	0.25 357	9.94 119	7	51	
10	9.68 784	23	9.74 673	29	0.25 327	9.94 112	7	50	
11	9.68 807	22	9.74 702	30	0.25 298	9.94 105	7	49	" 30 29 23
12	9.68 829	23	9.74 732	30	0.25 268	9.94 098	8	48	5 2.5 2.4 1.9
13	9.68 852	23	9.74 762	29	0.25 238	9.94 090	7	47	10 5.0 4.8 3.8
14	9.68 875	22	9.74 791	30	0.25 209	9.94 083	7	46	15 7.5 7.2 5.8
15	9.68 897	23	9.74 821	30	0.25 179	9.94 076	7	45	20 10.0 9.7 7.7
16	9.68 920	22	9.74 851	29	0.25 149	9.94 069	7	44	25 12.5 12.1 9.6
17	9.68 942	23	9.74 880	30	0.25 120	9.94 062	7	43	30 15.0 14.5 11.5
18	9.68 965	22	9.74 910	29	0.25 090	9.94 055	7	42	35 17.5 16.9 13.4
19	9.68 987	23	9.74 939	30	0.25 061	9.94 048	7	41	40 20.0 19.3 15.3
20	9.69 010	22	9.74 969	29	0.25 031	9.94 041	7	40	45 22.5 21.8 17.2
21	9.69 032	23	9.74 998	30	0.25 002	9.94 034	7	39	50 25.0 24.2 19.2
22	9.69 055	22	9.75 028	30	0.24 972	9.94 027	7	38	55 27.5 26.6 21.1
23	9.69 077	23	9.75 058	29	0.24 942	9.94 020	8	37	
24	9.69 100	22	9.75 087	30	0.24 913	9.94 012	7	36	
25	9.69 122	22	9.75 117	29	0.24 883	9.94 005	7	35	
26	9.69 144	23	9.75 146	30	0.24 854	9.93 998	7	34	
27	9.69 167	22	9.75 176	29	0.24 824	9.93 991	7	33	
28	9.69 189	23	9.75 205	30	0.24 795	9.93 984	7	32	
29	9.69 212	22	9.75 235	29	0.24 765	9.93 977	7	31	
30	9.69 234	22	9.75 264	30	0.24 736	9.93 970	7	30	
31	9.69 256	23	9.75 294	29	0.24 706	9.93 963	8	29	
32	9.69 279	22	9.75 323	30	0.24 677	9.93 955	7	28	
33	9.69 301	22	9.75 353	29	0.24 647	9.93 948	7	27	
34	9.69 323	22	9.75 382	29	0.24 618	9.93 941	7	26	
35	9.69 345	23	9.75 411	30	0.24 589	9.93 934	7	25	
36	9.69 368	22	9.75 441	29	0.24 559	9.93 927	7	24	
37	9.69 390	22	9.75 470	30	0.24 530	9.93 920	8	23	
38	9.69 412	22	9.75 500	29	0.24 500	9.93 912	7	22	
39	9.69 434	22	9.75 529	29	0.24 471	9.93 905	7	21	
40	9.69 456	23	9.75 558	30	0.24 442	9.93 898	7	20	
41	9.69 479	22	9.75 588	29	0.24 412	9.93 891	7	19	" 22 8 7
42	9.69 501	22	9.75 617	30	0.24 383	9.93 884	8	18	5 1.8 0.7 0.6
43	9.69 523	22	9.75 647	29	0.24 353	9.93 876	7	17	10 3.7 1.3 1.2
44	9.69 545	22	9.75 676	29	0.24 324	9.93 869	7	16	15 5.5 2.0 1.8
45	9.69 567	22	9.75 705	30	0.24 295	9.93 862	7	15	20 7.3 2.7 2.3
46	9.69 589	22	9.75 735	29	0.24 265	9.93 855	7	14	25 9.2 3.3 2.9
47	9.69 611	22	9.75 764	29	0.24 236	9.93 847	8	13	30 11.0 4.0 3.5
48	9.69 633	22	9.75 793	29	0.24 207	9.93 840	7	12	35 12.8 4.7 4.1
49	9.69 655	22	9.75 822	30	0.24 178	9.93 833	7	11	40 14.7 5.3 4.7
50	9.69 677	22	9.75 852	29	0.24 148	9.93 826	7	10	45 16.5 6.0 5.2
51	9.69 699	22	9.75 881	29	0.24 119	9.93 819	8	9	50 18.3 6.7 5.8
52	9.69 721	22	9.75 910	29	0.24 090	9.93 811	7	8	55 20.2 7.3 6.4
53	9.69 743	22	9.75 939	30	0.24 061	9.93 804	7	7	
54	9.69 765	22	9.75 969	29	0.24 031	9.93 797	8	6	
55	9.69 787	22	9.75 998	29	0.24 002	9.93 789	7	5	
56	9.69 809	22	9.76 027	29	0.23 973	9.93 782	7	4	
57	9.69 831	22	9.76 056	30	0.23 944	9.93 775	7	3	
58	9.69 853	22	9.76 085	29	0.23 914	9.93 768	8	2	
59	9.69 875	22	9.76 115	29	0.23 885	9.93 760	7	1	
60	9.69 897	22	9.76 144	29	0.23 856	9.93 753	7	0	
	L. Cos.	d.	L. Cot.	c.d.	L. Tan.	L. Sin.	d.	'	P.P.

	L. Sin.	d.	L. Tan.	c.d.	L. Cot.	L. Cos.	d.		P.P.
0	9.69 897		9.76 144		0.23 856	9.93 753		60	
1	9.69 919	22	9.76 173	29	0.23 827	9.93 746	7	59	
2	9.69 941	22	9.76 202	29	0.23 798	9.93 738	8	58	
3	9.69 963	22	9.76 231	29	0.23 769	9.93 731	7	57	
4	9.69 984	21	9.76 261	30	0.23 739	9.93 724	7	56	
5	9.70 006	22	9.76 290	29	0.23 710	9.93 717	7	55	
6	9.70 028	22	9.76 319	29	0.23 681	9.93 709	8	54	" 30 29 28
7	9.70 050	22	9.76 348	29	0.23 652	9.93 702	7	53	5 2.5 2.4 2.3
8	9.70 072	21	9.76 377	29	0.23 623	9.93 695	7	52	10 5.0 4.8 4.7
9	9.70 093	22	9.76 406	29	0.23 594	9.93 687	8	51	15 7.5 7.2 7.0
10	9.70 115	22	9.76 435	29	0.23 565	9.93 680	7	50	20 10.0 9.7 9.3
11	9.70 137	22	9.76 464	29	0.23 536	9.93 673	7	49	25 12.5 12.1 11.7
12	9.70 159	22	9.76 493	29	0.23 507	9.93 665	8	48	30 15.0 14.5 14.0
13	9.70 180	21	9.76 522	29	0.23 478	9.93 658	7	47	35 17.5 16.9 16.3
14	9.70 202	22	9.76 551	29	0.23 449	9.93 650	8	46	40 20.0 19.3 18.7
15	9.70 224	21	9.76 580	29	0.23 420	9.93 643	7	45	45 22.5 21.8 21.0
16	9.70 245	22	9.76 609	30	0.23 391	9.93 636	8	44	50 25.0 24.2 23.3
17	9.70 267	21	9.76 639	29	0.23 361	9.93 628	7	43	55 27.5 26.6 25.7
18	9.70 288	22	9.76 668	29	0.23 332	9.93 621	8	42	
19	9.70 310	22	9.76 697	28	0.23 303	9.93 614	7	41	
20	9.70 332	21	9.76 725	29	0.23 275	9.93 606	8	40	
21	9.70 353	22	9.76 754	29	0.23 246	9.93 599	7	39	
22	9.70 375	22	9.76 783	29	0.23 217	9.93 591	8	38	
23	9.70 396	21	9.76 812	29	0.23 188	9.93 584	7	37	
24	9.70 418	22	9.76 841	29	0.23 159	9.93 577	8	36	
25	9.70 439	21	9.76 870	29	0.23 130	9.93 569	7	35	
26	9.70 461	22	9.76 899	29	0.23 101	9.93 562	8	34	" 22 21
27	9.70 482	21	9.76 928	29	0.23 072	9.93 554	7	33	5 1.8 1.8
28	9.70 504	22	9.76 957	29	0.23 043	9.93 547	8	32	10 3.7 3.5
29	9.70 525	21	9.76 986	29	0.23 014	9.93 539	7	31	15 5.5 5.2
30	9.70 547	22	9.77 015	29	0.22 985	9.93 532	8	30	20 7.3 7.0
31	9.70 568	21	9.77 044	29	0.22 956	9.93 525	7	29	25 9.2 8.8
32	9.70 590	22	9.77 073	28	0.22 927	9.93 517	8	28	30 11.0 10.5
33	9.70 611	21	9.77 101	29	0.22 899	9.93 510	7	27	35 12.8 12.2
34	9.70 633	22	9.77 130	29	0.22 870	9.93 502	8	26	40 14.7 14.0
35	9.70 654	21	9.77 159	29	0.22 841	9.93 495	7	25	45 16.5 15.8
36	9.70 675	22	9.77 188	29	0.22 812	9.93 487	8	24	50 18.3 17.5
37	9.70 697	21	9.77 217	29	0.22 783	9.93 480	7	23	55 20.2 19.2
38	9.70 718	22	9.77 246	28	0.22 754	9.93 472	8	22	
39	9.70 739	21	9.77 274	29	0.22 726	9.93 465	7	21	
40	9.70 761	22	9.77 303	29	0.22 697	9.93 457	8	20	
41	9.70 782	21	9.77 332	29	0.22 668	9.93 450	7	19	
42	9.70 803	22	9.77 361	29	0.22 639	9.93 442	8	18	
43	9.70 824	21	9.77 390	28	0.22 610	9.93 435	7	17	
44	9.70 846	22	9.77 418	29	0.22 582	9.93 427	8	16	
45	9.70 867	21	9.77 447	29	0.22 553	9.93 420	7	15	
46	9.70 888	22	9.77 476	29	0.22 524	9.93 412	8	14	" 8 7
47	9.70 909	21	9.77 505	28	0.22 495	9.93 405	7	13	5 0.7 0.6
48	9.70 931	22	9.77 533	29	0.22 467	9.93 397	8	12	10 1.3 1.2
49	9.70 952	21	9.77 562	29	0.22 438	9.93 390	7	11	15 2.0 1.8
50	9.70 973	22	9.77 591	28	0.22 409	9.93 382	8	10	20 2.7 2.3
51	9.70 994	21	9.77 619	29	0.22 381	9.93 375	7	9	25 3.3 2.9
52	9.71 015	22	9.77 648	29	0.22 352	9.93 367	8	8	30 4.0 3.5
53	9.71 036	21	9.77 677	29	0.22 323	9.93 360	7	7	35 4.7 4.1
54	9.71 058	22	9.77 706	28	0.22 294	9.93 352	8	6	40 5.3 4.7
55	9.71 079	21	9.77 734	29	0.22 266	9.93 344	7	5	45 6.0 5.2
56	9.71 100	22	9.77 763	28	0.22 237	9.93 337	8	4	50 6.7 5.8
57	9.71 121	21	9.77 791	29	0.22 209	9.93 329	7	3	55 7.3 6.4
58	9.71 142	22	9.77 820	29	0.22 180	9.93 322	8	2	
59	9.71 163	21	9.77 849	28	0.22 151	9.93 314	7	1	
60	9.71 184	22	9.77 877	29	0.22 123	9.93 307	8	0	
	L. Cos.	d.	L. Cot.	c.d.	L. Tan.	L. Sin.	d.		P.P.

	L. Sin.	d.	L. Tan.	c.d.	L. Cot.	L. Cos.	d.		P.P.
0	9.71 184		9.77 877		0.22 123	9.93 307		60	
1	9.71 205	21	9.77 906	29	0.22 094	9.93 299	8	59	
2	9.71 226	21	9.77 935	29	0.22 065	9.93 291	8	58	
3	9.71 247	21	9.77 963	28	0.22 037	9.93 284	7	57	
4	9.71 268	21	9.77 992	29	0.22 008	9.93 276	8	56	
5	9.71 289	21	9.78 020	28	0.21 980	9.93 269	7	55	
6	9.71 310	21	9.78 049	29	0.21 951	9.93 261	8	54	
7	9.71 331	21	9.78 077	28	0.21 923	9.93 253	8	53	
8	9.71 352	21	9.78 106	29	0.21 894	9.93 246	7	52	
9	9.71 373	21	9.78 135	29	0.21 865	9.93 238	8	51	
10	9.71 393	20	9.78 163	28	0.21 837	9.93 230	8	50	
11	9.71 414	21	9.78 192	29	0.21 808	9.93 223	7	49	" 29 28 21
12	9.71 435	21	9.78 220	28	0.21 780	9.93 215	8	48	5 2.4 2.3 1.8
13	9.71 456	21	9.78 249	29	0.21 751	9.93 207	8	47	10 4.8 4.7 3.5
14	9.71 477	21	9.78 277	28	0.21 723	9.93 200	7	46	15 7.2 7.0 5.2
15	9.71 498	21	9.78 306	29	0.21 694	9.93 192	8	45	20 9.7 9.3 7.0
16	9.71 519	21	9.78 334	28	0.21 666	9.93 184	8	44	25 12.1 11.7 8.8
17	9.71 539	20	9.78 363	29	0.21 637	9.93 177	7	43	30 14.5 14.0 10.5
18	9.71 560	21	9.78 391	28	0.21 609	9.93 169	8	42	35 16.9 16.3 12.2
19	9.71 581	21	9.78 419	29	0.21 581	9.93 161	8	41	40 19.3 18.7 14.0
20	9.71 602	20	9.78 448	28	0.21 552	9.93 154	7	40	45 21.8 21.0 15.8
21	9.71 622	21	9.78 476	29	0.21 524	9.93 146	8	39	50 24.2 23.3 17.5
22	9.71 643	21	9.78 505	28	0.21 495	9.93 138	8	38	55 26.6 25.7 19.2
23	9.71 664	21	9.78 533	29	0.21 467	9.93 131	7	37	
24	9.71 685	20	9.78 562	28	0.21 438	9.93 123	8	36	
25	9.71 705	21	9.78 590	29	0.21 410	9.93 115	8	35	
26	9.71 726	21	9.78 618	28	0.21 382	9.93 108	7	34	
27	9.71 747	20	9.78 647	29	0.21 353	9.93 100	8	33	
28	9.71 767	21	9.78 675	28	0.21 325	9.93 092	8	32	
29	9.71 788	21	9.78 704	29	0.21 296	9.93 084	8	31	
30	9.71 809	20	9.78 732	28	0.21 268	9.93 077	7	30	
31	9.71 829	21	9.78 760	29	0.21 240	9.93 069	8	29	
32	9.71 850	20	9.78 789	28	0.21 211	9.93 061	8	28	
33	9.71 870	21	9.78 817	29	0.21 183	9.93 053	8	27	
34	9.71 891	20	9.78 845	28	0.21 155	9.93 046	7	26	
35	9.71 911	21	9.78 874	29	0.21 126	9.93 038	8	25	
36	9.71 932	21	9.78 902	28	0.21 098	9.93 030	8	24	
37	9.71 952	20	9.78 930	29	0.21 070	9.93 022	8	23	
38	9.71 973	21	9.78 959	28	0.21 041	9.93 014	8	22	
39	9.71 994	20	9.78 987	29	0.21 013	9.93 007	7	21	
40	9.72 014	20	9.79 015	28	0.20 985	9.92 999	8	20	
41	9.72 034	21	9.79 043	29	0.20 957	9.92 991	8	19	" 20 8 7
42	9.72 055	20	9.79 072	28	0.20 928	9.92 983	8	18	5 1.7 0.7 0.6
43	9.72 075	21	9.79 100	29	0.20 900	9.92 976	7	17	10 3.3 1.3 1.2
44	9.72 096	20	9.79 128	28	0.20 872	9.92 968	8	16	15 5.0 2.0 1.8
45	9.72 116	21	9.79 156	29	0.20 844	9.92 960	8	15	20 6.7 2.7 2.3
46	9.72 137	20	9.79 185	28	0.20 815	9.92 952	8	14	25 8.3 3.3 2.9
47	9.72 157	21	9.79 213	29	0.20 787	9.92 944	8	13	30 10.0 4.0 3.5
48	9.72 177	20	9.79 241	28	0.20 759	9.92 936	8	12	35 11.7 4.7 4.1
49	9.72 198	21	9.79 269	29	0.20 731	9.92 929	7	11	40 13.3 5.3 4.7
50	9.72 218	20	9.79 297	28	0.20 703	9.92 921	8	10	45 15.0 6.0 5.2
51	9.72 238	21	9.79 326	29	0.20 674	9.92 913	8	9	50 16.7 6.7 5.8
52	9.72 259	20	9.79 354	28	0.20 646	9.92 905	8	8	55 18.3 7.3 6.4
53	9.72 279	21	9.79 382	29	0.20 618	9.92 897	8	7	
54	9.72 299	20	9.79 410	28	0.20 590	9.92 889	8	6	
55	9.72 320	21	9.79 438	29	0.20 562	9.92 881	8	5	
56	9.72 340	20	9.79 466	28	0.20 534	9.92 874	7	4	
57	9.72 360	21	9.79 495	29	0.20 505	9.92 866	8	3	
58	9.72 381	20	9.79 523	28	0.20 477	9.92 858	8	2	
59	9.72 401	21	9.79 551	29	0.20 449	9.92 850	8	1	
60	9.72 421	20	9.79 579	28	0.20 421	9.92 842	8	0	
	L. Cos.	d.	L. Cot.	c.d.	L. Tan.	L. Sin.	d.		P.P.

	L. Sin.	d.	L. Tan.	c.d.	L. Cot.	L. Cos.	d.		P.P.
0	9.72 421		9.79 579		0.20 421	9.92 842		60	
1	9.72 441	20	9.79 607	28	0.20 393	9.92 834	8	59	
2	9.72 461	20	9.79 635	28	0.20 365	9.92 826	8	58	
3	9.72 482	20	9.79 663	28	0.20 337	9.92 818	8	57	
4	9.72 502	20	9.79 691	28	0.20 309	9.92 810	8	56	
5	9.72 522	20	9.79 719	28	0.20 281	9.92 803	7	55	
6	9.72 542	20	9.79 747	28	0.20 253	9.92 795	8	54	
7	9.72 562	20	9.79 776	28	0.20 224	9.92 787	8	53	" 29 28 27
8	9.72 582	20	9.79 804	28	0.20 196	9.92 779	8	52	5 2.4 2.3 2.2
9	9.72 602	20	9.79 832	28	0.20 168	9.92 771	8	51	10 4.8 4.7 4.5
10	9.72 622	20	9.79 860	28	0.20 140	9.92 763	8	50	15 7.2 7.0 6.8
11	9.72 643	20	9.79 888	28	0.20 112	9.92 755	8	49	20 9.7 9.3 9.0
12	9.72 663	20	9.79 916	28	0.20 084	9.92 747	8	48	25 12.1 11.7 11.2
13	9.72 683	20	9.79 944	28	0.20 056	9.92 739	8	47	30 14.5 14.0 13.5
14	9.72 703	20	9.79 972	28	0.20 028	9.92 731	8	46	35 16.9 16.3 15.8
15	9.72 723	20	9.80 000	28	0.20 000	9.92 723	8	45	40 19.3 18.7 18.0
16	9.72 743	20	9.80 028	28	0.19 972	9.92 715	8	44	45 21.8 21.0 20.2
17	9.72 763	20	9.80 056	28	0.19 944	9.92 707	8	43	50 24.2 23.3 22.5
18	9.72 783	20	9.80 084	28	0.19 916	9.92 699	8	42	55 26.6 25.7 24.8
19	9.72 803	20	9.80 112	28	0.19 888	9.92 691	8	41	
20	9.72 823	20	9.80 140	28	0.19 860	9.92 683	8	40	
21	9.72 843	20	9.80 168	27	0.19 832	9.92 675	8	39	
22	9.72 863	20	9.80 195	28	0.19 805	9.92 667	8	38	
23	9.72 883	19	9.80 223	28	0.19 777	9.92 659	8	37	
24	9.72 902	20	9.80 251	28	0.19 749	9.92 651	8	36	
25	9.72 922	20	9.80 279	28	0.19 721	9.92 643	8	35	
26	9.72 942	20	9.80 307	28	0.19 693	9.92 635	8	34	" 21 20 19
27	9.72 962	20	9.80 335	28	0.19 665	9.92 627	8	33	5 1.8 1.7 1.6
28	9.72 982	20	9.80 363	28	0.19 637	9.92 619	8	32	10 3.5 3.3 3.2
29	9.73 002	20	9.80 391	28	0.19 609	9.92 611	8	31	15 5.2 5.0 4.8
30	9.73 022	19	9.80 419	28	0.19 581	9.92 603	8	30	20 7.0 6.7 6.3
31	9.73 041	20	9.80 447	27	0.19 553	9.92 595	8	29	25 8.8 8.3 7.9
32	9.73 061	20	9.80 474	28	0.19 526	9.92 587	8	28	30 10.5 10.0 9.5
33	9.73 081	20	9.80 502	28	0.19 498	9.92 579	8	27	35 12.2 11.7 11.1
34	9.73 101	20	9.80 530	28	0.19 470	9.92 571	8	26	40 14.0 13.3 12.7
35	9.73 121	19	9.80 558	28	0.19 442	9.92 563	8	25	45 15.8 15.0 14.2
36	9.73 140	20	9.80 586	28	0.19 414	9.92 555	8	24	50 17.5 16.7 15.8
37	9.73 160	20	9.80 614	28	0.19 386	9.92 546	9	23	55 19.2 18.3 17.4
38	9.73 180	20	9.80 642	27	0.19 358	9.92 538	8	22	
39	9.73 200	19	9.80 669	28	0.19 331	9.92 530	8	21	
40	9.73 219	20	9.80 697	28	0.19 303	9.92 522	8	20	
41	9.73 239	20	9.80 725	28	0.19 275	9.92 514	8	19	
42	9.73 259	19	9.80 753	28	0.19 247	9.92 506	8	18	
43	9.73 278	20	9.80 781	27	0.19 219	9.92 498	8	17	
44	9.73 298	20	9.80 808	28	0.19 192	9.92 490	8	16	
45	9.73 318	19	9.80 836	28	0.19 164	9.92 482	9	15	
46	9.73 337	20	9.80 864	28	0.19 136	9.92 473	8	14	" 9 8 7
47	9.73 357	20	9.80 892	27	0.19 108	9.92 465	8	13	5 0.8 0.7 0.6
48	9.73 377	19	9.80 919	28	0.19 081	9.92 457	8	12	10 1.5 1.3 1.2
49	9.73 396	20	9.80 947	28	0.19 053	9.92 449	8	11	15 2.2 2.0 1.8
50	9.73 416	19	9.80 975	28	0.19 025	9.92 441	8	10	20 3.0 2.7 2.3
51	9.73 435	20	9.81 003	27	0.18 997	9.92 433	8	9	25 3.8 3.3 2.9
52	9.73 455	19	9.81 030	28	0.18 970	9.92 425	9	8	30 4.5 4.0 3.5
53	9.73 474	20	9.81 058	28	0.18 942	9.92 416	8	7	35 5.2 4.7 4.1
54	9.73 494	19	9.81 086	27	0.18 914	9.92 408	8	6	40 6.0 5.3 4.7
55	9.73 513	20	9.81 113	28	0.18 887	9.92 400	8	5	45 6.8 6.0 5.2
56	9.73 533	19	9.81 141	28	0.18 859	9.92 392	8	4	50 7.5 6.7 5.8
57	9.73 552	20	9.81 169	27	0.18 831	9.92 384	8	3	55 8.2 7.3 6.4
58	9.73 572	19	9.81 196	28	0.18 804	9.92 376	9	2	
59	9.73 591	20	9.81 224	28	0.18 776	9.92 367	8	1	
60	9.73 611		9.81 252		0.18 748	9.92 359		0	
	L. Cos.	d.	L. Cot.	c.d.	L. Tan.	L. Sin.	d.		P.P.

'	L. Sin.	d.	L. Tan.	c.d.	L. Cot.	L. Cos.	d.		P.P.
0	9.73 611	19	9.81 252	27	0.18 748	9.92 359	8	60	
1	9.73 630	20	9.81 279	28	0.18 721	9.92 351	8	59	
2	9.73 650	19	9.81 307	28	0.18 693	9.92 343	8	58	
3	9.73 669	20	9.81 335	27	0.18 665	9.92 335	9	57	
4	9.73 689	19	9.81 362	28	0.18 638	9.92 326	8	56	
5	9.73 708	19	9.81 390	28	0.18 610	9.92 318	8	55	
6	9.73 727	20	9.81 418	27	0.18 582	9.92 310	8	54	" 28 27
7	9.73 747	19	9.81 445	28	0.18 555	9.92 302	9	53	5 2.3 2.2
8	9.73 766	19	9.81 473	27	0.18 527	9.92 293	8	52	10 4.7 4.5
9	9.73 785	20	9.81 500	28	0.18 500	9.92 285	9	51	15 7.0 6.8
10	9.73 805	19	9.81 528	28	0.18 472	9.92 277	8	50	20 9.3 9.0
11	9.73 824	19	9.81 556	27	0.18 444	9.92 269	8	49	25 11.7 11.2
12	9.73 843	20	9.81 583	28	0.18 417	9.92 260	9	48	30 14.0 13.5
13	9.73 863	19	9.81 611	27	0.18 389	9.92 252	8	47	35 16.3 15.8
14	9.73 882	19	9.81 638	28	0.18 362	9.92 244	9	46	40 18.7 18.0
15	9.73 901	20	9.81 666	27	0.18 334	9.92 235	8	45	45 21.0 20.2
16	9.73 921	19	9.81 693	28	0.18 307	9.92 227	9	44	50 23.3 22.5
17	9.73 940	19	9.81 721	27	0.18 279	9.92 219	8	43	55 25.7 24.8
18	9.73 959	19	9.81 748	28	0.18 252	9.92 211	9	42	
19	9.73 978	20	9.81 776	27	0.18 224	9.92 202	8	41	
20	9.73 997	19	9.81 803	28	0.18 197	9.92 194	9	40	
21	9.74 017	19	9.81 831	27	0.18 169	9.92 186	8	39	
22	9.74 036	20	9.81 858	28	0.18 142	9.92 177	9	38	
23	9.74 055	19	9.81 886	27	0.18 114	9.92 169	8	37	
24	9.74 074	19	9.81 913	28	0.18 087	9.92 161	9	36	
25	9.74 093	20	9.81 941	27	0.18 059	9.92 152	8	35	
26	9.74 113	19	9.81 968	28	0.18 032	9.92 144	9	34	" 20 19 18
27	9.74 132	19	9.81 996	27	0.18 004	9.92 136	8	33	5 1.7 1.6 1.5
28	9.74 151	20	9.82 023	28	0.17 977	9.92 127	9	32	10 3.3 3.2 3.0
29	9.74 170	19	9.82 051	27	0.17 949	9.92 119	8	31	15 5.0 4.8 4.5
30	9.74 189	19	9.82 078	28	0.17 922	9.92 111	9	30	20 6.7 6.3 6.0
31	9.74 208	20	9.82 106	27	0.17 894	9.92 102	8	29	25 8.3 7.9 7.5
32	9.74 227	19	9.82 133	28	0.17 867	9.92 094	9	28	30 10.0 9.5 9.0
33	9.74 246	19	9.82 161	27	0.17 839	9.92 086	8	27	35 11.7 11.1 10.5
34	9.74 265	20	9.82 188	28	0.17 812	9.92 077	9	26	40 13.3 12.7 12.0
35	9.74 284	19	9.82 215	27	0.17 785	9.92 069	8	25	45 15.0 14.2 13.5
36	9.74 303	19	9.82 243	28	0.17 757	9.92 060	9	24	50 16.7 15.8 15.0
37	9.74 322	20	9.82 270	27	0.17 730	9.92 052	8	23	55 18.3 17.4 16.5
38	9.74 341	19	9.82 298	28	0.17 702	9.92 044	9	22	
39	9.74 360	19	9.82 325	27	0.17 675	9.92 035	8	21	
40	9.74 379	20	9.82 352	28	0.17 648	9.92 027	9	20	
41	9.74 398	19	9.82 380	27	0.17 620	9.92 018	8	19	
42	9.74 417	19	9.82 407	28	0.17 593	9.92 010	9	18	
43	9.74 436	20	9.82 435	27	0.17 565	9.92 002	8	17	
44	9.74 455	19	9.82 462	28	0.17 538	9.91 993	9	16	
45	9.74 474	19	9.82 489	27	0.17 511	9.91 985	8	15	
46	9.74 493	20	9.82 517	28	0.17 483	9.91 976	9	14	" 9 8
47	9.74 512	19	9.82 544	27	0.17 456	9.91 968	8	13	5 0.8 0.7
48	9.74 531	18	9.82 571	28	0.17 429	9.91 959	9	12	10 1.5 1.3
49	9.74 549	19	9.82 599	27	0.17 401	9.91 951	8	11	15 2.2 2.0
50	9.74 568	20	9.82 626	28	0.17 374	9.91 942	9	10	20 3.0 2.7
51	9.74 587	19	9.82 653	27	0.17 347	9.91 934	8	9	25 3.8 3.3
52	9.74 606	19	9.82 681	28	0.17 319	9.91 925	9	8	30 4.5 4.0
53	9.74 625	20	9.82 708	27	0.17 292	9.91 917	8	7	35 5.2 4.7
54	9.74 644	19	9.82 735	28	0.17 265	9.91 908	9	6	40 6.0 5.3
55	9.74 662	18	9.82 762	27	0.17 238	9.91 900	8	5	45 6.8 6.0
56	9.74 681	19	9.82 790	28	0.17 210	9.91 891	9	4	50 7.5 6.7
57	9.74 700	20	9.82 817	27	0.17 183	9.91 883	8	3	55 8.2 7.3
58	9.74 719	19	9.82 844	28	0.17 156	9.91 874	9	2	
59	9.74 737	18	9.82 871	27	0.17 129	9.91 866	8	1	
60	9.74 756	19	9.82 899	28	0.17 101	9.91 857	9	0	
	L. Cos.	d.	L. Cot.	c.d.	L. Tan.	L. Sin.	d.	'	P.P.

	L. Sin.	d.	L. Tan.	c.d.	L. Cot.	L. Cos.	d.		P.P.
0	9.74 756	19	9.82 899	27	0.17 101	9.91 857	8	60	
1	9.74 775	19	9.82 926	27	0.17 074	9.91 849	9	59	
2	9.74 794	18	9.82 953	27	0.17 047	9.91 840	8	58	
3	9.74 812	19	9.82 980	28	0.17 020	9.91 832	9	57	
4	9.74 831	19	9.83 008	27	0.16 992	9.91 823	8	56	
5	9.74 850	18	9.83 035	27	0.16 965	9.91 815	9	55	
6	9.74 868	19	9.83 062	27	0.16 938	9.91 806	8	54	" 28 27 26
7	9.74 887	19	9.83 089	28	0.16 911	9.91 798	9	53	5 2.3 2.2 2.2
8	9.74 906	18	9.83 117	27	0.16 883	9.91 789	8	52	10 4.7 4.5 4.3
9	9.74 924	19	9.83 144	27	0.16 856	9.91 781	9	51	15 7.0 6.8 6.5
10	9.74 943	18	9.83 171	27	0.16 829	9.91 772	9	50	20 9.3 9.0 8.7
11	9.74 961	19	9.83 198	27	0.16 802	9.91 763	8	49	25 11.7 11.2 10.8
12	9.74 980	19	9.83 225	27	0.16 775	9.91 755	9	48	30 14.0 13.5 13.0
13	9.74 999	18	9.83 252	28	0.16 748	9.91 746	8	47	35 16.3 15.8 15.2
14	9.75 017	19	9.83 280	27	0.16 720	9.91 738	9	46	40 18.7 18.0 17.3
15	9.75 036	18	9.83 307	27	0.16 693	9.91 729	8	45	45 21.0 20.2 19.5
16	9.75 054	19	9.83 334	27	0.16 666	9.91 720	9	44	50 23.3 22.5 21.7
17	9.75 073	18	9.83 361	27	0.16 639	9.91 712	8	43	55 25.7 24.8 23.8
18	9.75 091	19	9.83 388	27	0.16 612	9.91 703	9	42	
19	9.75 110	18	9.83 415	27	0.16 585	9.91 695	8	41	
20	9.75 128	19	9.83 442	28	0.16 558	9.91 686	9	40	
21	9.75 147	18	9.83 470	27	0.16 530	9.91 677	8	39	
22	9.75 165	19	9.83 497	27	0.16 503	9.91 669	9	38	
23	9.75 184	18	9.83 524	27	0.16 476	9.91 660	8	37	
24	9.75 202	19	9.83 551	27	0.16 449	9.91 651	9	36	
25	9.75 221	18	9.83 578	27	0.16 422	9.91 643	8	35	
26	9.75 239	19	9.83 605	27	0.16 395	9.91 634	9	34	" 19 18
27	9.75 258	18	9.83 632	27	0.16 368	9.91 625	8	33	5 1.6 1.5
28	9.75 276	19	9.83 659	27	0.16 341	9.91 617	9	32	10 3.2 3.0
29	9.75 294	18	9.83 686	27	0.16 314	9.91 608	8	31	15 4.8 4.5
30	9.75 313	19	9.83 713	27	0.16 287	9.91 599	9	30	20 6.3 6.0
31	9.75 331	18	9.83 740	28	0.16 260	9.91 591	8	29	25 7.9 7.5
32	9.75 350	19	9.83 768	27	0.16 232	9.91 582	9	28	30 9.5 9.0
33	9.75 368	18	9.83 795	27	0.16 205	9.91 573	8	27	35 11.1 10.5
34	9.75 386	19	9.83 822	27	0.16 178	9.91 565	9	26	40 12.7 12.0
35	9.75 405	18	9.83 849	27	0.16 151	9.91 556	8	25	45 14.2 13.5
36	9.75 423	19	9.83 876	27	0.16 124	9.91 547	9	24	50 15.8 15.0
37	9.75 441	18	9.83 903	27	0.16 097	9.91 538	8	23	55 17.4 16.5
38	9.75 459	19	9.83 930	27	0.16 070	9.91 530	9	22	
39	9.75 478	18	9.83 957	27	0.16 043	9.91 521	8	21	
40	9.75 496	19	9.83 984	27	0.16 016	9.91 512	9	20	
41	9.75 514	18	9.84 011	27	0.15 989	9.91 504	8	19	
42	9.75 533	19	9.84 038	27	0.15 962	9.91 495	9	18	
43	9.75 551	18	9.84 065	27	0.15 935	9.91 486	8	17	
44	9.75 569	19	9.84 092	27	0.15 908	9.91 477	9	16	
45	9.75 587	18	9.84 119	27	0.15 881	9.91 469	8	15	
46	9.75 605	19	9.84 146	27	0.15 854	9.91 460	9	14	" 9 8
47	9.75 624	18	9.84 173	27	0.15 827	9.91 451	8	13	5 0.8 0.7
48	9.75 642	19	9.84 200	27	0.15 800	9.91 442	9	12	10 1.5 1.3
49	9.75 660	18	9.84 227	27	0.15 773	9.91 433	8	11	15 2.2 2.0
50	9.75 678	19	9.84 254	26	0.15 746	9.91 425	9	10	20 3.0 2.7
51	9.75 696	18	9.84 280	27	0.15 720	9.91 416	8	9	25 3.8 3.3
52	9.75 714	19	9.84 307	27	0.15 693	9.91 407	9	8	30 4.5 4.0
53	9.75 733	18	9.84 334	27	0.15 666	9.91 398	8	7	35 5.2 4.7
54	9.75 751	19	9.84 361	27	0.15 639	9.91 389	9	6	40 6.0 5.3
55	9.75 769	18	9.84 388	27	0.15 612	9.91 381	8	5	45 6.8 6.0
56	9.75 787	19	9.84 415	27	0.15 585	9.91 372	9	4	50 7.5 6.7
57	9.75 805	18	9.84 442	27	0.15 558	9.91 363	8	3	55 8.2 7.3
58	9.75 823	19	9.84 469	27	0.15 531	9.91 354	9	2	
59	9.75 841	18	9.84 496	27	0.15 504	9.91 345	8	1	
60	9.75 859	19	9.84 523	27	0.15 477	9.91 336	9	0	
	L. Cos.	d.	L. Cot.	c.d.	L. Tan.	L. Sin.	d.		P.P.

	L. Sin.	d.	L. Tan.	c.d.	L. Cot.	L. Cos.	d.		P.P.
0	9.75 859		9.84 523		0.15 477	9.91 336		60	
1	9.75 877	18	9.84 550	27	0.15 450	9.91 328	8	59	
2	9.75 895	18	9.84 576	26	0.15 424	9.91 319	9	58	
3	9.75 913	18	9.84 603	27	0.15 397	9.91 310	9	57	
4	9.75 931	18	9.84 630	27	0.15 370	9.91 301	9	56	
5	9.75 949	18	9.84 657	27	0.15 343	9.91 292	9	55	
6	9.75 967	18	9.84 684	27	0.15 316	9.91 283	9	54	" 27 26
7	9.75 985	18	9.84 711	27	0.15 289	9.91 274	8	53	5 2.2 2.2
8	9.76 003	18	9.84 738	26	0.15 262	9.91 266	9	52	10 4.5 4.3
9	9.76 021	18	9.84 764	27	0.15 236	9.91 257	9	51	15 6.8 6.5
10	9.76 039	18	9.84 791	27	0.15 209	9.91 248	9	50	20 9.0 8.7
11	9.76 057	18	9.84 818	27	0.15 182	9.91 239	9	49	25 11.2 10.8
12	9.76 075	18	9.84 845	27	0.15 155	9.91 230	9	48	30 13.5 13.0
13	9.76 093	18	9.84 872	27	0.15 128	9.91 221	9	47	35 15.8 15.2
14	9.76 111	18	9.84 899	26	0.15 101	9.91 212	9	46	40 18.0 17.3
15	9.76 129	17	9.84 925	27	0.15 075	9.91 203	9	45	45 20.2 19.5
16	9.76 146	18	9.84 952	27	0.15 048	9.91 194	9	44	50 22.5 21.7
17	9.76 164	18	9.84 979	27	0.15 021	9.91 185	9	43	55 24.8 23.8
18	9.76 182	18	9.85 006	27	0.14 994	9.91 176	9	42	
19	9.76 200	18	9.85 033	26	0.14 967	9.91 167	9	41	
20	9.76 218	18	9.85 059	27	0.14 941	9.91 158	9	40	
21	9.76 236	17	9.85 086	27	0.14 914	9.91 149	8	39	
22	9.76 253	18	9.85 113	27	0.14 887	9.91 141	9	38	
23	9.76 271	18	9.85 140	26	0.14 860	9.91 132	9	37	
24	9.76 289	18	9.85 166	27	0.14 834	9.91 123	9	36	
25	9.76 307	17	9.85 193	27	0.14 807	9.91 114	9	35	
26	9.76 324	18	9.85 220	27	0.14 780	9.91 105	9	34	" 18 17
27	9.76 342	18	9.85 247	26	0.14 753	9.91 096	9	33	5 1.5 1.4
28	9.76 360	18	9.85 273	27	0.14 727	9.91 087	9	32	10 3.0 2.8
29	9.76 378	17	9.85 300	27	0.14 700	9.91 078	9	31	15 4.5 4.2
30	9.76 395	18	9.85 327	27	0.14 673	9.91 069	9	30	20 6.0 5.7
31	9.76 413	18	9.85 354	26	0.14 646	9.91 060	9	29	25 7.5 7.1
32	9.76 431	17	9.85 380	27	0.14 620	9.91 051	9	28	30 9.0 8.5
33	9.76 448	18	9.85 407	27	0.14 593	9.91 042	9	27	35 10.5 9.9
34	9.76 466	18	9.85 434	26	0.14 566	9.91 033	10	26	40 12.0 11.3
35	9.76 484	17	9.85 460	27	0.14 540	9.91 023	9	25	45 13.5 12.8
36	9.76 501	18	9.85 487	27	0.14 513	9.91 014	9	24	50 15.0 14.2
37	9.76 519	18	9.85 514	26	0.14 486	9.91 005	9	23	55 16.5 15.6
38	9.76 537	17	9.85 540	27	0.14 460	9.90 996	9	22	
39	9.76 554	18	9.85 567	27	0.14 433	9.90 987	9	21	
40	9.76 572	18	9.85 594	26	0.14 406	9.90 978	9	20	
41	9.76 590	17	9.85 620	27	0.14 380	9.90 969	9	19	
42	9.76 607	18	9.85 647	27	0.14 353	9.90 960	9	18	
43	9.76 625	17	9.85 674	26	0.14 326	9.90 951	9	17	
44	9.76 642	18	9.85 700	27	0.14 300	9.90 942	9	16	
45	9.76 660	17	9.85 727	27	0.14 273	9.90 933	9	15	
46	9.76 677	18	9.85 754	26	0.14 246	9.90 924	9	14	" 10 9 8
47	9.76 695	17	9.85 780	27	0.14 220	9.90 915	9	13	5 0.8 0.8 0.7
48	9.76 712	18	9.85 807	27	0.14 193	9.90 906	10	12	10 1.7 1.5 1.3
49	9.76 730	17	9.85 834	26	0.14 166	9.90 896	9	11	15 2.5 2.2 2.0
50	9.76 747	18	9.85 860	27	0.14 140	9.90 887	9	10	20 3.3 3.0 2.7
51	9.76 765	17	9.85 887	26	0.14 113	9.90 878	9	9	25 4.2 3.8 3.3
52	9.76 782	18	9.85 913	27	0.14 087	9.90 869	9	8	30 5.0 4.5 4.0
53	9.76 800	17	9.85 940	27	0.14 060	9.90 860	9	7	35 5.8 5.2 4.7
54	9.76 817	18	9.85 967	26	0.14 033	9.90 851	9	6	40 6.7 6.0 5.3
55	9.76 835	17	9.85 993	27	0.14 007	9.90 842	10	5	45 7.5 6.8 6.0
56	9.76 852	18	9.86 020	26	0.13 980	9.90 832	9	4	50 8.3 7.5 6.7
57	9.76 870	17	9.86 046	27	0.13 954	9.90 823	9	3	55 9.2 8.2 7.3
58	9.76 887	17	9.86 073	27	0.13 927	9.90 814	9	2	
59	9.76 904	18	9.86 100	26	0.13 900	9.90 805	9	1	
60	9.76 922		9.86 126		0.13 874	9.90 796		0	
	L. Cos.	d.	L. Cot.	c.d.	T. Tan.	L. Sin.	d.		P.P.

'	L. Sin.	d.	L. Tan.	c.d.	L. Cot.	L. Cos.	d.		P.P.
0	9.76 922	17	9.86 126	27	0.13 874	9.90 796	9	60	
1	9.76 939	18	9.86 153	26	0.13 847	9.90 787	10	59	
2	9.76 957	17	9.86 179	27	0.13 821	9.90 777	9	58	
3	9.76 974	17	9.86 206	26	0.13 794	9.90 768	9	57	
4	9.76 991	18	9.86 232	27	0.13 768	9.90 759	9	56	
5	9.77 009	17	9.86 259	26	0.13 741	9.90 750	9	55	
6	9.77 026	17	9.86 285	27	0.13 715	9.90 741	10	54	" 27 26
7	9.77 043	18	9.86 312	26	0.13 688	9.90 731	9	53	5 2.2 2.2
8	9.77 061	17	9.86 338	27	0.13 662	9.90 722	9	52	10 4.5 4.3
9	9.77 078	17	9.86 365	27	0.13 635	9.90 713	9	51	15 6.8 6.5
10	9.77 095	17	9.86 392	26	0.13 608	9.90 704	10	50	20 9.0 8.7
11	9.77 112	18	9.86 418	27	0.13 582	9.90 694	9	49	25 11.2 10.8
12	9.77 130	17	9.86 445	26	0.13 555	9.90 685	10	48	30 13.5 13.0
13	9.77 147	17	9.86 471	27	0.13 529	9.90 676	9	47	35 15.8 15.2
14	9.77 164	17	9.86 498	26	0.13 502	9.90 667	9	46	40 18.0 17.3
15	9.77 181	18	9.86 524	27	0.13 476	9.90 657	10	45	45 20.2 19.5
16	9.77 199	17	9.86 551	26	0.13 449	9.90 648	9	44	50 22.5 21.7
17	9.77 216	17	9.86 577	26	0.13 423	9.90 639	9	43	55 24.8 23.8
18	9.77 233	17	9.86 603	27	0.13 397	9.90 630	10	42	
19	9.77 250	18	9.86 630	26	0.13 370	9.90 620	9	41	
20	9.77 268	17	9.86 656	27	0.13 344	9.90 611	9	40	
21	9.77 285	17	9.86 683	26	0.13 317	9.90 602	10	39	
22	9.77 302	17	9.86 709	27	0.13 291	9.90 592	9	38	
23	9.77 319	17	9.86 736	26	0.13 264	9.90 583	9	37	
24	9.77 336	17	9.86 762	27	0.13 238	9.90 574	9	36	
25	9.77 353	17	9.86 789	26	0.13 211	9.90 565	10	35	" 18 17 16
26	9.77 370	17	9.86 815	27	0.13 185	9.90 555	9	34	5 1.5 1.4 1.3
27	9.77 387	18	9.86 842	26	0.13 158	9.90 546	9	33	10 3.0 2.8 2.7
28	9.77 405	17	9.86 868	26	0.13 132	9.90 537	10	32	15 4.5 4.2 4.0
29	9.77 422	17	9.86 894	27	0.13 106	9.90 527	9	31	20 6.0 5.7 5.3
30	9.77 439	17	9.86 921	26	0.13 079	9.90 518	9	30	25 7.5 7.1 6.7
31	9.77 456	17	9.86 947	27	0.13 053	9.90 509	10	29	30 9.0 8.5 8.0
32	9.77 473	17	9.86 974	26	0.13 026	9.90 499	9	28	35 10.5 9.9 9.3
33	9.77 490	17	9.87 000	27	0.13 000	9.90 490	10	27	40 12.0 11.3 10.7
34	9.77 507	17	9.87 027	26	0.12 973	9.90 480	9	26	45 13.5 12.8 12.0
35	9.77 524	17	9.87 053	26	0.12 947	9.90 471	9	25	50 15.0 14.2 13.3
36	9.77 541	17	9.87 079	27	0.12 921	9.90 462	10	24	55 16.5 15.6 14.7
37	9.77 558	17	9.87 106	26	0.12 894	9.90 452	9	23	
38	9.77 575	17	9.87 132	26	0.12 868	9.90 443	9	22	
39	9.77 592	17	9.87 158	27	0.12 842	9.90 434	10	21	
40	9.77 609	17	9.87 185	26	0.12 815	9.90 424	9	20	
41	9.77 626	17	9.87 211	27	0.12 789	9.90 415	10	19	
42	9.77 643	17	9.87 238	26	0.12 762	9.90 405	9	18	
43	9.77 660	17	9.87 264	26	0.12 736	9.90 396	10	17	
44	9.77 677	17	9.87 290	27	0.12 710	9.90 386	9	16	
45	9.77 694	17	9.87 317	26	0.12 683	9.90 377	9	15	
46	9.77 711	17	9.87 343	26	0.12 657	9.90 368	10	14	" 10 9
47	9.77 728	16	9.87 369	27	0.12 631	9.90 358	9	13	5 0.8 0.8
48	9.77 744	17	9.87 396	26	0.12 604	9.90 349	10	12	10 1.7 1.5
49	9.77 761	17	9.87 422	26	0.12 578	9.90 339	9	11	15 2.5 2.2
50	9.77 778	17	9.87 448	27	0.12 552	9.90 330	10	10	20 3.3 3.0
51	9.77 795	17	9.87 475	26	0.12 525	9.90 320	9	9	25 4.2 3.8
52	9.77 812	17	9.87 501	26	0.12 499	9.90 311	10	8	30 5.0 4.5
53	9.77 829	17	9.87 527	27	0.12 473	9.90 301	9	7	35 5.8 5.2
54	9.77 846	16	9.87 554	26	0.12 446	9.90 292	10	6	40 6.7 6.0
55	9.77 862	17	9.87 580	26	0.12 420	9.90 282	9	5	45 7.5 6.8
56	9.77 879	17	9.87 606	27	0.12 394	9.90 273	10	4	50 8.3 7.5
57	9.77 896	17	9.87 633	26	0.12 367	9.90 263	9	3	55 9.2 8.2
58	9.77 913	17	9.87 659	26	0.12 341	9.90 254	10	2	
59	9.77 930	16	9.87 685	26	0.12 315	9.90 244	9	1	
60	9.77 946		9.87 711		0.12 289	9.90 235		0	
	L. Cos.	d.	L. Cot.	c.d.	L. Tan.	L. Sin.	d.	'	P.P.

'	L. Sin.	d.	L. Tan.	c.d.	L. Cot.	L. Cos.	d.		P.P.
0	9.77 946	17	9.87 711	27	0.12 289	9.90 235	10	60	
1	9.77 963	17	9.87 738	26	0.12 262	9.90 225	9	59	
2	9.77 980	17	9.87 764	26	0.12 236	9.90 216	10	58	
3	9.77 997	16	9.87 790	27	0.12 210	9.90 206	9	57	
4	9.78 013	17	9.87 817	26	0.12 183	9.90 197	10	56	
5	9.78 030	17	9.87 843	26	0.12 157	9.90 187	9	55	
6	9.78 047	16	9.87 869	26	0.12 131	9.90 178	10	54	
7	9.78 063	17	9.87 895	27	0.12 105	9.90 168	9	53	
8	9.78 080	17	9.87 922	26	0.12 078	9.90 159	10	52	
9	9.78 097	16	9.87 948	26	0.12 052	9.90 149	10	51	
10	9.78 113	17	9.87 974	26	0.12 026	9.90 139	9	50	
11	9.78 130	17	9.88 000	27	0.12 000	9.90 130	10	49	" 27 26 17
12	9.78 147	16	9.88 027	26	0.11 973	9.90 120	9	48	5 2.2 2.2 1.4
13	9.78 163	17	9.88 053	26	0.11 947	9.90 111	10	47	10 4.5 4.3 2.8
14	9.78 180	17	9.88 079	26	0.11 921	9.90 101	10	46	15 6.8 6.5 4.2
15	9.78 197	16	9.88 105	26	0.11 895	9.90 091	10	45	20 9.0 8.7 5.7
16	9.78 213	17	9.88 131	27	0.11 869	9.90 082	9	44	25 11.2 10.8 7.1
17	9.78 230	16	9.88 158	26	0.11 842	9.90 072	10	43	30 13.5 13.0 8.5
18	9.78 246	17	9.88 184	26	0.11 816	9.90 063	9	42	35 15.8 15.2 9.9
19	9.78 263	17	9.88 210	26	0.11 790	9.90 053	10	41	40 18.0 17.3 11.3
20	9.78 280	16	9.88 236	26	0.11 764	9.90 043	10	40	45 20.2 19.5 12.8
21	9.78 296	17	9.88 262	27	0.11 738	9.90 034	9	39	50 22.5 21.7 14.2
22	9.78 313	16	9.88 289	26	0.11 711	9.90 024	10	38	55 24.8 23.8 15.6
23	9.78 329	17	9.88 315	26	0.11 685	9.90 014	10	37	
24	9.78 346	16	9.88 341	26	0.11 659	9.90 005	9	36	
25	9.78 362	17	9.88 367	26	0.11 633	9.89 995	10	35	
26	9.78 379	16	9.88 393	27	0.11 607	9.89 985	10	34	
27	9.78 395	17	9.88 420	26	0.11 580	9.89 976	9	33	
28	9.78 412	16	9.88 446	26	0.11 554	9.89 966	10	32	
29	9.78 428	17	9.88 472	26	0.11 528	9.89 956	10	31	
30	9.78 445	16	9.88 498	26	0.11 502	9.89 947	9	30	
31	9.78 461	17	9.88 524	26	0.11 476	9.89 937	10	29	
32	9.78 478	16	9.88 550	26	0.11 450	9.89 927	10	28	
33	9.78 494	16	9.88 577	27	0.11 423	9.89 918	9	27	
34	9.78 510	17	9.88 603	26	0.11 397	9.89 908	10	26	
35	9.78 527	16	9.88 629	26	0.11 371	9.89 898	10	25	
36	9.78 543	17	9.88 655	26	0.11 345	9.89 888	10	24	
37	9.78 560	16	9.88 681	26	0.11 319	9.89 879	9	23	
38	9.78 576	16	9.88 707	26	0.11 293	9.89 869	10	22	
39	9.78 592	17	9.88 733	26	0.11 267	9.89 859	10	21	
40	9.78 609	16	9.88 759	26	0.11 241	9.89 849	10	20	
41	9.78 625	17	9.88 786	27	0.11 214	9.89 840	9	19	" 16 10 9
42	9.78 642	16	9.88 812	26	0.11 188	9.89 830	10	18	5 1.3 0.8 0.8
43	9.78 658	16	9.88 838	26	0.11 162	9.89 820	10	17	10 2.7 1.7 1.5
44	9.78 674	17	9.88 864	26	0.11 136	9.89 810	10	16	15 4.0 2.5 2.2
45	9.78 691	16	9.88 890	26	0.11 110	9.89 801	10	15	20 5.3 3.3 3.0
46	9.78 707	16	9.88 916	26	0.11 084	9.89 791	9	14	25 6.7 4.2 3.8
47	9.78 723	16	9.88 942	26	0.11 058	9.89 781	10	13	30 8.0 5.0 4.5
48	9.78 739	17	9.88 968	26	0.11 032	9.89 771	10	12	35 9.3 5.8 5.2
49	9.78 756	16	9.88 994	26	0.11 006	9.89 761	10	11	40 10.7 6.7 6.0
50	9.78 772	16	9.89 020	26	0.10 980	9.89 752	9	10	45 12.0 7.5 6.8
51	9.78 788	17	9.89 046	27	0.10 954	9.89 742	10	9	50 13.3 8.3 7.5
52	9.78 805	16	9.89 073	26	0.10 927	9.89 732	10	8	55 14.7 9.2 8.2
53	9.78 821	16	9.89 099	26	0.10 901	9.89 722	10	7	
54	9.78 837	16	9.89 125	26	0.10 875	9.89 712	10	6	
55	9.78 853	16	9.89 151	26	0.10 849	9.89 702	10	5	
56	9.78 869	17	9.89 177	26	0.10 823	9.89 693	9	4	
57	9.78 886	16	9.89 203	26	0.10 797	9.89 683	10	3	
58	9.78 902	16	9.89 229	26	0.10 771	9.89 673	10	2	
59	9.78 918	16	9.89 255	26	0.10 745	9.89 663	10	1	
60	9.78 934	16	9.89 281	26	0.10 719	9.89 653	10	0	
	L. Cos.	d.	L. Cot.	c.d.	L. Tan.	L. Sin.	d.	'	P.P.

'	L. Sin.	d.	L. Tan.	c.d.	L. Cot.	L. Cos.	d.		P.P.
0	9.78 934		9.89 281		0.10 719	9.89 653		60	
1	9.78 950	16	9.89 307	26	0.10 693	9.89 643	10	59	
2	9.78 967	17	9.89 333	26	0.10 667	9.89 633	10	58	
3	9.78 983	16	9.89 359	26	0.10 641	9.89 624	9	57	
4	9.78 999	16	9.89 385	26	0.10 615	9.89 614	10	56	
5	9.79 015	16	9.89 411	26	0.10 589	9.89 604	10	55	
6	9.79 031	16	9.89 437	26	0.10 563	9.89 594	10	54	
7	9.79 047	16	9.89 463	26	0.10 537	9.89 584	10	53	" 26 25
8	9.79 063	16	9.89 489	26	0.10 511	9.89 574	10	52	5 2.2 2.1
9	9.79 079	16	9.89 515	26	0.10 485	9.89 564	10	51	10 4.3 4.2
10	9.79 095	16	9.89 541	26	0.10 459	9.89 554	10	50	15 6.5 6.2
11	9.79 111	16	9.89 567	26	0.10 433	9.89 544	10	49	20 8.7 8.3
12	9.79 128	17	9.89 593	26	0.10 407	9.89 534	10	48	25 10.8 10.4
13	9.79 144	16	9.89 619	26	0.10 381	9.89 524	10	47	30 13.0 12.5
14	9.79 160	16	9.89 645	26	0.10 355	9.89 514	10	46	35 15.2 14.6
15	9.79 176	16	9.89 671	26	0.10 329	9.89 504	10	45	40 17.3 16.7
16	9.79 192	16	9.89 697	26	0.10 303	9.89 495	9	44	45 19.5 18.8
17	9.79 208	16	9.89 723	26	0.10 277	9.89 485	10	43	50 21.7 20.8
18	9.79 224	16	9.89 749	26	0.10 251	9.89 475	10	42	55 23.8 22.9
19	9.79 240	16	9.89 775	26	0.10 225	9.89 465	10	41	
20	9.79 256	16	9.89 801	26	0.10 199	9.89 455	10	40	
21	9.79 272	16	9.89 827	26	0.10 173	9.89 445	10	39	
22	9.79 288	16	9.89 853	26	0.10 147	9.89 435	10	38	
23	9.79 304	15	9.89 879	26	0.10 121	9.89 425	10	37	
24	9.79 319	16	9.89 905	26	0.10 095	9.89 415	10	36	
25	9.79 335	16	9.89 931	26	0.10 069	9.89 405	10	35	
26	9.79 351	16	9.89 957	26	0.10 043	9.89 395	10	34	" 17 16 15
27	9.79 367	16	9.89 983	26	0.10 017	9.89 385	10	33	5 1.4 1.3 1.2
28	9.79 383	16	9.90 009	26	0.09 991	9.89 375	10	32	10 2.8 2.7 2.5
29	9.79 399	16	9.90 035	26	0.09 965	9.89 364	11	31	15 4.2 4.0 3.8
30	9.79 415	16	9.90 061	26	0.09 939	9.89 354	10	30	20 5.7 5.3 5.0
31	9.79 431	16	9.90 086	25	0.09 914	9.89 344	10	29	25 7.1 6.7 6.2
32	9.79 447	16	9.90 112	26	0.09 888	9.89 334	10	28	30 8.5 8.0 7.5
33	9.79 463	15	9.90 138	26	0.09 862	9.89 324	10	27	35 9.9 9.3 8.8
34	9.79 478	16	9.90 164	26	0.09 836	9.89 314	10	26	40 11.3 10.7 10.0
35	9.79 494	16	9.90 190	26	0.09 810	9.89 304	10	25	45 12.8 12.0 11.2
36	9.79 510	16	9.90 216	26	0.09 784	9.89 294	10	24	50 14.2 13.3 12.5
37	9.79 526	16	9.90 242	26	0.09 758	9.89 284	10	23	55 15.6 14.7 13.8
38	9.79 542	16	9.90 268	26	0.09 732	9.89 274	10	22	
39	9.79 558	16	9.90 294	26	0.09 706	9.89 264	10	21	
40	9.79 573	15	9.90 320	26	0.09 680	9.89 254	10	20	
41	9.79 589	16	9.90 346	26	0.09 654	9.89 244	10	19	
42	9.79 605	16	9.90 371	25	0.09 629	9.89 233	11	18	
43	9.79 621	16	9.90 397	26	0.09 603	9.89 223	10	17	
44	9.79 636	15	9.90 423	26	0.09 577	9.89 213	10	16	
45	9.79 652	16	9.90 449	26	0.09 551	9.89 203	10	15	
46	9.79 668	16	9.90 475	26	0.09 525	9.89 193	10	14	" 11 10 9
47	9.79 684	16	9.90 501	26	0.09 499	9.89 183	10	13	5 0.9 0.8 0.8
48	9.79 699	15	9.90 527	26	0.09 473	9.89 173	10	12	10 1.8 1.7 1.5
49	9.79 715	16	9.90 553	26	0.09 447	9.89 162	11	11	15 2.8 2.5 2.2
50	9.79 731	16	9.90 578	25	0.09 422	9.89 152	10	10	20 3.7 3.3 3.0
51	9.79 746	15	9.90 604	26	0.09 396	9.89 142	10	9	25 4.6 4.2 3.8
52	9.79 762	16	9.90 630	26	0.09 370	9.89 132	10	8	30 5.5 5.0 4.5
53	9.79 778	16	9.90 656	26	0.09 344	9.89 122	10	7	35 6.4 5.8 5.2
54	9.79 793	15	9.90 682	26	0.09 318	9.89 112	10	6	40 7.3 6.7 6.0
55	9.79 809	16	9.90 708	26	0.09 292	9.89 101	10	5	45 8.2 7.5 6.8
56	9.79 825	15	9.90 734	25	0.09 266	9.89 091	10	4	50 9.2 8.3 7.5
57	9.79 840	16	9.90 759	26	0.09 241	9.89 081	10	3	55 10.1 9.2 8.2
58	9.79 856	16	9.90 785	26	0.09 215	9.89 071	10	2	
59	9.79 872	15	9.90 811	26	0.09 189	9.89 060	10	1	
60	9.79 887		9.90 837		0.09 163	9.89 050		0	
	L. Cos.	d.	L. Cot.	c.d.	L. Tan.	L. Sin.	d.	'	P.P.

	L. Sin.	d.	L. Tan.	c.d.	L. Cot.	L. Cos.	d.		P.P.
0	9.79 887	16	9.90 837	26	0.09 163	9.89 050	10	60	
1	9.79 903	15	9.90 863	26	0.09 137	9.89 040	10	59	
2	9.79 918	16	9.90 889	25	0.09 111	9.89 030	10	58	
3	9.79 934	16	9.90 914	26	0.09 086	9.89 020	11	57	
4	9.79 950	15	9.90 940	26	0.09 060	9.89 009	10	56	
5	9.79 965	16	9.90 966	26	0.09 034	9.88 999	10	55	
6	9.79 981	16	9.90 992	26	0.09 008	9.88 989	11	54	
7	9.79 996	15	9.91 018	25	0.08 982	9.88 978	10	53	
8	9.80 012	15	9.91 043	26	0.08 957	9.88 968	10	52	
9	9.80 027	16	9.91 069	26	0.08 931	9.88 958	10	51	
10	9.80 043	15	9.91 095	26	0.08 905	9.88 948	11	50	
11	9.80 058	16	9.91 121	26	0.08 879	9.88 937	10	49	
12	9.80 074	15	9.91 147	25	0.08 853	9.88 927	10	48	
13	9.80 089	16	9.91 172	26	0.08 828	9.88 917	11	47	
14	9.80 105	15	9.91 198	26	0.08 802	9.88 906	10	46	
15	9.80 120	16	9.91 224	26	0.08 776	9.88 896	10	45	
16	9.80 136	15	9.91 250	26	0.08 750	9.88 886	11	44	
17	9.80 151	15	9.91 276	25	0.08 724	9.88 875	10	43	
18	9.80 166	16	9.91 301	26	0.08 699	9.88 865	10	42	
19	9.80 182	15	9.91 327	26	0.08 673	9.88 855	11	41	
20	9.80 197	16	9.91 353	26	0.08 647	9.88 844	10	40	
21	9.80 213	15	9.91 379	25	0.08 621	9.88 834	10	39	
22	9.80 228	16	9.91 404	26	0.08 596	9.88 824	11	38	
23	9.80 244	15	9.91 430	26	0.08 570	9.88 813	10	37	
24	9.80 259	16	9.91 456	26	0.08 544	9.88 803	10	36	
25	9.80 274	15	9.91 482	25	0.08 518	9.88 793	11	35	
26	9.80 290	16	9.91 507	26	0.08 493	9.88 782	10	34	
27	9.80 305	15	9.91 533	26	0.08 467	9.88 772	11	33	
28	9.80 320	16	9.91 559	26	0.08 441	9.88 761	10	32	
29	9.80 336	15	9.91 585	25	0.08 415	9.88 751	10	31	
30	9.80 351	16	9.91 610	26	0.08 390	9.88 741	11	30	
31	9.80 366	15	9.91 636	26	0.08 364	9.88 730	10	29	
32	9.80 382	16	9.91 662	26	0.08 338	9.88 720	11	28	
33	9.80 397	15	9.91 688	25	0.08 312	9.88 709	10	27	
34	9.80 412	16	9.91 713	26	0.08 287	9.88 699	11	26	
35	9.80 428	15	9.91 739	26	0.08 261	9.88 688	10	25	
36	9.80 443	16	9.91 765	26	0.08 235	9.88 678	10	24	
37	9.80 458	15	9.91 791	25	0.08 209	9.88 668	11	23	
38	9.80 473	16	9.91 816	26	0.08 184	9.88 657	10	22	
39	9.80 489	15	9.91 842	26	0.08 158	9.88 647	11	21	
40	9.80 504	16	9.91 868	25	0.08 132	9.88 636	10	20	
41	9.80 519	15	9.91 893	26	0.08 107	9.88 626	11	19	
42	9.80 534	16	9.91 919	26	0.08 081	9.88 615	10	18	
43	9.80 550	15	9.91 945	26	0.08 055	9.88 605	11	17	
44	9.80 565	16	9.91 971	25	0.08 029	9.88 594	10	16	
45	9.80 580	15	9.91 996	26	0.08 004	9.88 584	11	15	
46	9.80 595	16	9.92 022	26	0.07 978	9.88 573	10	14	
47	9.80 610	15	9.92 048	25	0.07 952	9.88 563	11	13	
48	9.80 625	16	9.92 073	26	0.07 927	9.88 552	10	12	
49	9.80 641	15	9.92 099	26	0.07 901	9.88 542	11	11	
50	9.80 656	16	9.92 125	25	0.07 875	9.88 531	10	10	
51	9.80 671	15	9.92 150	26	0.07 850	9.88 521	11	9	
52	9.80 686	16	9.92 176	26	0.07 824	9.88 510	11	8	
53	9.80 701	15	9.92 202	25	0.07 798	9.88 499	10	7	
54	9.80 716	16	9.92 227	26	0.07 773	9.88 489	11	6	
55	9.80 731	15	9.92 253	26	0.07 747	9.88 478	10	5	
56	9.80 746	16	9.92 279	25	0.07 721	9.88 468	11	4	
57	9.80 762	15	9.92 304	26	0.07 696	9.88 457	10	3	
58	9.80 777	16	9.92 330	26	0.07 670	9.88 447	11	2	
59	9.80 792	15	9.92 356	25	0.07 644	9.88 436	11	1	
60	9.80 807	16	9.92 381	25	0.07 619	9.88 425	11	0	
	L. Cos.	d.	L. Cot.	c.d.	L. Tan.	L. Sin.	d.		P.P.

"	26	25	16
5	2.2	2.1	1.3
10	4.3	4.2	2.7
15	6.5	6.2	4.0
20	8.7	8.3	5.3
25	10.8	10.4	6.7
30	13.0	12.5	8.0
35	15.2	14.6	9.3
40	17.3	16.7	10.7
45	19.5	18.8	12.0
50	21.7	20.8	13.3
55	23.8	22.9	14.7

"	15	11	10
5	1.2	0.9	0.8
10	2.5	1.8	1.7
15	3.8	2.8	2.5
20	5.0	3.7	3.3
25	6.2	4.6	4.2
30	7.5	5.5	5.0
35	8.8	6.4	5.8
40	10.0	7.3	6.7
45	11.2	8.2	7.5
50	12.5	9.2	8.3
55	13.8	10.1	9.2

	L. Sin.	d.	L. Tan.	c.d.	L. Cot.	L. Cos.	d.		P.P.
0	9.80 807		9.92 381		0.07 619	9.88 425		60	
1	9.80 822	15	9.92 407	26	0.07 593	9.88 415	10	59	
2	9.80 837	15	9.92 433	25	0.07 567	9.88 404	10	58	
3	9.80 852	15	9.92 458	26	0.07 542	9.88 394	11	57	
4	9.80 867	15	9.92 484	26	0.07 516	9.88 383	11	56	
5	9.80 882	15	9.92 510	25	0.07 490	9.88 372	10	55	
6	9.80 897	15	9.92 535	26	0.07 465	9.88 362	11	54	
7	9.80 912	15	9.92 561	26	0.07 439	9.88 351	11	53	
8	9.80 927	15	9.92 587	25	0.07 413	9.88 340	10	52	
9	9.80 942	15	9.92 612	26	0.07 388	9.88 330	11	51	
10	9.80 957	15	9.92 638	25	0.07 362	9.88 319	11	50	
11	9.80 972	15	9.92 663	26	0.07 337	9.88 308	10	49	
12	9.80 987	15	9.92 689	26	0.07 311	9.88 298	11	48	
13	9.81 002	15	9.92 715	25	0.07 285	9.88 287	11	47	
14	9.81 017	15	9.92 740	26	0.07 260	9.88 276	10	46	
15	9.81 032	15	9.92 766	26	0.07 234	9.88 266	11	45	
16	9.81 047	14	9.92 792	25	0.07 208	9.88 255	11	44	
17	9.81 061	15	9.92 817	26	0.07 183	9.88 244	10	43	
18	9.81 076	15	9.92 843	25	0.07 157	9.88 234	11	42	
19	9.81 091	15	9.92 868	26	0.07 132	9.88 223	11	41	
20	9.81 106	15	9.92 894	26	0.07 106	9.88 212	11	40	
21	9.81 121	15	9.92 920	25	0.07 080	9.88 201	10	39	
22	9.81 136	15	9.92 945	26	0.07 055	9.88 191	11	38	
23	9.81 151	15	9.92 971	25	0.07 029	9.88 180	11	37	
24	9.81 166	14	9.92 996	26	0.07 004	9.88 169	11	36	
25	9.81 180	15	9.93 022	26	0.06 978	9.88 158	10	35	
26	9.81 195	15	9.93 048	25	0.06 952	9.88 148	11	34	
27	9.81 210	15	9.93 073	26	0.06 927	9.88 137	11	33	
28	9.81 225	15	9.93 099	25	0.06 901	9.88 126	11	32	
29	9.81 240	14	9.93 124	26	0.06 876	9.88 115	10	31	
30	9.81 254	15	9.93 150	25	0.06 850	9.88 105	11	30	
31	9.81 269	15	9.93 175	26	0.06 825	9.88 094	11	29	
32	9.81 284	15	9.93 201	26	0.06 799	9.88 083	11	28	
33	9.81 299	15	9.93 227	25	0.06 773	9.88 072	11	27	
34	9.81 314	14	9.93 252	26	0.06 748	9.88 061	10	26	
35	9.81 328	15	9.93 278	25	0.06 722	9.88 051	11	25	
36	9.81 343	15	9.93 303	26	0.06 697	9.88 040	11	24	
37	9.81 358	14	9.93 329	25	0.06 671	9.88 029	11	23	
38	9.81 372	15	9.93 354	26	0.06 646	9.88 018	11	22	
39	9.81 387	15	9.93 380	26	0.06 620	9.88 007	11	21	
40	9.81 402	15	9.93 406	25	0.06 594	9.87 996	11	20	
41	9.81 417	14	9.93 431	26	0.06 569	9.87 985	10	19	
42	9.81 431	15	9.93 457	25	0.06 543	9.87 975	11	18	
43	9.81 446	15	9.93 482	26	0.06 518	9.87 964	11	17	
44	9.81 461	14	9.93 508	25	0.06 492	9.87 953	11	16	
45	9.81 475	15	9.93 533	26	0.06 467	9.87 942	11	15	
46	9.81 490	15	9.93 559	25	0.06 441	9.87 931	11	14	
47	9.81 505	14	9.93 584	26	0.06 416	9.87 920	11	13	
48	9.81 519	15	9.93 610	26	0.06 390	9.87 909	11	12	
49	9.81 534	15	9.93 636	25	0.06 364	9.87 898	11	11	
50	9.81 549	14	9.93 661	26	0.06 339	9.87 887	10	10	
51	9.81 563	15	9.93 687	25	0.06 313	9.87 877	11	9	
52	9.81 578	14	9.93 712	26	0.06 288	9.87 866	11	8	
53	9.81 592	15	9.93 738	25	0.06 262	9.87 855	11	7	
54	9.81 607	15	9.93 763	26	0.06 237	9.87 844	11	6	
55	9.81 622	14	9.93 789	25	0.06 211	9.87 833	11	5	
56	9.81 636	15	9.93 814	26	0.06 186	9.87 822	11	4	
57	9.81 651	14	9.93 840	25	0.06 160	9.87 811	11	3	
58	9.81 665	15	9.93 865	26	0.06 135	9.87 800	11	2	
59	9.81 680	14	9.93 891	25	0.06 109	9.87 789	11	1	
60	9.81 694		9.93 916		0.06 084	9.87 778		0	
	L. Cos.	d.	L. Cot.	c.d.	L. Tan.	L. Sin.	d.		P.P.

"	26	25	15
5	2.2	2.1	1.2
10	4.3	4.2	2.5
15	6.5	6.2	3.8
20	8.7	8.3	5.0
25	10.8	10.4	6.2
30	13.0	12.5	7.5
35	15.2	14.6	8.8
40	17.3	16.7	10.0
45	19.5	18.8	11.2
50	21.7	20.8	12.5
55	23.8	22.9	13.8

"	14	11	10
5	1.2	0.9	0.8
10	2.3	1.8	1.7
15	3.5	2.8	2.5
20	4.7	3.7	3.3
25	5.8	4.6	4.2
30	7.0	5.5	5.0
35	8.2	6.4	5.8
40	9.3	7.3	6.7
45	10.5	8.2	7.5
50	11.7	9.2	8.3
55	12.8	10.1	9.2

	L. Sin.	d.	L. Tan.	c.d.	L. Cot.	L. Cos.	d.		P.P.
0	9.81 694	15	9.93 916	26	0.06 084	9.87 778	11	60	
1	9.81 709	14	9.93 942	25	0.06 058	9.87 767	11	59	
2	9.81 723	15	9.93 967	26	0.06 033	9.87 756	11	58	
3	9.81 738	14	9.93 993	25	0.06 007	9.87 745	11	57	
4	9.81 752	15	9.94 018	26	0.05 982	9.87 734	11	56	
5	9.81 767	14	9.94 044	25	0.05 956	9.87 723	11	55	
6	9.81 781	15	9.94 069	26	0.05 931	9.87 712	11	54	
7	9.81 796	14	9.94 095	25	0.05 905	9.87 701	11	53	
8	9.81 810	15	9.94 120	26	0.05 880	9.87 690	11	52	
9	9.81 825	14	9.94 146	25	0.05 854	9.87 679	11	51	
10	9.81 839	15	9.94 171	26	0.05 829	9.87 668	11	50	
11	9.81 854	14	9.94 197	25	0.05 803	9.87 657	11	49	
12	9.81 868	15	9.94 222	26	0.05 778	9.87 646	11	48	
13	9.81 882	14	9.94 248	25	0.05 752	9.87 635	11	47	
14	9.81 897	15	9.94 273	26	0.05 727	9.87 624	11	46	
15	9.81 911	14	9.94 299	25	0.05 701	9.87 613	12	45	
16	9.81 926	15	9.94 324	26	0.05 676	9.87 601	11	44	
17	9.81 940	14	9.94 350	25	0.05 650	9.87 590	11	43	
18	9.81 955	15	9.94 375	26	0.05 625	9.87 579	11	42	
19	9.81 969	14	9.94 401	25	0.05 599	9.87 568	11	41	
20	9.81 983	15	9.94 426	26	0.05 574	9.87 557	11	40	
21	9.81 998	14	9.94 452	25	0.05 548	9.87 546	11	39	
22	9.82 012	15	9.94 477	26	0.05 523	9.87 535	11	38	
23	9.82 026	14	9.94 503	25	0.05 497	9.87 524	11	37	
24	9.82 041	15	9.94 528	26	0.05 472	9.87 513	12	36	
25	9.82 055	14	9.94 554	25	0.05 446	9.87 501	11	35	
26	9.82 069	15	9.94 579	26	0.05 421	9.87 490	11	34	
27	9.82 084	14	9.94 604	25	0.05 396	9.87 479	11	33	
28	9.82 098	15	9.94 630	26	0.05 370	9.87 468	11	32	
29	9.82 112	14	9.94 655	25	0.05 345	9.87 457	11	31	
30	9.82 126	15	9.94 681	26	0.05 319	9.87 446	12	30	
31	9.82 141	14	9.94 706	25	0.05 294	9.87 434	11	29	
32	9.82 155	15	9.94 732	26	0.05 268	9.87 423	11	28	
33	9.82 169	14	9.94 757	25	0.05 243	9.87 412	11	27	
34	9.82 184	15	9.94 783	26	0.05 217	9.87 401	11	26	
35	9.82 198	14	9.94 808	25	0.05 192	9.87 390	12	25	
36	9.82 212	15	9.94 834	26	0.05 166	9.87 378	11	24	
37	9.82 226	14	9.94 859	25	0.05 141	9.87 367	11	23	
38	9.82 240	15	9.94 884	26	0.05 116	9.87 356	11	22	
39	9.82 255	14	9.94 910	25	0.05 090	9.87 345	11	21	
40	9.82 269	15	9.94 935	26	0.05 065	9.87 334	12	20	
41	9.82 283	14	9.94 961	25	0.05 039	9.87 322	11	19	
42	9.82 297	15	9.94 986	26	0.05 014	9.87 311	11	18	
43	9.82 311	14	9.95 012	25	0.04 988	9.87 300	11	17	
44	9.82 326	15	9.95 037	26	0.04 963	9.87 288	11	16	
45	9.82 340	14	9.95 062	25	0.04 938	9.87 277	11	15	
46	9.82 354	15	9.95 088	26	0.04 912	9.87 266	11	14	
47	9.82 368	14	9.95 113	25	0.04 887	9.87 255	12	13	
48	9.82 382	15	9.95 139	26	0.04 861	9.87 243	11	12	
49	9.82 396	14	9.95 164	25	0.04 836	9.87 232	11	11	
50	9.82 410	15	9.95 190	26	0.04 810	9.87 221	12	10	
51	9.82 424	14	9.95 215	25	0.04 785	9.87 209	11	9	
52	9.82 439	15	9.95 240	26	0.04 760	9.87 198	11	8	
53	9.82 453	14	9.95 266	25	0.04 734	9.87 187	12	7	
54	9.82 467	15	9.95 291	26	0.04 709	9.87 175	11	6	
55	9.82 481	14	9.95 317	25	0.04 683	9.87 164	11	5	
56	9.82 495	15	9.95 342	26	0.04 658	9.87 153	12	4	
57	9.82 509	14	9.95 368	25	0.04 632	9.87 141	11	3	
58	9.82 523	15	9.95 393	26	0.04 607	9.87 130	11	2	
59	9.82 537	14	9.95 418	25	0.04 582	9.87 119	12	1	
60	9.82 551	15	9.95 444	26	0.04 556	9.87 107	11	0	
	L. Cos.	d.	L. Cot.	c.d.	L. Tan.	L. Sin.	d.		P.P.

"	26	25	15
5	2.2	2.1	1.2
10	4.3	4.2	2.5
15	6.5	6.2	3.8
20	8.7	8.3	5.0
25	10.8	10.4	6.2
30	13.0	12.5	7.5
35	15.2	14.6	8.8
40	17.3	16.7	10.0
45	19.5	18.8	11.2
50	21.7	20.8	12.5
55	23.8	22.9	13.8

"	14	12	11
5	1.2	1.0	0.9
10	2.3	2.0	1.8
15	3.5	3.0	2.8
20	4.7	4.0	3.7
25	5.8	5.0	4.6
30	7.0	6.0	5.5
35	8.2	7.0	6.4
40	9.3	8.0	7.3
45	10.5	9.0	8.2
50	11.7	10.0	9.2
55	12.8	11.0	10.1

	L. Sin.	d.	L. Tan.	c.d.	L. Cot.	L. Cos.	d.		P.P.
0	9.82 551	14	9.95 444	25	0.04 556	9.87 107	11	60	
1	9.82 565	14	9.95 469	26	0.04 531	9.87 096	11	59	
2	9.82 579	14	9.95 495	25	0.04 505	9.87 085	12	58	
3	9.82 593	14	9.95 520	25	0.04 480	9.87 073	11	57	
4	9.82 607	14	9.95 545	26	0.04 455	9.87 062	12	56	
5	9.82 621	14	9.95 571	25	0.04 429	9.87 050	11	55	
6	9.82 635	14	9.95 596	26	0.04 404	9.87 039	11	54	
7	9.82 649	14	9.95 622	25	0.04 378	9.87 028	12	53	
8	9.82 663	14	9.95 647	25	0.04 353	9.87 016	11	52	
9	9.82 677	14	9.95 672	26	0.04 328	9.87 005	12	51	
10	9.82 691	14	9.95 698	25	0.04 302	9.86 993	11	50	
11	9.82 705	14	9.95 723	25	0.04 277	9.86 982	12	49	
12	9.82 719	14	9.95 748	26	0.04 252	9.86 970	11	48	" 26 25 14
13	9.82 733	14	9.95 774	25	0.04 226	9.86 959	12	47	5 2.2 2.1 1.2
14	9.82 747	14	9.95 799	26	0.04 201	9.86 947	11	46	10 4.3 4.2 2.3
15	9.82 761	14	9.95 825	25	0.04 175	9.86 936	12	45	15 6.5 6.2 3.5
16	9.82 775	13	9.95 850	25	0.04 150	9.86 924	11	44	20 8.7 8.3 4.7
17	9.82 788	14	9.95 875	26	0.04 125	9.86 913	12	43	25 10.8 10.4 5.8
18	9.82 802	14	9.95 901	25	0.04 099	9.86 902	11	42	30 13.0 12.5 7.0
19	9.82 816	14	9.95 926	26	0.04 074	9.86 890	12	41	35 15.2 14.6 8.2
20	9.82 830	14	9.95 952	25	0.04 048	9.86 879	11	40	40 17.3 16.7 9.3
21	9.82 844	14	9.95 977	25	0.04 023	9.86 867	12	39	45 19.5 18.8 10.5
22	9.82 858	14	9.96 002	26	0.03 998	9.86 855	11	38	50 21.7 20.8 11.7
23	9.82 872	13	9.96 028	25	0.03 972	9.86 844	12	37	55 23.8 22.9 12.8
24	9.82 885	14	9.96 053	25	0.03 947	9.86 832	11	36	
25	9.82 899	14	9.96 078	26	0.03 922	9.86 821	12	35	
26	9.82 913	14	9.96 104	25	0.03 896	9.86 809	11	34	
27	9.82 927	14	9.96 129	26	0.03 871	9.86 798	12	33	
28	9.82 941	14	9.96 155	25	0.03 845	9.86 786	11	32	
29	9.82 955	13	9.96 180	25	0.03 820	9.86 775	12	31	
30	9.82 968	14	9.96 205	26	0.03 795	9.86 763	11	30	
31	9.82 982	14	9.96 231	25	0.03 769	9.86 752	12	29	
32	9.82 996	14	9.96 256	25	0.03 744	9.86 740	12	28	
33	9.83 010	13	9.96 281	26	0.03 719	9.86 728	11	27	
34	9.83 023	14	9.96 307	25	0.03 693	9.86 717	12	26	
35	9.83 037	14	9.96 332	25	0.03 668	9.86 705	11	25	
36	9.83 051	14	9.96 357	26	0.03 643	9.86 694	12	24	
37	9.83 065	13	9.96 383	25	0.03 617	9.86 682	12	23	
38	9.83 078	14	9.96 408	25	0.03 592	9.86 670	11	22	
39	9.83 092	14	9.96 433	26	0.03 567	9.86 659	12	21	
40	9.83 106	14	9.96 459	25	0.03 541	9.86 647	12	20	
41	9.83 120	13	9.96 484	26	0.03 516	9.86 635	11	19	" 13 12 11
42	9.83 133	14	9.96 510	25	0.03 490	9.86 624	12	18	5 1.1 1.0 0.9
43	9.83 147	14	9.96 535	25	0.03 465	9.86 612	12	17	10 2.2 2.0 1.8
44	9.83 161	13	9.96 560	26	0.03 440	9.86 600	11	16	15 3.2 3.0 2.8
45	9.83 174	14	9.96 586	25	0.03 414	9.86 589	12	15	20 4.3 4.0 3.7
46	9.83 188	14	9.96 611	25	0.03 389	9.86 577	12	14	25 5.4 5.0 4.6
47	9.83 202	13	9.96 636	26	0.03 364	9.86 565	11	13	30 6.5 6.0 5.5
48	9.83 215	14	9.96 662	25	0.03 338	9.86 554	12	12	35 7.6 7.0 6.4
49	9.83 229	13	9.96 687	25	0.03 313	9.86 542	12	11	40 8.7 8.0 7.3
50	9.83 242	14	9.96 712	26	0.03 288	9.86 530	12	10	45 9.8 9.0 8.2
51	9.83 256	14	9.96 738	25	0.03 262	9.86 518	11	9	50 10.8 10.0 9.2
52	9.83 270	13	9.96 763	25	0.03 237	9.86 507	12	8	55 11.9 11.0 10.1
53	9.83 283	14	9.96 788	26	0.03 212	9.86 495	12	7	
54	9.83 297	13	9.96 814	25	0.03 186	9.86 483	11	6	
55	9.83 310	14	9.96 839	25	0.03 161	9.86 472	12	5	
56	9.83 324	14	9.96 864	26	0.03 136	9.86 460	12	4	
57	9.83 338	13	9.96 890	25	0.03 110	9.86 448	12	3	
58	9.83 351	14	9.96 915	25	0.03 085	9.86 436	11	2	
59	9.83 365	13	9.96 940	26	0.03 060	9.86 425	12	1	
60	9.83 378		9.96 966		0.03 034	9.86 413		0	
	L. Cos.	d.	L. Cot.	c.d.	L. Tan.	L. Sin.	d.		P.P.

	L. Sin.	d.	L. Tan.	c.d.	L. Cot.	L. Cos.	d.		P.P.
0	9.83 378		9.96 966		0.03 034	9.86 413		60	
1	9.83 392	14	9.96 991	25	0.03 009	9.86 401	12	59	
2	9.83 405	13	9.97 016	25	0.02 984	9.86 389	12	58	
3	9.83 419	14	9.97 042	26	0.02 958	9.86 377	12	57	
4	9.83 432	13	9.97 067	25	0.02 933	9.86 366	11	56	
5	9.83 446	14	9.97 092	25	0.02 908	9.86 354	12	55	
6	9.83 459	13	9.97 118	26	0.02 882	9.86 342	12	54	
7	9.83 473	14	9.97 143	25	0.02 857	9.86 330	12	53	
8	9.83 486	13	9.97 168	25	0.02 832	9.86 318	12	52	
9	9.83 500	14	9.97 193	25	0.02 807	9.86 306	12	51	
		13		26			11		
10	9.83 513		9.97 219		0.02 781	9.86 295		50	
		14		25			12		
11	9.83 527	13	9.97 244	25	0.02 756	9.86 283	12	49	
12	9.83 540	14	9.97 269	25	0.02 731	9.86 271	12	48	
13	9.83 554	14	9.97 295	26	0.02 705	9.86 259	12	47	
14	9.83 567	13	9.97 320	25	0.02 680	9.86 247	12	46	
15	9.83 581	14	9.97 345	25	0.02 655	9.86 235	12	45	
16	9.83 594	13	9.97 371	26	0.02 629	9.86 223	12	44	
		14		25			12		
17	9.83 608	13	9.97 396	25	0.02 604	9.86 211	11	43	
18	9.83 621	14	9.97 421	25	0.02 579	9.86 200	12	42	
19	9.83 634	13	9.97 447	26	0.02 553	9.86 188	12	41	
		14		25			12		
20	9.83 648		9.97 472		0.02 528	9.86 176		40	
		13		25			12		
21	9.83 661	13	9.97 497	26	0.02 503	9.86 164	12	39	
22	9.83 674	14	9.97 523	25	0.02 477	9.86 152	12	38	
23	9.83 688	14	9.97 548	25	0.02 452	9.86 140	12	37	
		13		25			12		
24	9.83 701	14	9.97 573	25	0.02 427	9.86 128	12	36	
25	9.83 715	13	9.97 598	25	0.02 402	9.86 116	12	35	
26	9.83 728	13	9.97 624	26	0.02 376	9.86 104	12	34	
27	9.83 741	14	9.97 649	25	0.02 351	9.86 092	12	33	
28	9.83 755	13	9.97 674	25	0.02 326	9.86 080	12	32	
29	9.83 768	14	9.97 700	26	0.02 300	9.86 068	12	31	
		13		25			12		
30	9.83 781		9.97 725		0.02 275	9.86 056		30	
		14		25			12		
31	9.83 795	13	9.97 750	26	0.02 250	9.86 044	12	29	
32	9.83 808	13	9.97 776	25	0.02 224	9.86 032	12	28	
33	9.83 821	13	9.97 801	25	0.02 199	9.86 020	12	27	
34	9.83 834	14	9.97 826	25	0.02 174	9.86 008	12	26	
35	9.83 848	13	9.97 851	26	0.02 149	9.85 996	12	25	
36	9.83 861	13	9.97 877	26	0.02 123	9.85 984	12	24	
		14		25			12		
37	9.83 874	13	9.97 902	25	0.02 098	9.85 972	12	23	
38	9.83 887	13	9.97 927	25	0.02 073	9.85 960	12	22	
39	9.83 901	14	9.97 953	26	0.02 047	9.85 948	12	21	
		13		25			12		
40	9.83 914		9.97 978		0.02 022	9.85 936		20	
		13		25			12		
41	9.83 927	13	9.98 003	26	0.01 997	9.85 924	12	19	
42	9.83 940	14	9.98 029	25	0.01 971	9.85 912	12	18	
43	9.83 954	13	9.98 054	25	0.01 946	9.85 900	12	17	
44	9.83 967	13	9.98 079	25	0.01 921	9.85 888	12	16	
45	9.83 980	13	9.98 104	25	0.01 896	9.85 876	12	15	
46	9.83 993	13	9.98 130	26	0.01 870	9.85 864	12	14	
		14		25			13		
47	9.84 006	14	9.98 155	25	0.01 845	9.85 851	12	13	
48	9.84 020	13	9.98 180	25	0.01 820	9.85 839	12	12	
49	9.84 033	13	9.98 206	26	0.01 794	9.85 827	12	11	
		14		25			12		
50	9.84 046		9.98 231		0.01 769	9.85 815		10	
		13		25			12		
51	9.84 059	13	9.98 256	25	0.01 744	9.85 803	12	9	
52	9.84 072	13	9.98 281	26	0.01 719	9.85 791	12	8	
53	9.84 085	13	9.98 307	25	0.01 693	9.85 779	12	7	
54	9.84 098	13	9.98 332	25	0.01 668	9.85 766	13	6	
55	9.84 112	14	9.98 357	25	0.01 643	9.85 754	12	5	
56	9.84 125	13	9.98 383	26	0.01 617	9.85 742	12	4	
57	9.84 138	13	9.98 408	25	0.01 592	9.85 730	12	3	
58	9.84 151	13	9.98 433	25	0.01 567	9.85 718	12	2	
59	9.84 164	13	9.98 458	25	0.01 542	9.85 706	12	1	
		14		26			13		
60	9.84 177		9.98 484		0.01 516	9.85 693		0	
		13							
	L. Cos.	d.	L. Cot.	c.d.	L. Tan.	L. Sin.	d.		P.P.

"	26	25	14
5	2.2	2.1	1.2
10	4.3	4.2	2.3
15	6.5	6.2	3.5
20	8.7	8.3	4.7
25	10.8	10.4	5.8
30	13.0	12.5	7.0
35	15.2	14.6	8.2
40	17.3	16.7	9.3
45	19.5	18.8	10.5
50	21.7	20.8	11.7
55	23.8	22.9	12.8

"	13	12	11
5	1.1	1.0	0.9
10	2.2	2.0	1.8
15	3.2	3.0	2.8
20	4.3	4.0	3.7
25	5.4	5.0	4.6
30	6.5	6.0	5.5
35	7.6	7.0	6.4
40	8.7	8.0	7.3
45	9.8	9.0	8.2
50	10.8	10.0	9.2
55	11.9	11.0	10.1

	L. Sin.	d.	L. Tan.	c.d.	L. Cot.	L. Cos.	d.		P.P.
0	9.84 177		9.98 484	25	0.01 516	9.85 693	12	60	
1	9.84 190	13	9.98 509	25	0.01 491	9.85 681	12	59	
2	9.84 203	13	9.98 534	26	0.01 466	9.85 669	12	58	
3	9.84 216	13	9.98 560	25	0.01 440	9.85 657	12	57	
4	9.84 229	13	9.98 585	25	0.01 415	9.85 645	13	56	
5	9.84 242	13	9.98 610	25	0.01 390	9.85 632	12	55	
6	9.84 255	14	9.98 635	26	0.01 365	9.85 620	12	54	
7	9.84 269	13	9.98 661	25	0.01 339	9.85 608	12	53	
8	9.84 282	13	9.98 686	25	0.01 314	9.85 596	13	52	
9	9.84 295	13	9.98 711	26	0.01 289	9.85 583	12	51	
10	9.84 308	13	9.98 737	25	0.01 263	9.85 571	12	50	
11	9.84 321	13	9.98 762	25	0.01 238	9.85 559	12	49	
12	9.84 334	13	9.98 787	25	0.01 213	9.85 547	13	48	" 26 25
13	9.84 347	13	9.98 812	26	0.01 188	9.85 534	12	47	5 2.2 2.1
14	9.84 360	13	9.98 838	25	0.01 162	9.85 522	12	46	10 4.3 4.2
15	9.84 373	12	9.98 863	25	0.01 137	9.85 510	13	45	15 6.5 6.2
16	9.84 385	13	9.98 888	25	0.01 112	9.85 497	12	44	20 8.7 8.3
17	9.84 398	13	9.98 913	26	0.01 087	9.85 485	12	43	25 10.8 10.4
18	9.84 411	13	9.98 939	25	0.01 061	9.85 473	13	42	30 13.0 12.5
19	9.84 424	13	9.98 964	25	0.01 036	9.85 460	12	41	35 15.2 14.6
20	9.84 437	13	9.98 989	26	0.01 011	9.85 448	12	40	40 17.3 16.7
21	9.84 450	13	9.99 015	25	0.00 985	9.85 436	13	39	45 19.5 18.8
22	9.84 463	13	9.99 040	25	0.00 960	9.85 423	12	38	50 21.7 20.8
23	9.84 476	13	9.99 065	25	0.00 935	9.85 411	12	37	55 23.8 22.9
24	9.84 489	13	9.99 090	26	0.00 910	9.85 399	13	36	
25	9.84 502	13	9.99 116	25	0.00 884	9.85 386	12	35	
26	9.84 515	13	9.99 141	25	0.00 859	9.85 374	13	34	
27	9.84 528	12	9.99 166	25	0.00 834	9.85 361	12	33	
28	9.84 540	13	9.99 191	26	0.00 809	9.85 349	12	32	
29	9.84 553	13	9.99 217	25	0.00 783	9.85 337	13	31	
30	9.84 566	13	9.99 242	25	0.00 758	9.85 324	12	30	
31	9.84 579	13	9.99 267	26	0.00 733	9.85 312	13	29	
32	9.84 592	13	9.99 293	25	0.00 707	9.85 299	12	28	
33	9.84 605	13	9.99 318	25	0.00 682	9.85 287	13	27	
34	9.84 618	12	9.99 343	25	0.00 657	9.85 274	12	26	
35	9.84 630	13	9.99 368	26	0.00 632	9.85 262	12	25	
36	9.84 643	13	9.99 394	25	0.00 606	9.85 250	13	24	
37	9.84 656	13	9.99 419	25	0.00 581	9.85 237	12	23	
38	9.84 669	13	9.99 444	25	0.00 556	9.85 225	13	22	
39	9.84 682	12	9.99 469	26	0.00 531	9.85 212	12	21	
40	9.84 694	13	9.99 495	25	0.00 505	9.85 200	13	20	" 14 13 12
41	9.84 707	13	9.99 520	25	0.00 480	9.85 187	12	19	5 1.2 1.1 1.0
42	9.84 720	13	9.99 545	25	0.00 455	9.85 175	13	18	10 2.3 2.2 2.0
43	9.84 733	12	9.99 570	26	0.00 430	9.85 162	12	17	15 3.5 3.2 3.0
44	9.84 745	13	9.99 596	25	0.00 404	9.85 150	13	16	20 4.7 4.3 4.0
45	9.84 758	13	9.99 621	25	0.00 379	9.85 137	12	15	25 5.8 5.4 5.0
46	9.84 771	13	9.99 646	26	0.00 354	9.85 125	13	14	30 7.0 6.5 6.0
47	9.84 784	12	9.99 672	25	0.00 328	9.85 112	12	13	35 8.2 7.6 7.0
48	9.84 796	13	9.99 697	25	0.00 303	9.85 100	13	12	40 9.3 8.7 8.0
49	9.84 809	13	9.99 722	25	0.00 278	9.85 087	12	11	45 10.5 9.8 9.0
50	9.84 822	13	9.99 747	26	0.00 253	9.85 074	13	10	50 11.7 10.8 10.0
51	9.84 835	12	9.99 773	25	0.00 227	9.85 062	12	9	55 12.8 11.9 11.0
52	9.84 847	13	9.99 798	25	0.00 202	9.85 049	13	8	
53	9.84 860	13	9.99 823	25	0.00 177	9.85 037	12	7	
54	9.84 873	12	9.99 848	26	0.00 152	9.85 024	13	6	
55	9.84 885	13	9.99 874	25	0.00 126	9.85 012	12	5	
56	9.84 898	13	9.99 899	25	0.00 101	9.84 999	13	4	
57	9.84 911	12	9.99 924	25	0.00 076	9.84 986	12	3	
58	9.84 923	13	9.99 949	26	0.00 051	9.84 974	13	2	
59	9.84 936	13	9.99 975	25	0.00 025	9.84 961	12	1	
60	9.84 949		0.00 000		0.00 000	9.84 949		0	
	L. Cos.	d.	L. Cot.	c.d.	L. Tan.	L. Sin.	d.		P.P.

TABLE III.

NATURAL TRIGONOMETRIC
FUNCTIONS.

	N. Sin.	N. Tan.	N. Cot.	N. Cos.	
0	.00000	.00000	∞	1.0000	60
1	029	029	3437.7	000	59
2	058	058	1718.9	000	58
3	087	087	1145.9	000	57
4	116	116	859.44	000	56
5	.00145	.00145	687.55	1.0000	55
6	175	175	572.96	000	54
7	204	204	491.11	000	53
8	233	233	429.72	000	52
9	262	262	381.97	000	51
10	.00291	.00291	343.77	1.0000	50
11	320	320	312.52	.99999	49
12	349	349	286.48	999	48
13	378	378	264.44	999	47
14	407	407	245.55	999	46
15	.00436	.00436	229.18	.99999	45
16	465	465	214.86	999	44
17	495	495	202.22	999	43
18	524	524	190.98	999	42
19	553	553	180.93	998	41
20	.00582	.00582	171.89	.99998	40
21	611	611	163.70	998	39
22	640	640	156.26	998	38
23	669	669	149.47	998	37
24	698	698	143.24	998	36
25	.00727	.00727	137.51	.99997	35
26	756	756	132.22	997	34
27	785	785	127.32	997	33
28	814	815	122.77	997	32
29	844	844	118.54	996	31
30	.00873	.00873	114.59	.99996	30
31	902	902	110.89	996	29
32	931	931	107.43	996	28
33	960	960	104.17	995	27
34	.00989	.00989	101.11	995	26
35	.01018	.01018	98.218	.99995	25
36	047	047	95.489	995	24
37	076	076	92.908	994	23
38	105	105	90.463	994	22
39	134	135	88.144	994	21
40	.01164	.01164	85.940	.99993	20
41	193	193	83.844	993	19
42	222	222	81.847	993	18
43	251	251	79.943	992	17
44	280	280	78.126	992	16
45	.01309	.01309	76.390	.99991	15
46	338	338	74.729	991	14
47	367	367	73.139	991	13
48	396	396	71.615	990	12
49	425	425	70.153	990	11
50	.01454	.01455	68.750	.99989	10
51	483	484	67.402	989	9
52	513	513	66.105	989	8
53	542	542	64.858	988	7
54	571	571	63.657	988	6
55	.01600	.01600	62.499	.99987	5
56	629	629	61.383	987	4
57	658	658	60.306	986	3
58	687	687	59.266	986	2
59	716	716	58.261	985	1
60	.01745	.01746	57.290	.99985	0
	N. Cos.	N. Cot.	N. Tan.	N. Sin.	

	N. Sin.	N. Tan.	N. Cot.	N. Cos.	
0	.01745	.01746	57.290	.99985	60
1	774	775	56.351	984	59
2	803	804	55.442	984	58
3	832	833	54.561	983	57
4	862	862	53.709	983	56
5	.01891	.01891	52.882	.99982	55
6	920	920	52.081	982	54
7	949	949	51.303	981	53
8	.01978	.01978	50.549	980	52
9	.02007	.02007	49.816	980	51
10	.02036	.02036	49.104	.99979	50
11	065	066	48.412	979	49
12	094	095	47.740	978	48
13	123	124	47.085	977	47
14	152	153	46.449	977	46
15	.02181	.02182	45.829	.99976	45
16	211	211	45.226	976	44
17	240	240	44.639	975	43
18	269	269	44.066	974	42
19	298	298	43.508	974	41
20	.02327	.02328	42.964	.99973	40
21	356	357	42.433	972	39
22	385	386	41.916	972	38
23	414	415	41.411	971	37
24	443	444	40.917	970	36
25	.02472	.02473	40.436	.99969	35
26	501	502	39.965	969	34
27	530	531	39.506	968	33
28	560	560	39.057	967	32
29	589	589	38.618	966	31
30	.02618	.02619	38.188	.99966	30
31	647	648	37.769	965	29
32	676	677	37.358	964	28
33	705	706	36.956	963	27
34	734	735	36.563	963	26
35	.02763	.02764	36.178	.99962	25
36	792	793	35.801	961	24
37	821	822	35.431	960	23
38	850	851	35.070	959	22
39	879	881	34.715	959	21
40	.02908	.02910	34.368	.99958	20
41	938	939	34.027	957	19
42	967	968	33.694	956	18
43	.02996	.02997	33.366	955	17
44	.03025	.03026	33.045	954	16
45	.03054	.03055	32.730	.99953	15
46	083	084	32.421	952	14
47	112	114	32.118	952	13
48	141	143	31.821	951	12
49	170	172	31.528	950	11
50	.03199	.03201	31.242	.99949	10
51	228	230	30.960	948	9
52	257	259	30.683	947	8
53	286	288	30.412	946	7
54	316	317	30.145	945	6
55	.03345	.03346	29.882	.99944	5
56	374	376	29.624	943	4
57	403	405	29.371	942	3
58	432	434	29.122	941	2
59	461	463	28.877	940	1
60	.03490	.03492	28.636	.99939	0
	N. Cos.	N. Cot.	N. Tan.	N. Sin.	

	N. Sin.	N. Tan.	N. Cot.	N. Cos.	
0	.03490	.03492	28.636	.99939	60
1	519	521	.399	938	59
2	548	550	28.166	937	58
3	577	579	27.937	936	57
4	606	609	.712	935	56
5	.03635	.03638	27.490	.99934	55
6	664	667	.271	933	54
7	693	696	27.057	932	53
8	723	725	26.845	931	52
9	752	754	.637	930	51
10	.03781	.03783	26.432	.99929	50
11	810	812	.230	927	49
12	839	842	26.031	926	48
13	868	871	25.835	925	47
14	897	900	.642	924	46
15	.03926	.03929	25.452	.99923	45
16	955	958	.264	922	44
17	.03984	.03987	25.080	921	43
18	.04013	.04016	24.898	919	42
19	042	046	.719	918	41
20	.04071	.04075	24.542	.99917	40
21	100	104	.368	916	39
22	129	133	.196	915	38
23	159	162	24.026	913	37
24	188	191	23.859	912	36
25	.04217	.04220	23.695	.99911	35
26	246	250	.532	910	34
27	275	279	.372	909	33
28	304	308	.214	907	32
29	333	337	23.058	906	31
30	.04362	.04366	22.904	.99905	30
31	391	395	.752	904	29
32	420	424	.602	902	28
33	449	454	.454	901	27
34	478	483	.308	900	26
35	.04507	.04512	22.164	.99898	25
36	536	541	22.022	897	24
37	565	570	21.881	896	23
38	594	599	.743	894	22
39	623	628	.606	893	21
40	.04653	.04658	21.470	.99892	20
41	682	687	.337	890	19
42	711	716	.205	889	18
43	740	745	21.075	888	17
44	769	774	20.946	886	16
45	.04798	.04803	20.819	.99885	15
46	827	833	.693	883	14
47	856	862	.569	882	13
48	885	891	.446	881	12
49	914	920	.325	879	11
50	.04943	.04949	20.206	.99878	10
51	.04972	.04978	20.087	876	9
52	.05001	.05007	19.970	875	8
53	030	037	.855	873	7
54	059	066	.740	872	6
55	.05088	.05095	19.627	.99870	5
56	117	124	.516	869	4
57	146	153	.405	867	3
58	175	182	.296	866	2
59	205	212	.188	864	1
60	.05234	.05241	19.081	.99863	0
	N. Cos.	N. Cot.	N. Tan.	N. Sin.	

	N. Sin.	N. Tan.	N. Cot.	N. Cos.	
0	.05234	.05241	19.081	.99863	60
1	263	270	18.976	861	59
2	292	299	.871	860	58
3	321	328	.768	858	57
4	350	357	.666	857	56
5	.05379	.05387	18.564	.99855	55
6	408	416	.464	854	54
7	437	445	.366	852	53
8	466	474	.268	851	52
9	495	503	.171	849	51
10	.05524	.05533	18.075	.99847	50
11	553	562	17.980	846	49
12	582	591	.886	844	48
13	611	620	.793	842	47
14	640	649	.702	841	46
15	.05669	.05678	17.611	.99839	45
16	698	708	.521	838	44
17	727	737	.431	836	43
18	756	766	.343	834	42
19	785	795	.256	833	41
20	.05814	.05824	17.169	.99831	40
21	844	854	17.084	829	39
22	873	883	16.999	827	38
23	902	912	.915	826	37
24	931	941	.832	824	36
25	.05960	.05970	16.750	.99822	35
26	989	999	.668	821	34
27	.06018	.06029	.587	819	33
28	047	058	.507	817	32
29	076	087	.428	815	31
30	.06105	.06116	16.350	.99813	30
31	134	145	.272	812	29
32	163	175	.195	810	28
33	192	204	.119	808	27
34	221	233	16.043	806	26
35	.06250	.06262	15.969	.99804	25
36	279	291	.895	803	24
37	308	321	.821	801	23
38	337	350	.748	799	22
39	366	379	.676	797	21
40	.06395	.06408	15.605	.99795	20
41	424	438	.534	793	19
42	453	467	.464	792	18
43	482	496	.394	790	17
44	511	525	.325	788	16
45	.06540	.06554	15.257	.99786	15
46	569	584	.189	784	14
47	598	613	.122	782	13
48	627	642	15.056	780	12
49	656	671	14.990	778	11
50	.06685	.06700	14.924	.99776	10
51	714	730	.860	774	9
52	743	759	.795	772	8
53	773	788	.732	770	7
54	802	817	.669	768	6
55	.06831	.06847	14.606	.99766	5
56	860	876	.544	764	4
57	889	905	.482	762	3
58	918	934	.421	760	2
59	947	963	.361	758	1
60	.06976	.06993	14.301	.99756	0
	N. Cos.	N. Cot.	N. Tan.	N. Sin.	

	N. Sin.	N. Tan.	N. Cot.	N. Cos.	
0	.06976	.06993	14.301	.99756	60
1	.07005	.07022	.241	754	59
2	.034	.051	.182	752	58
3	.063	.080	.124	750	57
4	.092	.110	.065	748	56
5	.07121	.07139	14.008	.99746	55
6	.150	.168	13.951	744	54
7	.179	.197	.894	742	53
8	.208	.227	.838	740	52
9	.237	.256	.782	738	51
10	.07266	.07285	13.727	.99736	50
11	.295	.314	.672	734	49
12	.324	.344	.617	731	48
13	.353	.373	.563	729	47
14	.382	.402	.510	727	46
15	.07411	.07431	13.457	.99725	45
16	.440	.461	.404	723	44
17	.469	.490	.352	721	43
18	.498	.519	.300	719	42
19	.527	.548	.248	716	41
20	.07556	.07578	13.197	.99714	40
21	.585	.607	.146	712	39
22	.614	.636	.096	710	38
23	.643	.665	13.046	708	37
24	.672	.695	12.996	705	36
25	.07701	.07724	12.947	.99703	35
26	.730	.753	.898	701	34
27	.759	.782	.850	699	33
28	.788	.812	.801	696	32
29	.817	.841	.754	694	31
30	.07846	.07870	12.706	.99692	30
31	.875	.899	.659	689	29
32	.904	.929	.612	687	28
33	.933	.958	.566	685	27
34	.962	.07987	.520	683	26
35	.07991	.08017	12.474	.99680	25
36	.08020	.046	.429	678	24
37	.049	.075	.384	676	23
38	.078	.104	.339	673	22
39	.107	.134	.295	671	21
40	.08136	.08163	12.251	.99668	20
41	.165	.192	.207	666	19
42	.194	.221	.163	664	18
43	.223	.251	.120	661	17
44	.252	.280	.077	659	16
45	.08281	.08309	12.035	.99657	15
46	.310	.339	11.992	654	14
47	.339	.368	.950	652	13
48	.368	.397	.909	649	12
49	.397	.427	.867	647	11
50	.08426	.08456	11.826	.99644	10
51	.455	.485	.785	642	9
52	.484	.514	.745	639	8
53	.513	.544	.705	637	7
54	.542	.573	.664	635	6
55	.08571	.08602	11.625	.99632	5
56	.600	.632	.585	630	4
57	.629	.661	.546	627	3
58	.658	.690	.507	625	2
59	.687	.720	.468	622	1
60	.08716	.08749	11.430	.99619	0
	N. Cos.	N. Cot.	N. Tan.	N. Sin.	

	N. Sin.	N. Tan.	N. Cot.	N. Cos.	
0	.08716	.08749	11.430	.99619	60
1	.745	.778	.392	617	59
2	.774	.807	.354	614	58
3	.803	.837	.316	612	57
4	.831	.866	.279	609	56
5	.08860	.08895	11.242	.99607	55
6	.889	.925	.205	604	54
7	.918	.954	.168	602	53
8	.947	.08983	.132	599	52
9	.08976	.09013	.095	596	51
10	.09005	.09042	11.059	.99594	50
11	.034	.071	11.024	591	49
12	.063	.101	10.988	588	48
13	.092	.130	.953	586	47
14	.121	.159	.918	583	46
15	.09150	.09189	10.883	.99580	45
16	.179	.218	.848	578	44
17	.208	.247	.814	575	43
18	.237	.277	.780	572	42
19	.266	.306	.746	570	41
20	.09295	.09335	10.712	.99567	40
21	.324	.365	.678	564	39
22	.353	.394	.645	562	38
23	.382	.423	.612	559	37
24	.411	.453	.579	556	36
25	.09440	.09482	10.546	.99553	35
26	.469	.511	.514	551	34
27	.498	.541	.481	548	33
28	.527	.570	.449	545	32
29	.556	.600	.417	542	31
30	.09585	.09629	10.385	.99540	30
31	.614	.658	.354	537	29
32	.642	.688	.322	534	28
33	.671	.717	.291	531	27
34	.700	.746	.260	528	26
35	.09729	.09776	10.229	.99526	25
36	.758	.805	.199	523	24
37	.787	.834	.168	520	23
38	.816	.864	.138	517	22
39	.845	.893	.108	514	21
40	.09874	.09923	10.078	.99511	20
41	.903	.952	.048	508	19
42	.932	.09981	10.019	506	18
43	.961	.10011	9.9893	503	17
44	.09990	.040	.9601	500	16
45	.10019	.10069	9.9310	.99497	15
46	.048	.099	.9021	494	14
47	.077	.128	.8734	491	13
48	.106	.158	.8448	488	12
49	.135	.187	.8164	485	11
50	.10164	.10216	9.7882	.99482	10
51	.192	.246	.7601	479	9
52	.221	.275	.7322	476	8
53	.250	.305	.7044	473	7
54	.279	.334	.6768	470	6
55	.10308	.10363	9.6493	.99467	5
56	.337	.393	.6220	464	4
57	.366	.422	.5949	461	3
58	.395	.452	.5679	458	2
59	.424	.481	.5411	455	1
60	.10453	.10510	9.5144	.99452	0
	N. Cos.	N. Cot.	N. Tan.	N. Sin.	

'	N. Sin.	N. Tan.	N. Cot.	N. Cos.	
0	.10453	.10510	9.5144	.99452	60
1	482	540	.4878	449	59
2	511	569	.4614	446	58
3	540	599	.4352	443	57
4	569	628	.4090	440	56
5	.10597	.10657	9.3831	.99437	55
6	626	687	.3572	434	54
7	655	716	.3315	431	53
8	684	746	.3060	428	52
9	713	775	.2806	424	51
10	.10742	.10805	9.2553	.99421	50
11	771	834	.2302	418	49
12	800	863	.2052	415	48
13	829	893	.1803	412	47
14	858	922	.1555	409	46
15	.10887	.10952	9.1309	.99406	45
16	916	.10981	.1065	402	44
17	945	.11011	.0821	399	43
18	.10973	040	.0579	396	42
19	.11002	070	.0338	393	41
20	.11031	.11099	9.0098	.99390	40
21	060	128	8.9860	386	39
22	089	158	.9623	383	38
23	118	187	.9387	380	37
24	147	217	.9152	377	36
25	.11176	.11246	8.8919	.99374	35
26	205	276	.8686	370	34
27	234	305	.8455	367	33
28	263	335	.8225	364	32
29	291	364	.7996	360	31
30	.11320	.11394	8.7769	.99357	30
31	349	423	.7542	354	29
32	378	452	.7317	351	28
33	407	482	.7093	347	27
34	436	511	.6870	344	26
35	.11465	.11541	8.6648	.99341	25
36	494	570	.6427	337	24
37	523	600	.6208	334	23
38	552	629	.5989	331	22
39	580	659	.5772	327	21
40	.11609	.11688	8.5555	.99324	20
41	638	718	.5340	320	19
42	667	747	.5126	317	18
43	696	777	.4913	314	17
44	725	806	.4701	310	16
45	.11754	.11836	8.4490	.99307	15
46	783	865	.4280	303	14
47	812	895	.4071	300	13
48	840	924	.3863	297	12
49	869	954	.3656	293	11
50	.11898	.11983	8.3450	.99290	10
51	927	.12013	.3245	286	9
52	956	042	.3041	283	8
53	.11985	072	.2838	279	7
54	.12014	101	.2636	276	6
55	.12043	.12131	8.2434	.99272	5
56	071	160	.2234	269	4
57	100	190	.2035	265	3
58	129	219	.1837	262	2
59	158	249	.1640	258	1
60	.12187	.12278	8.1443	.99255	0
	N. Cos.	N. Cot.	N. Tan.	N. Sin.	'

'	N. Sin.	N. Tan.	N. Cot.	N. Cos.	
0	.12187	.12278	8.1443	.99255	60
1	216	308	.1248	251	59
2	245	338	.1054	248	58
3	274	367	.0860	244	57
4	302	397	.0667	240	56
5	.12331	.12426	8.0476	.99237	55
6	360	456	.0285	233	54
7	389	485	8.0095	230	53
8	418	515	7.9906	226	52
9	447	544	.9718	222	51
10	.12476	.12574	7.9530	.99219	50
11	504	603	.9344	215	49
12	533	633	.9158	211	48
13	562	662	.8973	208	47
14	591	692	.8789	204	46
15	.12620	.12722	7.8606	.99200	45
16	649	751	.8424	197	44
17	678	781	.8243	193	43
18	706	810	.8062	189	42
19	735	840	.7882	186	41
20	.12764	.12869	7.7704	.99182	40
21	793	899	.7525	178	39
22	822	929	.7348	175	38
23	851	958	.7171	171	37
24	880	.12988	.6996	167	36
25	.12908	.13017	7.6821	.99163	35
26	937	047	.6647	160	34
27	966	076	.6473	156	33
28	.12995	106	.6301	152	32
29	.13024	136	.6129	148	31
30	.13053	.13165	7.5958	.99144	30
31	081	195	.5787	141	29
32	110	224	.5618	137	28
33	139	254	.5449	133	27
34	168	284	.5281	129	26
35	.13197	.13313	7.5113	.99125	25
36	226	343	.4947	122	24
37	254	372	.4781	118	23
38	283	402	.4615	114	22
39	312	432	.4451	110	21
40	.13341	.13461	7.4287	.99106	20
41	370	491	.4124	102	19
42	399	521	.3962	098	18
43	427	550	.3800	094	17
44	456	580	.3639	091	16
45	.13485	.13609	7.3479	.99087	15
46	514	639	.3319	083	14
47	543	669	.3160	079	13
48	572	698	.3002	075	12
49	600	728	.2844	071	11
50	.13629	.13758	7.2687	.99067	10
51	658	787	.2531	063	9
52	687	817	.2375	059	8
53	716	846	.2220	055	7
54	744	876	.2066	051	6
55	.13773	.13906	7.1912	.99047	5
56	802	935	.1759	043	4
57	831	965	.1607	039	3
58	860	.13995	.1455	035	2
59	889	.14024	.1304	031	1
60	.13917	.14054	7.1154	.99027	0
	N. Cos.	N. Cot.	N. Tan.	N. Sin.	'

	N. Sin.	N. Tan.	N. Cot.	N. Cos.	
0	.13917	.14054	7.1154	.99027	60
1	946	084	.1004	023	59
2	.13975	113	.0855	019	58
3	.14004	143	.0706	015	57
4	033	173	.0558	011	56
5	.14061	.14202	7.0410	.99006	55
6	090	232	.0264	.99002	54
7	119	262	7.0117	.98998	53
8	148	291	6.9972	994	52
9	177	321	.9827	990	51
10	.14205	.14351	6.9682	.98986	50
11	234	381	.9538	982	49
12	263	410	.9395	978	48
13	292	440	.9252	973	47
14	320	470	.9110	969	46
15	.14349	.14499	6.8969	.98965	45
16	378	529	.8828	961	44
17	407	559	.8687	957	43
18	436	588	.8548	953	42
19	464	618	.8408	948	41
20	.14493	.14648	6.8269	.98944	40
21	522	678	.8131	940	39
22	551	707	.7994	936	38
23	580	737	.7856	931	37
24	608	767	.7720	927	36
25	.14637	.14796	6.7584	.98923	35
26	666	826	.7448	919	34
27	695	856	.7313	914	33
28	723	886	.7179	910	32
29	752	915	.7045	906	31
30	.14781	.14945	6.6912	.98902	30
31	810	.14975	.6779	897	29
32	838	.15005	.6646	893	28
33	867	034	.6514	889	27
34	896	064	.6383	884	26
35	.14925	.15094	6.6252	.98880	25
36	954	124	.6122	876	24
37	.14982	153	.5992	871	23
38	.15011	183	.5863	867	22
39	040	213	.5734	863	21
40	.15069	.15243	6.5606	.98858	20
41	097	272	.5478	854	19
42	126	302	.5350	849	18
43	155	332	.5223	845	17
44	184	362	.5097	841	16
45	.15212	.15391	6.4971	.98836	15
46	241	421	.4846	832	14
47	270	451	.4721	827	13
48	299	481	.4596	823	12
49	327	511	.4472	818	11
50	.15356	.15540	6.4348	.98814	10
51	385	570	.4225	809	9
52	414	600	.4103	805	8
53	442	630	.3980	800	7
54	471	660	.3859	796	6
55	.15500	.15689	6.3737	.98791	5
56	529	719	.3617	787	4
57	557	749	.3496	782	3
58	586	779	.3376	778	2
59	615	809	.3257	773	1
60	.15643	.15838	6.3138	.98769	0
	N. Cos.	N. Cot.	N. Tan.	N. Sin.	

	N. Sin.	N. Tan.	N. Cot.	N. Cos.	
0	.15643	.15838	6.3138	.98769	60
1	672	868	.3019	764	59
2	701	898	.2901	760	58
3	730	928	.2783	755	57
4	758	958	.2666	751	56
5	.15787	.15988	6.2549	.98746	55
6	816	.16017	.2432	741	54
7	845	047	.2316	737	53
8	873	077	.2200	732	52
9	902	107	.2085	728	51
10	.15931	.16137	6.1970	.98723	50
11	959	167	.1856	718	49
12	.15988	196	.1742	714	48
13	.16017	226	.1628	709	47
14	046	256	.1515	704	46
15	.16074	.16286	6.1402	.98700	45
16	103	316	.1290	695	44
17	132	346	.1178	690	43
18	160	376	.1066	686	42
19	189	405	.0955	681	41
20	.16218	.16435	6.0844	.98676	40
21	246	465	.0734	671	39
22	275	495	.0624	667	38
23	304	525	.0514	662	37
24	333	555	.0405	657	36
25	.16361	.16585	6.0296	.98652	35
26	390	615	.0188	648	34
27	419	645	6.0080	643	33
28	447	674	5.9972	638	32
29	476	704	.9865	633	31
30	.16505	.16734	5.9758	.98629	30
31	533	764	.9651	624	29
32	562	794	.9545	619	28
33	591	824	.9439	614	27
34	620	854	.9333	609	26
35	.16648	.16884	5.9228	.98604	25
36	677	914	.9124	600	24
37	706	944	.9019	595	23
38	734	.16974	.8915	590	22
39	763	.17004	.8811	585	21
40	.16792	.17033	5.8708	.98580	20
41	820	063	.8605	575	19
42	849	093	.8502	570	18
43	878	123	.8400	565	17
44	906	153	.8298	561	16
45	.16935	.17183	5.8197	.98556	15
46	964	213	.8095	551	14
47	.16992	243	.7994	546	13
48	.17021	273	.7894	541	12
49	050	303	.7794	536	11
50	.17078	.17333	5.7694	.98531	10
51	107	363	.7594	526	9
52	136	393	.7495	521	8
53	164	423	.7396	516	7
54	193	453	.7297	511	6
55	.17222	.17483	5.7199	.98506	5
56	250	513	.7101	501	4
57	279	543	.7004	496	3
58	308	573	.6906	491	2
59	336	603	.6809	486	1
60	.17365	.17633	5.6713	.98481	0
	N. Cos.	N. Cot.	N. Tan.	N. Sin.	

	N. Sin.	N. Tan.	N. Cot.	N. Cos.	
0	.17365	.17633	5.6713	.98481	60
1	393	663	.6617	476	59
2	422	693	.6521	471	58
3	451	723	.6425	466	57
4	479	753	.6329	461	56
5	.17508	.17783	5.6234	.98455	55
6	537	813	.6140	450	54
7	565	843	.6045	445	53
8	594	873	.5951	440	52
9	623	903	.5857	435	51
10	.17651	.17933	5.5764	.98430	50
11	680	963	.5671	425	49
12	708	.17993	.5578	420	48
13	737	.18023	.5485	414	47
14	766	053	.5393	409	46
15	.17794	.18083	5.5301	.98404	45
16	823	113	.5209	399	44
17	852	143	.5118	394	43
18	880	173	.5026	389	42
19	909	203	.4936	383	41
20	.17937	.18233	5.4845	.98378	40
21	966	263	.4755	373	39
22	.17995	293	.4665	368	38
23	.18023	323	.4575	362	37
24	052	353	.4486	357	36
25	.18081	.18384	5.4397	.98352	35
26	109	414	.4308	347	34
27	138	444	.4219	341	33
28	166	474	.4131	336	32
29	195	504	.4043	331	31
30	.18224	.18534	5.3955	.98325	30
31	252	564	.3868	320	29
32	281	594	.3781	315	28
33	309	624	.3694	310	27
34	338	654	.3607	304	26
35	.18367	.18684	5.3521	.98299	25
36	395	714	.3435	294	24
37	424	745	.3349	288	23
38	452	775	.3263	283	22
39	481	805	.3178	277	21
40	.18509	.18835	5.3093	.98272	20
41	538	865	.3008	267	19
42	567	895	.2924	261	18
43	595	925	.2839	256	17
44	624	955	.2755	250	16
45	.18652	.18986	5.2672	.98245	15
46	681	.19016	.2588	240	14
47	710	046	.2505	234	13
48	738	076	.2422	229	12
49	767	106	.2339	223	11
50	.18795	.19136	5.2257	.98218	10
51	824	166	.2174	212	9
52	852	197	.2092	207	8
53	881	227	.2011	201	7
54	910	257	.1929	196	6
55	.18938	.19287	5.1848	.98190	5
56	967	317	.1767	185	4
57	.18995	347	.1686	179	3
58	.19024	378	.1606	174	2
59	052	408	.1526	168	1
60	.19081	.19438	5.1446	.98163	0
	N. Cos.	N. Cot.	N. Tan.	N. Sin.	

	N. Sin.	N. Tan.	N. Cot.	N. Cos.	
0	.19081	.19438	5.1446	.98163	60
1	109	468	.1366	157	59
2	138	498	.1286	152	58
3	167	529	.1207	146	57
4	195	559	.1128	140	56
5	.19224	.19589	5.1049	.98135	55
6	252	619	.1070	129	54
7	281	649	.0892	124	53
8	309	680	.0814	118	52
9	338	710	.0736	112	51
10	.19366	.19740	5.0658	.98107	50
11	395	770	.0581	101	49
12	423	801	.0504	096	48
13	452	831	.0427	090	47
14	481	861	.0350	084	46
15	.19509	.19891	5.0273	.98079	45
16	538	921	.0197	073	44
17	566	952	.0121	067	43
18	595	.19982	5.0045	061	42
19	623	.20012	4.9969	056	41
20	.19652	.20042	4.9894	.98050	40
21	680	073	.9819	044	39
22	709	103	.9744	039	38
23	737	133	.9669	033	37
24	766	164	.9594	027	36
25	.19794	.20194	4.9520	.98021	35
26	823	224	.9446	016	34
27	851	254	.9372	010	33
28	880	285	.9298	.98004	32
29	908	315	.9225	.97998	31
30	.19937	.20345	4.9152	.97992	30
31	965	376	.9078	987	29
32	.19994	406	.9006	981	28
33	.20022	436	.8933	975	27
34	051	466	.8860	969	26
35	.20079	.20497	4.8788	.97963	25
36	108	527	.8716	958	24
37	136	557	.8644	952	23
38	165	588	.8573	946	22
39	193	618	.8501	940	21
40	.20222	.20648	4.8430	.97934	20
41	250	679	.8359	928	19
42	279	709	.8288	922	18
43	307	739	.8218	916	17
44	336	770	.8147	910	16
45	.20364	.20800	4.8077	.97905	15
46	393	830	.8007	899	14
47	421	861	.7937	893	13
48	450	891	.7867	887	12
49	478	921	.7798	881	11
50	.20507	.20952	4.7729	.97875	10
51	535	.20982	.7659	869	9
52	563	.21013	.7591	863	8
53	592	043	.7522	857	7
54	620	073	.7453	851	6
55	.20649	.21104	4.7385	.97845	5
56	677	134	.7317	839	4
57	706	164	.7249	833	3
58	734	195	.7181	827	2
59	763	225	.7114	821	1
60	.20791	.21256	4.7046	.97815	0
	N. Cos.	N. Cot.	N. Tan.	N. Sin.	

	N. Sin.	N. Tan.	N. Cot.	N. Cos.	
0	.20791	.21256	4.7046	.97815	60
1	820	286	.6979	809	59
2	848	316	.6912	803	58
3	877	347	.6845	797	57
4	905	377	.6779	791	56
5	.20933	.21408	4.6712	.97784	55
6	962	438	.6646	778	54
7	.20990	469	.6580	772	53
8	.21019	499	.6514	766	52
9	047	529	.6448	760	51
10	.21076	.21560	4.6382	.97754	50
11	104	590	.6317	748	49
12	132	621	.6252	742	48
13	161	651	.6187	735	47
14	189	682	.6122	729	46
15	.21218	.21712	4.6057	.97723	45
16	246	743	.5993	717	44
17	275	773	.5928	711	43
18	303	804	.5864	705	42
19	331	834	.5800	698	41
20	.21360	.21864	4.5736	.97692	40
21	388	895	.5673	686	39
22	417	925	.5609	680	38
23	445	956	.5546	673	37
24	474	.21986	.5483	667	36
25	.21502	.22017	4.5420	.97661	35
26	530	047	.5357	655	34
27	559	078	.5294	648	33
28	587	108	.5232	642	32
29	616	139	.5169	636	31
30	.21644	.22169	4.5107	.97630	30
31	672	200	.5045	623	29
32	701	231	.4983	617	28
33	729	261	.4922	611	27
34	758	292	.4860	604	26
35	.21786	.22322	4.4799	.97598	25
36	814	353	.4737	592	24
37	843	383	.4676	585	23
38	871	414	.4615	579	22
39	899	444	.4555	573	21
40	.21928	.22475	4.4494	.97566	20
41	956	505	.4434	560	19
42	.21985	536	.4373	553	18
43	.22013	567	.4313	547	17
44	041	597	.4253	541	16
45	.22070	.22628	4.4194	.97534	15
46	098	658	.4134	528	14
47	126	689	.4075	521	13
48	155	719	.4015	515	12
49	183	750	.3956	508	11
50	.22212	.22781	4.3897	.97502	10
51	240	811	.3838	496	9
52	268	842	.3779	489	8
53	297	872	.3721	483	7
54	325	903	.3662	476	6
55	.22353	.22934	4.3604	.97470	5
56	382	964	.3546	463	4
57	410	.22995	.3488	457	3
58	438	.23026	.3430	450	2
59	467	056	.3372	444	1
60	.22495	.23087	4.3315	.97437	0
	N. Cos.	N. Cot.	N. Tan.	N. Sin.	'

	N. Sin.	N. Tan.	N. Cot.	N. Cos.	
0	.22495	.23087	4.3315	.97437	60
1	523	117	.3257	430	59
2	552	148	.3200	424	58
3	580	179	.3143	417	57
4	608	209	.3086	411	56
5	.22637	.23240	4.3029	.97404	55
6	665	271	.2972	398	54
7	693	301	.2916	391	53
8	722	332	.2859	384	52
9	750	363	.2803	378	51
10	.22778	.23393	4.2747	.97371	50
11	807	424	.2691	365	49
12	835	455	.2635	358	48
13	863	485	.2580	351	47
14	892	516	.2524	345	46
15	.22920	.23547	4.2468	.97338	45
16	948	578	.2413	331	44
17	.22977	608	.2358	325	43
18	.23005	639	.2303	318	42
19	033	670	.2248	311	41
20	.23062	.23700	4.2193	.97304	40
21	090	731	.2139	298	39
22	118	762	.2084	291	38
23	146	793	.2030	284	37
24	175	823	.1976	278	36
25	.23203	.23854	4.1922	.97271	35
26	231	885	.1868	264	34
27	260	916	.1814	257	33
28	288	946	.1760	251	32
29	316	.23977	.1706	244	31
30	.23345	.24008	4.1653	.97237	30
31	373	039	.1600	230	29
32	401	069	.1547	223	28
33	429	100	.1493	217	27
34	458	131	.1441	210	26
35	.23486	.24162	4.1388	.97203	25
36	514	193	.1335	196	24
37	542	223	.1282	189	23
38	571	254	.1230	182	22
39	599	285	.1178	176	21
40	.23627	.24316	4.1126	.97169	20
41	656	347	.1074	162	19
42	684	377	.1022	155	18
43	712	408	.0970	148	17
44	740	439	.0918	141	16
45	.23769	.24470	4.0867	.97134	15
46	797	501	.0815	127	14
47	825	532	.0764	120	13
48	853	562	.0713	113	12
49	882	593	.0662	106	11
50	.23910	.24624	4.0611	.97100	10
51	938	655	.0560	093	9
52	966	686	.0509	086	8
53	.23995	717	.0459	079	7
54	.24023	747	.0408	072	6
55	.24051	.24778	4.0358	.97065	5
56	079	809	.0308	058	4
57	108	840	.0257	051	3
58	136	871	.0207	044	2
59	164	902	.0158	037	1
60	.24192	.24933	4.0108	.97030	0
	N. Cos.	N. Cot.	N. Tan.	N. Sin.	'

	N. Sin.	N. Tan.	N. Cot.	N. Cos.	
0	.24192	.24933	4.0108	.97030	60
1	220	964	.0058	.023	59
2	249	.24995	4.0009	.015	58
3	277	.25026	3.9959	.008	57
4	305	056	.9910	.97001	56
5	.24333	.25087	3.9861	.96994	55
6	362	118	.9812	.987	54
7	390	149	.9763	.980	53
8	418	180	.9714	.973	52
9	446	211	.9665	.966	51
10	.24474	.25242	3.9617	.96959	50
11	503	273	.9568	.952	49
12	531	304	.9520	.945	48
13	559	335	.9471	.937	47
14	587	366	.9423	.930	46
15	.24615	.25397	3.9375	.96923	45
16	644	428	.9327	.916	44
17	672	459	.9279	.909	43
18	700	490	.9232	.902	42
19	728	521	.9184	.894	41
20	.24756	.25552	3.9135	.96887	40
21	784	583	.9089	.880	39
22	813	614	.9042	.873	38
23	841	645	.8995	.866	37
24	869	676	.8947	.858	36
25	.24897	.25707	3.8900	.96851	35
26	925	738	.8854	.844	34
27	954	769	.8807	.837	33
28	.24982	800	.8760	.829	32
29	.25010	831	.8714	.822	31
30	.25038	.25862	3.8667	.96815	30
31	066	893	.8621	.807	29
32	094	924	.8575	.800	28
33	122	955	.8528	.793	27
34	151	.25986	.8482	.786	26
35	.25179	.26017	3.8436	.96778	25
36	207	048	.8391	.771	24
37	235	079	.8345	.764	23
38	263	110	.8299	.756	22
39	291	141	.8254	.749	21
40	.25320	.26172	3.8208	.96742	20
41	348	203	.8163	.734	19
42	376	235	.8118	.727	18
43	404	266	.8073	.719	17
44	432	297	.8028	.712	16
45	.25460	.26328	3.7983	.96705	15
46	488	359	.7938	.697	14
47	516	390	.7893	.690	13
48	545	421	.7848	.682	12
49	573	452	.7804	.675	11
50	.25601	.26483	3.7760	.96667	10
51	629	515	.7715	.660	9
52	657	546	.7671	.653	8
53	685	577	.7627	.645	7
54	713	608	.7583	.638	6
55	.25741	.26639	3.7539	.96630	5
56	769	670	.7495	.623	4
57	798	701	.7451	.615	3
58	826	733	.7408	.608	2
59	854	764	.7364	.600	1
60	.25882	.26795	3.7321	.96593	0
	N. Cos.	N. Cot.	N. Tan.	N. Sin.	

	N. Sin.	N. Tan.	N. Cot.	N. Cos.	
0	.25882	.26795	3.7321	.96593	60
1	910	826	.7277	.585	59
2	938	857	.7234	.578	58
3	966	888	.7191	.570	57
4	.25994	920	.7148	.562	56
5	.26022	.26951	3.7105	.96555	55
6	050	.26982	.7062	.547	54
7	079	.27013	.7019	.540	53
8	107	044	.6976	.532	52
9	135	076	.6933	.524	51
10	.26163	.27107	3.6891	.96517	50
11	191	138	.6848	.509	49
12	219	169	.6806	.502	48
13	247	201	.6764	.494	47
14	275	232	.6722	.486	46
15	.26303	.27263	3.6680	.96479	45
16	331	294	.6638	.471	44
17	359	326	.6596	.463	43
18	387	357	.6554	.456	42
19	415	388	.6512	.448	41
20	.26443	.27419	3.6470	.96440	40
21	471	451	.6429	.433	39
22	500	482	.6387	.425	38
23	528	513	.6346	.417	37
24	556	545	.6305	.410	36
25	.26584	.27576	3.6264	.96402	35
26	612	607	.6222	.394	34
27	640	638	.6181	.386	33
28	668	670	.6140	.379	32
29	696	701	.6100	.371	31
30	.26724	.27732	3.6059	.96363	30
31	752	764	.6018	.355	29
32	780	795	.5978	.347	28
33	808	826	.5937	.340	27
34	836	858	.5897	.332	26
35	.26864	.27889	3.5856	.96324	25
36	892	921	.5816	.316	24
37	920	952	.5776	.308	23
38	948	.27983	.5736	.301	22
39	.26976	.28015	.5696	.293	21
40	.27004	.28046	3.5656	.96285	20
41	032	077	.5616	.277	19
42	060	109	.5576	.269	18
43	088	140	.5536	.261	17
44	116	172	.5497	.253	16
45	.27144	.28203	3.5457	.96246	15
46	172	234	.5418	.238	14
47	200	266	.5379	.230	13
48	228	297	.5339	.222	12
49	256	329	.5300	.214	11
50	.27284	.28360	3.5261	.96206	10
51	312	391	.5222	.198	9
52	340	423	.5183	.190	8
53	368	454	.5144	.182	7
54	396	486	.5105	.174	6
55	.27424	.28517	3.5067	.96166	5
56	452	549	.5028	.158	4
57	480	580	.4989	.150	3
58	508	612	.4951	.142	2
59	536	643	.4912	.134	1
60	.27564	.28675	3.4874	.96126	0
	N. Cos.	N. Cot.	N. Tan.	N. Sin.	

'	N. Sin.	N. Tan.	N. Cot.	N. Cos.	
0	.27564	.28675	3.4874	.96126	60
1	592	706	.4836	118	59
2	620	738	.4798	110	58
3	648	769	.4760	102	57
4	676	801	.4722	094	56
5	.27704	.28832	3.4684	.96086	55
6	731	864	.4646	078	54
7	759	895	.4608	070	53
8	787	927	.4570	062	52
9	815	958	.4533	054	51
10	.27843	.28990	3.4495	.96046	50
11	871	.29021	.4458	037	49
12	899	053	.4420	029	48
13	927	084	.4383	021	47
14	955	116	.4346	013	46
15	.27983	.29147	3.4308	.96005	45
16	.28011	179	.4271	.95997	44
17	039	210	.4234	989	43
18	067	242	.4197	981	42
19	095	274	.4160	972	41
20	.28123	.29305	3.4124	.95964	40
21	150	337	.4087	956	39
22	178	368	.4050	948	38
23	206	400	.4014	940	37
24	234	432	.3977	931	36
25	.28262	.29463	3.3941	.95923	35
26	290	495	.3904	915	34
27	318	526	.3868	907	33
28	346	558	.3832	898	32
29	374	590	.3796	890	31
30	.28402	.29621	3.3759	.95882	30
31	429	653	.3723	874	29
32	457	685	.3687	865	28
33	485	716	.3652	857	27
34	513	748	.3616	849	26
35	.28541	.29780	3.3580	.95841	25
36	569	811	.3544	832	24
37	597	843	.3509	824	23
38	625	875	.3473	816	22
39	652	906	.3438	807	21
40	.28680	.29938	3.3402	.95799	20
41	708	.29970	.3367	791	19
42	736	.30001	.3332	782	18
43	764	033	.3297	774	17
44	792	065	.3261	766	16
45	.28820	.30097	3.3226	.95757	15
46	847	128	.3191	749	14
47	875	160	.3156	740	13
48	903	192	.3122	732	12
49	931	224	.3087	724	11
50	.28959	.30255	3.3052	.95715	10
51	.28987	237	.3017	707	9
52	.29015	319	.2983	698	8
53	042	351	.2948	690	7
54	070	382	.2914	681	6
55	.29098	.30414	3.2879	.95673	5
56	126	446	.2845	664	4
57	154	478	.2811	656	3
58	182	509	.2777	647	2
59	209	541	.2743	639	1
60	.29237	.30573	3.2709	.95630	0
	N. Cos.	N. Cot.	N. Tan.	N. Sin.	'

'	N. Sin.	N. Tan.	N. Cot.	N. Cos.	
0	.29237	.30573	3.2709	.95630	60
1	265	605	.2675	622	59
2	293	637	.2641	613	58
3	321	669	.2607	605	57
4	348	700	.2573	596	56
5	.29376	.30732	3.2539	.95588	55
6	404	764	.2506	579	54
7	432	796	.2472	571	53
8	460	828	.2438	562	52
9	487	860	.2405	554	51
10	.29515	.30891	3.2371	.95545	50
11	543	923	.2338	536	49
12	571	955	.2305	528	48
13	599	.30987	.2272	519	47
14	626	.31019	.2238	511	46
15	.29654	.31051	3.2205	.95502	45
16	682	083	.2172	493	44
17	710	115	.2139	485	43
18	737	147	.2106	476	42
19	765	178	.2073	467	41
20	.29793	.31210	3.2041	.95459	40
21	821	242	.2008	450	39
22	849	274	.1975	441	38
23	876	306	.1943	433	37
24	904	338	.1910	424	36
25	.29932	.31370	3.1878	.95415	35
26	960	402	.1845	407	34
27	.29987	434	.1813	398	33
28	.30015	466	.1780	389	32
29	043	498	.1748	380	31
30	.30071	.31530	3.1716	.95372	30
31	098	562	.1684	363	29
32	126	594	.1652	354	28
33	154	626	.1620	345	27
34	182	658	.1588	337	26
35	.30209	.31690	3.1556	.95328	25
36	237	722	.1524	319	24
37	265	754	.1492	310	23
38	292	786	.1460	301	22
39	320	818	.1429	293	21
40	.30348	.31850	3.1397	.95284	20
41	376	882	.1366	275	19
42	403	914	.1334	266	18
43	431	946	.1303	257	17
44	459	.31978	.1271	248	16
45	.30486	.32010	3.1240	.95240	15
46	514	042	.1209	231	14
47	542	074	.1178	222	13
48	570	106	.1146	213	12
49	597	139	.1115	204	11
50	.30625	.32171	3.1084	.95195	10
51	653	203	.1053	186	9
52	680	235	.1022	177	8
53	708	267	.0991	168	7
54	736	299	.0961	159	6
55	.30763	.32331	3.0930	.95150	5
56	791	363	.0899	142	4
57	819	396	.0868	133	3
58	846	428	.0838	124	2
59	874	460	.0807	115	1
60	.30902	.32492	3.0777	.95106	0
	N. Cos.	N. Cot.	N. Tan.	N. Sin.	'

	N. Sin.	N. Tan.	N. Cot.	N. Cos.	
0	.30902	.32492	3.0777	.95106	60
1	929	524	.0746	097	59
2	957	556	.0716	088	58
3	.30985	588	.0686	079	57
4	.31012	621	.0655	070	56
5	.31040	.32653	3.0625	.95061	55
6	068	685	.0595	052	54
7	095	717	.0565	043	53
8	123	749	.0535	033	52
9	151	782	.0505	024	51
10	.31178	.32814	3.0475	.95015	50
11	206	846	.0445	.95006	49
12	233	878	.0415	.94997	48
13	261	911	.0385	988	47
14	289	943	.0356	979	46
15	.31316	.32975	3.0326	.94970	45
16	344	.33007	.0296	961	44
17	372	040	.0267	952	43
18	399	072	.0237	943	42
19	427	104	.0208	933	41
20	.31454	.33136	3.0178	.94924	40
21	482	169	.0149	915	39
22	510	201	.0120	906	38
23	537	233	.0090	897	37
24	565	266	.0061	888	36
25	.31593	.33298	3.0032	.94878	35
26	620	330	3.0003	869	34
27	648	363	2.9974	860	33
28	675	395	.9945	851	32
29	703	427	.9916	842	31
30	.31730	.33460	2.9887	.94832	30
31	758	492	.9858	823	29
32	786	524	.9829	814	28
33	813	557	.9800	805	27
34	841	589	.9772	795	26
35	.31868	.33621	2.9743	.94786	25
36	896	654	.9714	777	24
37	923	686	.9686	768	23
38	951	718	.9657	758	22
39	.31979	751	.9629	749	21
40	.32006	.33783	2.9600	.94740	20
41	034	816	.9572	730	19
42	061	848	.9544	721	18
43	089	881	.9515	712	17
44	116	913	.9487	702	16
45	.32144	.33945	2.9459	.94693	15
46	171	.33978	.9431	684	14
47	199	.34010	.9403	674	13
48	227	043	.9375	665	12
49	254	075	.9347	656	11
50	.32282	.34108	2.9319	.94646	10
51	309	140	.9291	637	9
52	337	173	.9263	627	8
53	364	205	.9235	618	7
54	392	238	.9208	609	6
55	.32419	.34270	2.9180	.94599	5
56	447	303	.9152	590	4
57	474	335	.9125	580	3
58	502	368	.9097	571	2
59	529	400	.9070	561	1
60	.32557	.34433	2.9042	.94552	0
	N. Cos.	N. Cot.	N. Tan.	N. Sin.	

	N. Sin.	N. Tan.	N. Cot.	N. Cos.	
0	.32557	.34433	2.9042	.94552	60
1	584	465	.9015	542	59
2	612	498	.8987	533	58
3	639	530	.8960	523	57
4	667	563	.8933	514	56
5	.32694	.34596	2.8905	.94504	55
6	722	628	.8878	495	54
7	749	661	.8851	485	53
8	777	693	.8824	476	52
9	804	726	.8797	466	51
10	.32832	.34758	2.8770	.94457	50
11	859	791	.8743	447	49
12	887	824	.8716	438	48
13	914	856	.8689	428	47
14	942	889	.8662	418	46
15	.32969	.34922	2.8636	.94409	45
16	.32997	954	.8609	399	44
17	.33024	.34987	.8582	390	43
18	051	.35020	.8556	380	42
19	079	052	.8529	370	41
20	.33106	.35085	2.8502	.94361	40
21	134	118	.8476	351	39
22	161	150	.8449	342	38
23	189	183	.8423	332	37
24	216	216	.8397	322	36
25	.33244	.35248	2.8370	.94313	35
26	271	281	.8344	303	34
27	298	314	.8318	293	33
28	326	346	.8291	284	32
29	353	379	.8265	274	31
30	.33381	.35412	2.8239	.94264	30
31	408	445	.8213	254	29
32	436	477	.8187	245	28
33	463	510	.8161	235	37
34	490	543	.8135	225	26
35	.33518	.35576	2.8109	.94215	25
36	545	608	.8083	206	24
37	573	641	.8057	196	23
38	600	674	.8032	186	22
39	627	707	.8006	176	21
40	.33655	.35740	2.7980	.94167	20
41	682	772	.7955	157	19
42	710	805	.7929	147	18
43	737	838	.7903	137	17
44	764	871	.7878	127	16
45	.33792	.35904	2.7852	.94118	15
46	819	937	.7827	108	14
47	846	.35969	.7801	098	13
48	874	.36002	.7776	088	12
49	901	035	.7751	078	11
50	.33929	.36068	2.7725	.94068	10
51	956	101	.7700	058	9
52	.33983	134	.7675	049	8
53	.34011	167	.7650	039	7
54	038	199	.7625	029	6
55	.34065	.36232	2.7600	.94019	5
56	093	265	.7575	.94009	4
57	120	298	.7550	.93999	3
58	147	331	.7525	989	2
59	175	364	.7500	979	1
60	.34202	.36397	2.7475	.93969	0
	N. Cos.	N. Cot.	N. Tan.	N. Sin.	

	N. Sin.	N. Tan.	N. Cot.	N. Cos.	
0	.34202	.36397	2.7475	.93969	60
1	229	430	.7450	959	59
2	257	463	.7425	949	58
3	284	496	.7400	939	57
4	311	529	.7376	929	56
5	.34339	.36562	2.7351	.93919	55
6	366	595	.7326	909	54
7	393	628	.7302	899	53
8	421	661	.7277	889	52
9	448	694	.7253	879	51
10	.34475	.36727	2.7228	.93869	50
11	503	760	.7204	859	49
12	530	793	.7179	849	48
13	557	826	.7155	839	47
14	584	859	.7130	829	46
15	.34612	.36892	2.7106	.93819	45
16	639	925	.7082	809	44
17	666	958	.7058	799	43
18	694	.36991	.7034	789	42
19	721	.37024	.7009	779	41
20	.34748	.37057	2.6985	.93769	40
21	775	090	.6961	759	39
22	803	123	.6937	748	38
23	830	157	.6913	738	37
24	857	190	.6889	728	36
25	.34884	.37223	2.6865	.93718	35
26	912	256	.6841	708	34
27	939	289	.6818	698	33
28	966	322	.6794	688	32
29	.34993	355	.6770	677	31
30	.35021	.37388	2.6746	.93667	30
31	048	422	.6723	657	29
32	075	455	.6699	647	28
33	102	488	.6675	637	27
34	130	521	.6652	626	26
35	.35157	.37554	2.6628	.93616	25
36	184	588	.6605	606	24
37	211	621	.6581	596	23
38	239	654	.6558	585	22
39	266	687	.6534	575	21
40	.35293	.37720	2.6511	.93565	20
41	320	754	.6488	555	19
42	347	787	.6464	544	18
43	375	820	.6441	534	17
44	402	853	.6418	524	16
45	.35429	.37887	2.6395	.93514	15
46	456	920	.6371	503	14
47	484	953	.6348	493	13
48	511	.37986	.6325	483	12
49	538	.38020	.6302	472	11
50	.35565	.38053	2.6279	.93462	10
51	592	086	.6256	452	9
52	619	120	.6233	441	8
53	647	153	.6210	431	7
54	674	186	.6187	420	6
55	.35701	.38220	2.6165	.93410	5
56	728	253	.6142	400	4
57	755	286	.6119	389	3
58	782	320	.6096	379	2
59	810	353	.6074	368	1
60	.35837	.38386	2.6051	.93358	0
	N. Cos.	N. Cot.	N. Tan.	N. Sin.	

	N. Sin.	N. Tan.	N. Cot.	N. Cos.	
0	.35837	.38386	2.6051	.93358	60
1	864	420	.6028	348	59
2	891	453	.6006	337	58
3	918	487	.5983	327	57
4	945	520	.5961	316	56
5	.35973	.38553	2.5938	.93306	55
6	.36000	587	.5916	295	54
7	027	620	.5893	285	53
8	054	654	.5871	274	52
9	081	687	.5848	264	51
10	.36108	.38721	2.5826	.93253	50
11	135	754	.5804	243	49
12	162	787	.5782	232	48
13	190	821	.5759	222	47
14	217	854	.5737	211	46
15	.36244	.38888	2.5715	.93201	45
16	271	921	.5693	190	44
17	298	955	.5671	180	43
18	325	.38988	.5649	169	42
19	352	.39022	.5627	159	41
20	.36379	.39055	2.5605	.93148	40
21	406	089	.5583	137	39
22	434	122	.5561	127	38
23	461	156	.5539	116	37
24	488	190	.5517	106	36
25	.36515	.39223	2.5495	.93095	35
26	542	257	.5473	084	34
27	569	290	.5452	074	33
28	596	324	.5430	063	32
29	623	357	.5408	052	31
30	.36650	.39391	2.5386	.93042	30
31	677	425	.5365	031	29
32	704	458	.5343	020	28
33	731	492	.5322	.93010	27
34	758	526	.5300	.92999	26
35	.36785	.39559	2.5279	.92988	25
36	812	593	.5257	978	24
37	839	626	.5236	967	23
38	867	660	.5214	956	22
39	894	694	.5193	945	21
40	.36921	.39727	2.5172	.92935	20
41	948	761	.5150	924	19
42	.36975	795	.5129	913	18
43	.37002	829	.5108	902	17
44	029	862	.5086	892	16
45	.37056	.39896	2.5065	.92881	15
46	083	930	.5044	870	14
47	110	963	.5023	859	13
48	137	.39997	.5002	849	12
49	164	.40031	.4981	838	11
50	.37191	.40065	2.4960	.92827	10
51	218	098	.4939	816	9
52	245	132	.4918	805	8
53	272	166	.4897	794	7
54	299	200	.4876	784	6
55	.37326	.40234	2.4855	.92773	5
56	353	267	.4834	762	4
57	380	301	.4813	751	3
58	407	335	.4792	740	2
59	434	369	.4772	729	1
60	.37461	.40403	2.4751	.92718	0
	N. Cos.	N. Cot.	N. Tan.	N. Sin.	

	N. Sin.	N. Tan.	N. Cot.	N. Cos.	
0	.37461	.40403	2.4751	.92718	60
1	488	436	.4730	707	59
2	515	470	.4709	697	58
3	542	504	.4689	686	57
4	569	538	.4668	675	56
5	.37595	.40572	2.4648	.92664	55
6	622	606	.4627	653	54
7	649	640	.4606	642	53
8	676	674	.4586	631	52
9	703	707	.4566	620	51
10	.37730	.40741	2.4545	.92609	50
11	757	775	.4525	598	49
12	784	809	.4504	587	48
13	811	843	.4484	576	47
14	838	877	.4464	565	46
15	.37865	.40911	2.4443	.92554	45
16	892	945	.4423	543	44
17	919	.40979	.4403	532	43
18	946	.41013	.4383	521	42
19	973	047	.4362	510	41
20	.37999	.41081	2.4342	.92499	40
21	.38026	115	.4322	488	39
22	053	149	.4302	477	38
23	080	183	.4282	466	37
24	107	217	.4262	455	36
25	.38134	.41251	2.4242	.92444	35
26	161	285	.4222	432	34
27	188	319	.4202	421	33
28	215	353	.4182	410	32
29	241	387	.4162	399	31
30	.38268	.41421	2.4142	.92388	30
31	295	455	.4122	377	29
32	322	490	.4102	366	28
33	349	524	.4083	355	27
34	376	558	.4063	343	26
35	.38403	.41592	2.4043	.92332	25
36	430	626	.4023	321	24
37	456	660	.4004	310	23
38	483	694	.3984	299	22
39	510	728	.3964	287	21
40	.38537	.41763	2.3945	.92276	20
41	564	797	.3925	265	19
42	591	831	.3906	254	18
43	617	865	.3886	243	17
44	644	899	.3867	231	16
45	.38671	.41933	2.3847	.92220	15
46	698	.41968	.3828	209	14
47	725	.42002	.3808	198	13
48	752	036	.3789	186	12
49	778	070	.3770	175	11
50	.38805	.42105	2.3750	.92164	10
51	832	139	.3731	152	9
52	859	173	.3712	141	8
53	886	207	.3693	130	7
54	912	242	.3673	119	6
55	.38939	.42276	2.3654	.92107	5
56	966	310	.3635	096	4
57	.38993	345	.3616	085	3
58	.39020	379	.3597	073	2
59	046	413	.3578	062	1
60	.39073	.42447	2.3559	.92050	0
	N. Cos.	N. Cot.	N. Tan.	N. Sin.	

	N. Sin.	N. Tan.	N. Cot.	N. Cos.	
0	.39073	.42447	2.3559	.92050	60
1	100	482	.3539	039	59
2	127	516	.3520	028	58
3	153	551	.3501	016	57
4	180	585	.3483	.92005	56
5	.39207	.42619	2.3464	.91994	55
6	234	654	.3445	982	54
7	260	688	.3426	971	53
8	287	722	.3407	959	52
9	314	757	.3388	948	51
10	.39341	.42791	2.3369	.91936	50
11	367	826	.3351	925	49
12	394	860	.3332	914	48
13	421	894	.3313	902	47
14	448	929	.3294	891	46
15	.39474	.42963	2.3276	.91879	45
16	501	.42998	.3257	868	44
17	528	.43032	.3238	856	43
18	555	067	.3220	845	42
19	581	101	.3201	833	41
20	.39608	.43136	2.3183	.91822	40
21	635	170	.3164	810	39
22	661	205	.3146	799	38
23	688	239	.3127	787	37
24	715	274	.3109	775	36
25	.39741	.43308	2.3090	.91764	35
26	768	343	.3072	752	34
27	795	378	.3053	741	33
28	822	412	.3035	729	32
29	848	447	.3017	718	31
30	.39875	.43481	2.2998	.91706	30
31	902	516	.2980	694	29
32	928	550	.2962	683	28
33	955	585	.2944	671	27
34	.39982	620	.2925	660	26
35	.40008	.43654	2.2907	.91648	25
36	035	689	.2889	636	24
37	062	724	.2871	625	23
38	088	758	.2853	613	22
39	115	793	.2835	601	21
40	.40141	.43828	2.2817	.91590	20
41	168	862	.2799	578	19
42	195	897	.2781	566	18
43	221	932	.2763	555	17
44	248	.43966	.2745	543	16
45	.40275	.44001	2.2727	.91531	15
46	301	036	.2709	519	14
47	328	071	.2691	508	13
48	355	105	.2673	496	12
49	381	140	.2655	484	11
50	.40408	.44175	2.2637	.91472	10
51	434	210	.2620	461	9
52	461	244	.2602	449	8
53	488	279	.2584	437	7
54	514	314	.2566	425	6
55	.40541	.44349	2.2549	.91414	5
56	567	384	.2531	402	4
57	594	418	.2513	390	3
58	621	453	.2496	378	2
59	647	488	.2478	366	1
60	.40674	.44523	2.2460	.91355	0
	N. Cos.	N. Cot.	N. Tan.	N. Sin.	

'	N. Sin.	N. Tan.	N. Cot.	N. Cos.	
0	.40674	.44523	2.2460	.91355	60
1	700	558	.2443	343	59
2	727	593	.2425	331	58
3	753	627	.2408	319	57
4	780	662	.2390	307	56
5	.40806	.44697	2.2373	.91295	55
6	833	732	.2355	283	54
7	860	767	.2338	272	53
8	886	802	.2320	260	52
9	913	837	.2303	248	51
10	.40939	.44872	2.2286	.91236	50
11	966	907	.2268	224	49
12	.40992	942	.2251	212	48
13	.41019	.44977	.2234	200	47
14	045	.45012	.2216	188	46
15	.41072	.45047	2.2199	.91176	45
16	098	082	.2182	164	44
17	125	117	.2165	152	43
18	151	152	.2148	140	42
19	178	187	.2130	128	41
20	.41204	.45222	2.2113	.91116	40
21	231	257	.2096	104	39
22	257	292	.2079	092	38
23	284	327	.2062	080	37
24	310	362	.2045	068	36
25	.41337	.45397	2.2028	.91056	35
26	363	432	.2011	044	34
27	390	467	.1994	032	33
28	416	502	.1977	020	32
29	443	538	.1960	.91008	31
30	.41469	.45573	2.1943	.90996	30
31	496	608	.1926	984	29
32	522	643	.1909	972	28
33	549	678	.1892	960	27
34	575	713	.1876	948	26
35	.41602	.45748	2.1859	.90936	25
36	628	784	.1842	924	24
37	655	819	.1825	911	23
38	681	854	.1808	899	22
39	707	889	.1792	887	21
40	.41734	.45924	2.1775	.90875	20
41	760	960	.1758	863	19
42	787	.45995	.1742	851	18
43	813	.46030	.1725	839	17
44	840	065	.1708	826	16
45	.41866	.46101	2.1692	.90814	15
46	892	136	.1675	802	14
47	919	171	.1659	790	13
48	945	206	.1642	778	12
49	972	242	.1625	766	11
50	.41998	.46277	2.1609	.90753	10
51	.42024	312	.1592	741	9
52	051	348	.1576	729	8
53	077	383	.1560	717	7
54	104	418	.1543	704	6
55	.42130	.46454	2.1527	.90692	5
56	156	489	.1510	680	4
57	183	525	.1494	668	3
58	209	560	.1478	655	2
59	235	595	.1461	643	1
60	.42262	.46631	2.1445	.90631	0
	N. Cos.	N. Cot.	N. Tan.	N. Sin.	'

'	N. Sin.	N. Tan.	N. Cot.	N. Cos.	
0	.42262	.46631	2.1445	.90631	60
1	288	666	.1429	618	59
2	315	702	.1413	606	58
3	341	737	.1396	594	57
4	367	772	.1380	582	56
5	.42394	.46808	2.1364	.90569	55
6	420	843	.1348	557	54
7	446	879	.1332	545	53
8	473	914	.1315	532	52
9	499	950	.1299	520	51
10	.42525	.46985	2.1283	.90507	50
11	552	.47021	.1267	495	49
12	578	056	.1251	483	48
13	604	092	.1235	470	47
14	631	128	.1219	458	46
15	.42657	.47163	2.1203	.90446	45
16	683	199	.1187	433	44
17	709	234	.1171	421	43
18	736	270	.1155	408	42
19	762	305	.1139	396	41
20	.42788	.47341	2.1123	.90383	40
21	815	377	.1107	371	39
22	841	412	.1092	358	38
23	867	448	.1076	346	37
24	894	483	.1060	334	36
25	.42920	.47519	2.1044	.90321	35
26	946	555	.1028	309	34
27	972	590	.1013	296	33
28	.42999	626	.0997	284	32
29	.43025	662	.0981	271	31
30	.43051	.47698	2.0965	.90259	30
31	077	733	.0950	246	29
32	104	769	.0934	233	28
33	130	805	.0918	221	27
34	156	840	.0903	208	26
35	.43182	.47876	2.0887	.90196	25
36	209	912	.0872	183	24
37	235	948	.0856	171	23
38	261	.47984	.0840	158	22
39	287	.48019	.0825	146	21
40	.43313	.48055	2.0809	.90133	20
41	340	091	.0794	120	19
42	366	127	.0778	108	18
43	392	163	.0763	095	17
44	418	198	.0748	082	16
45	.43445	.48234	2.0732	.90070	15
46	471	270	.0717	057	14
47	497	306	.0701	045	13
48	523	342	.0686	032	12
49	549	378	.0671	019	11
50	.43575	.48414	2.0655	.90007	10
51	602	450	.0640	.89994	9
52	628	486	.0625	981	8
53	654	521	.0609	968	7
54	680	557	.0594	956	6
55	.43706	.48593	2.0579	.89943	5
56	733	629	.0564	930	4
57	759	665	.0549	918	3
58	785	701	.0533	905	2
59	811	737	.0518	892	1
60	.43837	.48773	2.0503	.89879	0
	N. Cos.	N. Cot.	N. Tan.	N. Sin.	'

'	N. Sin.	N. Tan.	N. Cot.	N. Cos.	
0	.43837	.48773	2.0503	.89879	60
1	863	809	.0488	867	59
2	889	845	.0473	854	58
3	916	881	.0458	841	57
4	942	917	.0443	828	56
5	.43968	.48953	2.0428	.89816	55
6	.43994	.48989	.0413	803	54
7	.44020	.49026	.0398	790	53
8	046	062	.0383	777	52
9	072	098	.0368	764	51
10	.44098	.49134	2.0353	.89752	50
11	124	170	.0338	739	49
12	151	206	.0323	726	48
13	177	242	.0308	713	47
14	203	278	.0293	700	46
15	.44229	.49315	2.0278	.89687	45
16	255	351	.0263	674	44
17	281	387	.0248	662	43
18	307	423	.0233	649	42
19	333	459	.0219	636	41
20	.44359	.49495	2.0204	.89623	40
21	385	532	.0189	610	39
22	411	568	.0174	597	38
23	437	604	.0160	584	37
24	464	640	.0145	571	36
25	.44490	.49677	2.0130	.89558	35
26	516	713	.0115	545	34
27	542	749	.0101	532	33
28	568	786	.0086	519	32
29	594	822	.0072	506	31
30	.44620	.49858	2.0057	.89493	30
31	646	894	.0042	480	29
32	672	931	.0028	467	28
33	698	.49967	2.0013	454	27
34	724	.50004	1.9999	441	26
35	.44750	.50040	1.9984	.89428	25
36	776	076	.9970	415	24
37	802	113	.9955	402	23
38	828	149	.9941	389	22
39	854	185	.9926	376	21
40	.44880	.50222	1.9912	.89363	20
41	906	258	.9897	350	19
42	932	295	.9883	337	18
43	958	331	.9868	324	17
44	.44984	368	.9854	311	16
45	.45010	.50404	1.9840	.89298	15
46	036	441	.9825	285	14
47	062	477	.9811	272	13
48	088	514	.9797	259	12
49	114	550	.9782	245	11
50	.45140	.50587	1.9768	.89232	10
51	166	623	.9754	219	9
52	192	660	.9740	206	8
53	218	696	.9725	193	7
54	243	733	.9711	180	6
55	.45269	.50769	1.9697	.89167	5
56	295	806	.9683	153	4
57	321	843	.9669	140	3
58	347	879	.9654	127	2
59	373	916	.9640	114	1
60	.45399	.50953	1.9626	.89101	0
	N. Cos.	N. Cot.	N. Tan.	N. Sin.	'

'	N. Sin.	N. Tan.	N. Cot.	N. Cos.	
0	.45399	.50953	1.9626	.89101	60
1	425	.50989	.9612	087	59
2	451	.51026	.9598	074	58
3	477	063	.9584	061	57
4	503	099	.9570	048	56
5	.45529	.51136	1.9556	.89035	55
6	554	173	.9542	021	54
7	580	209	.9528	.89008	53
8	606	246	.9514	.88995	52
9	632	283	.9500	981	51
10	.45658	.51319	1.9486	.88968	50
11	684	356	.9472	955	49
12	710	393	.9458	942	48
13	736	430	.9444	928	47
14	762	467	.9430	915	46
15	.45787	.51503	1.9416	.88902	45
16	813	540	.9402	888	44
17	839	577	.9388	875	43
18	865	614	.9375	862	42
19	891	651	.9361	848	41
20	.45917	.51688	1.9347	.88835	40
21	942	724	.9333	822	39
22	968	761	.9319	808	38
23	.45994	798	.9306	795	37
24	.46020	835	.9292	782	36
25	.46046	.51872	1.9278	.88768	35
26	072	909	.9265	755	34
27	097	946	.9251	741	33
28	123	.51983	.9237	728	32
29	149	.52020	.9223	715	31
30	.46175	.52057	1.9210	.88701	30
31	201	094	.9196	688	29
32	226	131	.9183	674	28
33	252	168	.9169	661	27
34	278	205	.9155	647	26
35	.46304	.52242	1.9142	.88634	25
36	330	279	.9128	620	24
37	355	316	.9115	607	23
38	381	353	.9101	593	22
39	407	390	.9088	580	21
40	.46433	.52427	1.9074	.88566	20
41	458	464	.9061	553	19
42	484	501	.9047	539	18
43	510	538	.9034	526	17
44	536	575	.9020	512	16
45	.46561	.52613	1.9007	.88499	15
46	587	650	.8993	485	14
47	613	687	.8980	472	13
48	639	724	.8967	458	12
49	664	761	.8953	445	11
50	.46690	.52798	1.8940	.88431	10
51	716	836	.8927	417	9
52	742	873	.8913	404	8
53	767	910	.8900	390	7
54	793	947	.8887	377	6
55	.46819	.52985	1.8873	.88363	5
56	844	.53022	.8860	349	4
57	870	059	.8847	336	3
58	896	096	.8834	322	2
59	921	134	.8820	308	1
60	.46947	.53171	1.8807	.88295	0
	N. Cos.	N. Cot.	N. Tan.	N. Sin.	'

	N. Sin.	N. Tan.	N. Cot.	N. Cos.	
0	.46947	.53171	1.8807	.88295	60
1	973	208	.8794	281	59
2	.46999	246	.8781	267	58
3	.47024	283	.8768	254	57
4	050	320	.8755	240	56
5	.47076	.53358	1.8741	.88226	55
6	101	395	.8728	213	54
7	127	432	.8715	199	53
8	153	470	.8702	185	52
9	178	507	.8689	172	51
10	.47204	.53545	1.8676	.88158	50
11	229	582	.8663	144	49
12	255	620	.8650	130	48
13	281	657	.8637	117	47
14	306	694	.8624	103	46
15	.47332	.53732	1.8611	.88089	45
16	358	769	.8598	075	44
17	383	807	.8585	062	43
18	409	844	.8572	048	42
19	434	882	.8559	034	41
20	.47460	.53920	1.8546	.88020	40
21	486	957	.8533	.88006	39
22	511	.53995	.8520	.87993	38
23	537	.54032	.8507	979	37
24	562	070	.8495	965	36
25	.47588	.54107	1.8482	.87951	35
26	614	145	.8469	937	34
27	639	183	.8456	923	33
28	665	220	.8443	909	32
29	690	258	.8430	896	31
30	.47716	.54296	1.8418	8.7882	30
31	741	333	.8405	868	29
32	767	371	.8392	854	28
33	793	409	.8379	840	27
34	818	446	.8367	826	26
35	.47844	.54484	1.8354	8.7812	25
36	869	522	.8341	798	24
37	895	560	.8329	784	23
38	920	597	.8316	770	22
39	946	635	.8303	756	21
40	.47971	.54673	1.8291	8.7743	20
41	.47997	711	.8278	729	19
42	.48022	748	.8265	715	18
43	048	786	.8253	701	17
44	073	824	.8240	687	16
45	.48099	.54862	1.8228	8.7673	15
46	124	900	.8215	659	14
47	150	938	.8202	645	13
48	175	.54975	.8190	631	12
49	.201	.55013	.8177	617	11
50	.48226	.55051	1.8165	.87603	10
51	252	089	.8152	589	9
52	277	127	.8140	575	8
53	303	165	.8127	561	7
54	328	203	.8115	546	6
55	.48354	.55241	1.8103	.87532	5
56	379	279	.8090	518	4
57	405	317	.8078	504	3
58	430	355	.8065	490	2
59	456	393	.8053	476	1
60	.48481	.55431	1.8040	.87462	0
	N. Cos.	N. Cot.	N. Tan.	N. Sin.	

	N. Sin.	N. Tan.	N. Cot.	N. Cos.	
0	.48481	.55431	1.8040	.87462	60
1	506	469	.8028	448	59
2	532	507	.8016	434	58
3	557	545	.8003	420	57
4	583	583	.7991	406	56
5	.48608	.55621	1.7979	.87391	55
6	634	659	.7966	377	54
7	659	697	.7954	363	53
8	684	736	.7942	349	52
9	710	774	.7930	335	51
10	.48735	.55812	1.7917	.87321	50
11	761	850	.7905	306	49
12	786	888	.7893	292	48
13	811	926	.7881	278	47
14	837	.55964	.7868	264	46
15	.48862	.56003	1.7856	.87250	45
16	888	041	.7844	235	44
17	913	079	.7832	221	43
18	938	117	.7820	207	42
19	964	156	.7808	193	41
20	.48989	.56194	1.7796	.87178	40
21	.49014	232	.7783	164	39
22	040	270	.7771	150	38
23	065	309	.7759	136	37
24	090	347	.7747	121	36
25	.49116	.56385	1.7735	.87107	35
26	141	424	.7723	093	34
27	166	462	.7711	079	33
28	192	501	.7699	064	32
29	217	539	.7687	050	31
30	.49242	.56577	1.7675	.87036	30
31	268	616	.7663	021	29
32	293	654	.7651	.87007	28
33	318	693	.7639	.86993	27
34	344	731	.7627	978	26
35	.49369	.56769	1.7615	.86964	25
36	394	808	.7603	949	24
37	419	846	.7591	935	23
38	445	885	.7579	921	22
39	470	923	.7567	906	21
40	.49495	.56962	1.7556	.86892	20
41	521	.57000	.7544	878	19
42	546	039	.7532	863	18
43	571	078	.7520	849	17
44	596	116	.7508	834	16
45	.49622	.57155	1.7496	.86820	15
46	647	193	.7485	805	14
47	672	232	.7473	791	13
48	697	271	.7461	777	12
49	723	309	.7449	762	11
50	.49748	.57348	1.7437	.86748	10
51	773	386	.7426	733	9
52	798	425	.7414	719	8
53	824	464	.7402	704	7
54	849	503	.7391	690	6
55	.49874	.57541	1.7379	.86675	5
56	899	580	.7367	661	4
57	924	619	.7355	646	3
58	950	657	.7344	632	2
59	.49975	696	.7332	617	1
60	.50000	.57735	1.7321	.86603	0
	N. Cos.	N. Cot.	N. Tan.	N. Sin.	

'	N. Sin.	N. Tan.	N. Cot.	N. Cos.	
0	.50000	.57735	1.7321	.86603	60
1	025	774	.7309	588	59
2	050	813	.7297	573	58
3	076	851	.7286	559	57
4	101	890	.7274	544	56
5	.50126	.57929	1.7262	.86530	55
6	151	.57968	.7251	515	54
7	176	.58007	.7239	501	53
8	201	046	.7228	486	52
9	227	085	.7216	471	51
10	.50252	.58124	1.7205	.86457	50
11	277	162	.7193	442	49
12	302	201	.7182	427	48
13	327	240	.7170	413	47
14	352	279	.7159	398	46
15	.50377	.58318	1.7147	.86384	45
16	403	357	.7136	369	44
17	428	396	.7124	354	43
18	453	435	.7113	340	42
19	478	474	.7102	325	41
20	.50503	.58513	1.7090	.86310	40
21	528	552	.7079	295	39
22	553	591	.7067	281	38
23	578	631	.7056	266	37
24	603	670	.7045	251	36
25	.50628	.58709	1.7033	.86237	35
26	654	748	.7022	222	34
27	679	787	.7011	207	33
28	704	826	.6999	192	32
29	729	865	.6988	178	31
30	.50754	.58905	1.6977	.86163	30
31	779	944	.6965	148	29
32	804	.58983	.6954	133	28
33	829	.59022	.6943	119	27
34	854	061	.6932	104	26
35	.50879	.59101	1.6920	.86089	25
36	904	140	.6909	074	24
37	929	179	.6898	059	23
38	954	218	.6887	045	22
39	.50979	258	.6875	030	21
40	.51004	.59297	1.6864	.86015	20
41	029	336	.6853	.86000	19
42	054	376	.6842	.85985	18
43	079	415	.6831	970	17
44	104	454	.6820	956	16
45	.51129	.59494	1.6808	.85941	15
46	154	533	.6797	926	14
47	179	573	.6786	911	13
48	204	612	.6775	896	12
49	229	651	.6764	881	11
50	.51254	.59691	1.6753	.85866	10
51	279	730	.6742	851	9
52	304	770	.6731	836	8
53	329	809	.6720	821	7
54	354	849	.6709	806	6
55	.51379	.59888	1.6698	.85792	5
56	404	928	.6687	777	4
57	429	.59967	.6676	762	3
58	454	.60007	.6665	747	2
59	479	046	.6654	732	1
60	.51504	.60086	1.6643	.85717	0
	N. Cos.	N. Cot.	N. Tan.	N. Sin.	'

'	N. Sin.	N. Tan.	N. Cot.	N. Cos.	
0	.51504	.60086	1.6643	.85717	60
1	529	126	.6632	702	59
2	554	165	.6621	687	58
3	579	205	.6610	672	57
4	604	245	.6599	657	56
5	.51628	.60284	1.6588	.85642	55
6	653	324	.6577	627	54
7	678	364	.6566	612	53
8	703	403	.6555	597	52
9	728	443	.6545	582	51
10	.51753	.60483	1.6534	.85567	50
11	778	522	.6523	551	49
12	803	562	.6512	536	48
13	828	602	.6501	521	47
14	852	642	.6490	506	46
15	.51877	.60681	1.6479	.85491	45
16	902	721	.6469	476	44
17	927	761	.6458	461	43
18	952	801	.6447	446	42
19	.51977	841	.6436	431	41
20	.52002	.60881	1.6426	.85416	40
21	026	921	.6415	401	39
22	051	.60960	.6404	385	38
23	076	.61000	.6393	370	37
24	101	040	.6383	355	36
25	.52126	.61080	1.6372	.85340	35
26	151	120	.6361	325	34
27	175	160	.6351	310	33
28	200	200	.6340	294	32
29	225	240	.6329	279	31
30	.52250	.61280	1.6319	.85264	30
31	275	320	.6308	249	29
32	299	360	.6297	234	28
33	324	400	.6287	218	27
34	349	440	.6276	203	26
35	.52374	.61480	1.6265	.85188	25
36	399	520	.6255	173	24
37	423	561	.6244	157	23
38	448	601	.6234	142	22
39	473	641	.6223	127	21
40	.52498	.61681	1.6212	.85112	20
41	522	721	.6202	096	19
42	547	761	.6191	081	18
43	572	801	.6181	066	17
44	597	842	.6170	051	16
45	.52621	.61882	1.6160	.85035	15
46	646	922	.6149	020	14
47	671	.61962	.6139	.85005	13
48	696	.62003	.6128	.84989	12
49	720	043	.6118	974	11
50	.52745	.62083	1.6107	.84959	10
51	770	124	.6097	943	9
52	794	164	.6087	928	8
53	819	204	.6076	913	7
54	844	245	.6066	897	6
55	.52869	.62285	1.6055	.84882	5
56	893	325	.6045	866	4
57	918	366	.6034	851	3
58	943	406	.6024	836	2
59	967	446	.6014	820	1
60	.52992	.62487	1.6003	.84805	0
	N. Cos.	N. Cot.	N. Tan.	N. Sin.	'

'	N. Sin.	N. Tan.	N. Cot.	N. Cos.	
0	.52992	.62487	1.6003	.84805	60
1	.53017	527	.5993	789	59
2	041	568	.5983	774	58
3	066	608	.5972	759	57
4	091	649	.5962	743	56
5	.53115	.62689	1.5952	.84728	55
6	140	730	.5941	712	54
7	164	770	.5931	697	53
8	189	811	.5921	681	52
9	214	852	.5911	666	51
10	.53238	.62892	1.5900	.84650	50
11	263	933	.5890	635	49
12	288	.62973	.5880	619	48
13	312	.63014	.5869	604	47
14	337	055	.5859	588	46
15	.53361	.63095	1.5849	.84573	45
16	386	136	.5839	557	44
17	411	177	.5829	542	43
18	435	217	.5818	526	42
19	460	258	.5808	511	41
20	.53484	.63299	1.5798	.84495	40
21	509	340	.5788	480	39
22	534	380	.5778	464	38
23	558	421	.5768	448	37
24	583	462	.5757	433	36
25	.53607	.63503	1.5747	.84417	35
26	632	544	.5737	402	34
27	656	584	.5727	386	33
28	681	625	.5717	370	32
29	705	666	.5707	355	31
30	.53730	.63707	1.5697	.84339	30
31	754	748	.5687	324	29
32	779	789	.5677	308	28
33	804	830	.5667	292	27
34	828	871	.5657	277	26
35	.53853	.63912	1.5647	.84261	25
36	877	953	.5637	245	24
37	902	.63994	.5627	230	23
38	926	.64035	.5617	214	22
39	951	076	.5607	198	21
40	.53975	.64117	1.5597	.84182	20
41	.54000	158	.5587	167	19
42	024	199	.5577	151	18
43	049	240	.5567	135	17
44	073	281	.5557	120	16
45	.54097	.64322	1.5547	.84104	15
46	122	363	.5537	088	14
47	146	404	.5527	072	13
48	171	446	.5517	057	12
49	195	487	.5507	041	11
50	.54220	.64528	1.5497	.84025	10
51	244	569	.5487	.84009	9
52	269	610	.5477	.83994	8
53	293	652	.5468	978	7
54	317	693	.5458	962	6
55	.54342	.64734	1.5448	.83946	5
56	366	775	.5438	930	4
57	391	817	.5428	915	3
58	415	858	.5418	899	2
59	440	899	.5408	883	1
60	.54464	.64941	1.5399	.83867	0
	N. Cos.	N. Cot.	N. Tan.	N. Sin.	'

'	N. Sin.	N. Tan.	N. Cot.	N. Cos.	
0	.54464	.64941	1.5399	.83867	60
1	488	.64982	.5389	851	59
2	513	.65024	.5379	835	58
3	537	065	.5369	819	57
4	561	106	.5359	804	56
5	.54586	.65148	1.5350	.83788	55
6	610	189	.5340	772	54
7	635	231	.5330	756	53
8	659	272	.5320	740	52
9	683	314	.5311	724	51
10	.54708	.65355	1.5301	.83708	50
11	732	397	.5291	692	49
12	756	438	.5282	676	48
13	781	480	.5272	660	47
14	805	521	.5262	645	46
15	.54829	.65563	1.5253	.83629	45
16	854	604	.5243	613	44
17	878	646	.5233	597	43
18	902	688	.5224	581	42
19	927	729	.5214	565	41
20	.54951	.65771	1.5204	.83549	40
21	975	813	.5195	533	39
22	.54999	854	.5185	517	38
23	.55024	896	.5175	501	37
24	048	938	.5166	485	36
25	.55072	.65980	1.5156	.83469	35
26	097	.66021	.5147	453	34
27	121	063	.5137	437	33
28	145	105	.5127	421	32
29	169	147	.5118	405	31
30	.55194	.66189	1.5108	.83389	30
31	218	230	.5099	373	29
32	242	272	.5089	356	28
33	266	314	.5080	340	27
34	291	356	.5070	324	26
35	.55315	.66398	1.5061	.83308	25
36	339	440	.5051	292	24
37	363	482	.5042	276	23
38	388	524	.5032	260	22
39	412	566	.5023	244	21
40	.55436	.66608	1.5013	.83228	20
41	460	650	.5004	212	19
42	484	692	.4994	195	18
43	509	734	.4985	179	17
44	533	776	.4975	163	16
45	.55557	.66818	1.4966	.83147	15
46	581	860	.4957	131	14
47	605	902	.4947	115	13
48	630	944	.4938	098	12
49	654	.66986	.4928	082	11
50	.55678	.67028	1.4919	.83066	10
51	702	071	.4910	050	9
52	726	113	.4900	034	8
53	750	155	.4891	017	7
54	775	197	.4882	.83001	6
55	.55799	.67239	1.4872	.82985	5
56	823	282	.4863	969	4
57	847	324	.4854	953	3
58	871	366	.4844	936	2
59	895	409	.4835	920	1
60	.55919	.67451	1.4826	.82904	0
	N. Cos.	N. Cot.	N. Tan.	N. Sin.	'

	N. Sin.	N. Tan.	N. Cot.	N. Cos.	
0	.55919	.67451	1.4826	.82904	60
1	943	493	.4816	887	59
2	968	536	.4807	871	58
3	.55992	578	.4798	855	57
4	.56016	620	.4788	839	56
5	.56040	.67663	1.4779	.82822	55
6	064	705	.4770	806	54
7	088	748	.4761	790	53
8	112	790	.4751	773	52
9	136	832	.4742	757	51
10	.56160	.67875	1.4733	.82741	50
11	184	917	.4724	724	49
12	208	.67960	.4715	708	48
13	232	.68002	.4705	692	47
14	256	045	.4696	675	46
15	.56280	.68088	1.4687	.82659	45
16	305	130	.4678	643	44
17	329	173	.4669	626	43
18	353	215	.4659	610	42
19	377	258	.4650	593	41
20	.56401	.68301	1.4641	.82577	40
21	425	343	.4632	561	39
22	449	386	.4623	544	38
23	473	429	.4614	528	37
24	497	471	.4605	511	36
25	.56521	.68514	1.4596	.82495	35
26	545	557	.4586	478	34
27	569	600	.4577	462	33
28	593	642	.4568	446	32
29	617	685	.4559	429	31
30	.56641	.68728	1.4550	.82413	30
31	665	771	.4541	396	29
32	689	814	.4532	380	28
33	713	857	.4523	363	27
34	736	900	.4514	347	26
35	.56760	.68942	1.4505	.82330	25
36	784	.68985	.4496	314	24
37	808	.69028	.4487	297	23
38	832	071	.4478	281	22
39	856	114	.4469	264	21
40	.56880	.69157	1.4460	.82248	20
41	904	200	.4451	231	19
42	928	243	.4442	214	18
43	952	286	.4433	198	17
44	.56976	329	.4424	181	16
45	.57000	.69372	1.4415	.82165	15
46	024	416	.4406	148	14
47	047	459	.4397	132	13
48	071	502	.4388	115	12
49	095	545	.4379	098	11
50	.57119	.69588	1.4370	.82082	10
51	143	631	.4361	065	9
52	167	675	.4352	048	8
53	191	718	.4344	032	7
54	215	761	.4335	.82015	6
55	.57238	.69804	1.4326	.81999	5
56	262	847	.4317	982	4
57	286	891	.4308	965	3
58	310	934	.4299	949	2
59	334	.69977	.4290	932	1
60	.57358	.70021	1.4281	.81915	0
	N. Cos.	N. Cot.	N. Tan.	N. Sin.	

	N. Sin.	N. Tan.	N. Cot.	N. Cos.	
0	.57358	.70021	1.4281	.81915	60
1	381	064	.4273	899	59
2	405	107	.4264	882	58
3	429	151	.4255	865	57
4	453	194	.4246	848	56
5	.57477	.70238	1.4237	.81832	55
6	501	281	.4229	815	54
7	524	325	.4220	798	53
8	548	368	.4211	782	52
9	572	412	.4202	765	51
10	.57596	.70455	1.4193	.81748	50
11	619	499	.4185	731	49
12	643	542	.4176	714	48
13	667	586	.4167	698	47
14	691	629	.4158	681	46
15	.57715	.70673	1.4150	.81664	45
16	738	717	.4141	647	44
17	762	760	.4132	631	43
18	786	804	.4124	614	42
19	810	848	.4115	597	41
20	.57833	.70891	1.4106	.81580	40
21	857	935	.4097	563	39
22	881	.70979	.4089	546	38
23	904	.71023	.4080	530	37
24	928	066	.4071	513	36
25	.57952	.71110	1.4063	.81496	35
26	976	154	.4054	479	34
27	.57999	198	.4045	462	33
28	.58023	242	.4037	445	32
29	047	285	.4028	428	31
30	.58070	.71329	1.4019	.81412	30
31	094	373	.4011	395	29
32	118	417	.4002	378	28
33	141	461	.3994	361	27
34	165	505	.3985	344	26
35	.58189	.71549	1.3976	.81327	25
36	212	593	.3968	310	24
37	236	637	.3959	293	23
38	260	681	.3951	276	22
39	283	725	.3942	259	21
40	.58307	.71769	1.3934	.81242	20
41	330	813	.3925	225	19
42	354	857	.3916	208	18
43	378	901	.3908	191	17
44	401	946	.3899	174	16
45	.58425	.71990	1.3891	.81157	15
46	449	.72034	.3882	140	14
47	472	078	.3874	123	13
48	496	122	.3865	106	12
49	519	167	.3857	089	11
50	.58543	.72211	1.3848	.81072	10
51	567	255	.3840	055	9
52	590	299	.3831	038	8
53	614	344	.3823	021	7
54	637	388	.3814	.81004	6
55	.58661	.72432	1.3806	.80987	5
56	684	477	.3798	970	4
57	708	521	.3789	953	3
58	731	565	.3781	936	2
59	755	610	.3772	919	1
60	.58779	.72654	1.3764	.80902	0
	N. Cos.	N. Cot.	N. Tan.	N. Sin.	

'	N. Sin.	N. Tan.	N. Cot.	N. Cos.	
0	.58779	.72654	1.3764	.80902	60
1	802	699	.3755	885	59
2	826	743	.3747	867	58
3	849	788	.3739	850	57
4	873	832	.3730	833	56
5	.58896	.72877	1.3722	.80816	55
6	920	921	.3713	799	54
7	943	.72966	.3705	782	53
8	967	.73010	.3697	765	52
9	.58990	055	.3688	748	51
10	.59014	.73100	1.3680	.80730	50
11	037	144	.3672	713	49
12	061	189	.3663	696	48
13	084	234	.3655	679	47
14	108	278	.3647	662	46
15	.59131	.73323	1.3638	.80644	45
16	154	368	.3630	627	44
17	178	413	.3622	610	43
18	201	457	.3613	593	42
19	225	502	.3605	576	41
20	.59248	.73547	1.3597	.80558	40
21	272	592	.3588	541	39
22	295	637	.3580	524	38
23	318	681	.3572	507	37
24	342	726	.3564	489	36
25	.59365	.73771	1.3555	.80472	35
26	389	816	.3547	455	34
27	412	861	.3539	438	33
28	436	906	.3531	420	32
29	459	951	.3522	403	31
30	.59482	.73996	1.3514	.80386	30
31	506	.74041	.3506	368	29
32	529	086	.3498	351	28
33	552	131	.3490	334	27
34	576	176	.3481	316	26
35	.59599	.74221	1.3473	.80299	25
36	622	267	.3465	282	24
37	646	312	.3457	264	23
38	669	357	.3449	247	22
39	693	402	.3440	230	21
40	.59716	.74447	1.3432	.80212	20
41	739	492	.3424	195	19
42	763	538	.3416	178	18
43	786	583	.3408	160	17
44	809	628	.3400	143	16
45	.59832	.74674	1.3392	.80125	15
46	856	719	.3384	108	14
47	879	764	.3375	091	13
48	902	810	.3367	073	12
49	926	855	.3359	056	11
50	.59949	.74900	1.3351	.80038	10
51	972	946	.3343	021	9
52	.59995	.74991	.3335	.80003	8
53	.60019	.75037	.3327	.79986	7
54	042	082	.3319	968	6
55	.60065	.75128	1.3311	.79951	5
56	089	173	.3303	934	4
57	112	219	.3295	916	3
58	135	264	.3287	899	2
59	158	310	.3278	881	1
60	.60182	.75355	1.3270	.79864	0
	N. Cos.	N. Cot.	N. Tan.	N. Sin.	'

'	N. Sin.	N. Tan.	N. Cot.	N. Cos.	
0	.60182	.75355	1.3270	.79864	60
1	205	401	.3262	846	59
2	228	447	.3254	829	58
3	251	492	.3246	811	57
4	274	538	.3238	793	56
5	.60298	.75584	1.3230	.79776	55
6	321	629	.3222	758	54
7	344	675	.3214	741	53
8	367	721	.3206	723	52
9	390	767	.3198	706	51
10	.60414	.75812	1.3190	.79688	50
11	437	858	.3182	671	49
12	460	904	.3175	653	48
13	483	950	.3167	635	47
14	506	.75996	.3159	618	46
15	.60529	.76042	1.3151	.79600	45
16	553	088	.3143	583	44
17	576	134	.3135	565	43
18	599	180	.3127	547	42
19	622	226	.3119	530	41
20	.60645	.76272	1.3111	.79512	40
21	668	318	.3103	494	39
22	691	364	.3095	477	38
23	714	410	.3087	459	37
24	738	456	.3079	441	36
25	.60761	.76502	1.3072	.79424	35
26	784	548	.3064	406	34
27	807	594	.3056	388	33
28	830	640	.3048	371	32
29	853	686	.3040	353	31
30	.60876	.76733	1.3032	.79335	30
31	899	779	.3024	318	29
32	922	825	.3017	300	28
33	945	871	.3009	282	27
34	968	918	.3001	264	26
35	.60991	.76964	1.2993	.79247	25
36	.61015	.77010	.2985	229	24
37	038	057	.2977	211	23
38	061	103	.2970	193	22
39	084	149	.2962	176	21
40	.61107	.77196	1.2954	.79158	20
41	130	242	.2946	140	19
42	153	289	.2938	122	18
43	176	335	.2931	105	17
44	199	382	.2923	087	16
45	.61222	.77428	1.2915	.79069	15
46	245	475	.2907	051	14
47	268	521	.2900	033	13
48	291	568	.2892	.79016	12
49	314	615	.2884	.78998	11
50	.61337	.77661	1.2876	.78980	10
51	360	708	.2869	962	9
52	383	754	.2861	944	8
53	406	801	.2853	926	7
54	429	848	.2846	908	6
55	.61451	.77895	1.2838	.78891	5
56	474	941	.2830	873	4
57	497	.77988	.2822	855	3
58	520	.78035	.2815	837	2
59	543	082	.2807	819	1
60	.61566	.78129	1.2799	.78801	0
	N. Cos.	N. Cot.	N. Tan.	N. Sin.	'

	N. Sin.	N. Tan.	N. Cot.	N. Cos.	
0	.61566	.78129	1.2799	.78801	60
1	589	175	.2792	783	59
2	612	222	.2784	765	58
3	635	269	.2776	747	57
4	658	316	.2769	729	56
5	.61681	.78363	1.2761	.78711	55
6	704	410	.2753	694	54
7	726	457	.2746	676	53
8	749	504	.2738	658	52
9	772	551	.2731	640	51
10	.61795	.78598	1.2723	.78622	50
11	818	645	.2715	604	49
12	841	692	.2708	586	48
13	864	739	.2700	568	47
14	887	786	.2693	550	46
15	.61909	.78834	1.2685	.78532	45
16	932	881	.2677	514	44
17	955	928	.2670	496	43
18	.61978	.78975	.2662	478	42
19	.62001	.79022	.2655	460	41
20	.62024	.79070	1.2647	.78442	40
21	046	117	.2640	424	39
22	069	164	.2632	405	38
23	092	212	.2624	387	37
24	115	259	.2617	369	36
25	.62138	.79306	1.2609	.78351	35
26	160	354	.2602	333	34
27	183	401	.2594	315	33
28	206	449	.2587	297	32
29	229	496	.2579	279	31
30	.62251	.79544	1.2572	.78261	30
31	274	591	.2564	243	29
32	297	639	.2557	225	28
33	320	686	.2549	206	27
34	342	734	.2542	188	26
35	.62365	.79781	1.2534	.78170	25
36	388	829	.2527	152	24
37	411	877	.2519	134	23
38	433	924	.2512	116	22
39	456	.79972	.2504	098	21
40	.62479	.80020	1.2497	.78079	20
41	502	067	.2489	061	19
42	524	115	.2482	043	18
43	547	163	.2475	025	17
44	570	211	.2467	.78007	16
45	.62592	.80258	1.2460	.77988	15
46	615	306	.2452	970	14
47	638	354	.2445	952	13
48	660	402	.2437	934	12
49	683	450	.2430	916	11
50	.62706	.80498	1.2423	.77897	10
51	728	546	.2415	879	9
52	751	594	.2408	861	8
53	774	642	.2401	843	7
54	796	690	.2393	824	6
55	.62819	.80738	1.2386	.77806	5
56	842	786	.2378	788	4
57	864	834	.2371	769	3
58	887	882	.2364	751	2
59	909	930	.2356	733	1
60	.62932	.80978	1.2349	.77715	0
	N. Cos.	N. Cot.	N. Tan.	N. Sin.	'

	N. Sin.	N. Tan.	N. Cot.	N. Cos.	
0	.62932	.80978	1.2349	.77715	60
1	955	.81027	.2342	696	59
2	.62977	075	.2334	678	58
3	.63000	123	.2327	660	57
4	022	171	.2320	641	56
5	.63045	.81220	1.2312	.77623	55
6	068	268	.2305	605	54
7	090	316	.2298	586	53
8	113	364	.2290	568	52
9	135	413	.2283	550	51
10	.63158	.81461	1.2276	.77531	50
11	180	510	.2268	513	49
12	203	558	.2261	494	48
13	225	606	.2254	476	47
14	248	655	.2247	458	46
15	.63271	.81703	1.2239	.77439	45
16	293	752	.2232	421	44
17	316	800	.2225	402	43
18	338	849	.2218	384	42
19	361	898	.2210	366	41
20	.63383	.81946	1.2203	.77347	40
21	406	.81995	.2196	329	39
22	428	.82044	.2189	310	38
23	451	092	.2181	292	37
24	473	141	.2174	273	36
25	.63496	.82190	1.2167	.77255	35
26	518	238	.2160	236	34
27	540	287	.2153	218	33
28	563	336	.2145	199	32
29	585	385	.2138	181	31
30	.63608	.82434	1.2131	.77162	30
31	630	483	.2124	144	29
32	653	531	.2117	125	28
33	675	580	.2109	107	27
34	698	629	.2102	088	26
35	.63720	.82678	1.2095	.77070	25
36	742	727	.2088	051	24
37	765	776	.2081	033	23
38	787	825	.2074	.77014	22
39	810	874	.2066	.76996	21
40	.63832	.82923	1.2059	.76977	20
41	854	.82972	.2052	959	19
42	877	.83022	.2045	940	18
43	899	071	.2038	921	17
44	922	120	.2031	903	16
45	.63944	.83169	1.2024	.76884	15
46	966	218	.2017	866	14
47	.63989	268	.2009	847	13
48	.64011	317	.2002	828	12
49	033	366	.1995	810	11
50	.64056	.83415	1.1988	.76791	10
51	078	465	.1981	772	9
52	100	514	.1974	754	8
53	123	564	.1967	735	7
54	145	613	.1960	717	6
55	.64167	.83662	1.1953	.76698	5
56	190	712	.1946	679	4
57	212	761	.1939	661	3
58	234	811	.1932	642	2
59	256	860	.1925	623	1
60	.64279	.83910	1.1918	.76604	0
	N. Cos.	N. Cot.	N. Tan.	N. Sin.	'

190						40°					
	N. Sin.	N. Tan.	N. Cot.	N. Cos.							
0	.64279	.83910	1.1918	.76604	60						
1	301	.83960	.1910	586	59						
2	323	.84009	.1903	567	58						
3	346	059	.1896	548	57						
4	368	108	.1889	530	56						
5	.64390	.84158	1.1882	.76511	55						
6	412	208	.1875	492	54						
7	435	258	.1868	473	53						
8	457	307	.1861	455	52						
9	479	357	.1854	436	51						
10	.64501	.84407	1.1847	.76417	50						
11	524	457	.1840	398	49						
12	546	507	.1833	380	48						
13	568	556	.1826	361	47						
14	590	606	.1819	342	46						
15	.64612	.84656	1.1812	.76323	45						
16	635	706	.1806	304	44						
17	657	756	.1799	286	43						
18	679	806	.1792	267	42						
19	701	856	.1785	248	41						
20	.64723	.84906	1.1778	.76229	40						
21	746	.84956	.1771	210	39						
22	768	.85006	.1764	192	38						
23	790	057	.1757	173	37						
24	812	107	.1750	154	36						
25	.64834	.85157	1.1743	.76135	35						
26	856	207	.1736	116	34						
27	878	257	.1729	097	33						
28	901	308	.1722	078	32						
29	923	358	.1715	059	31						
30	.64945	.85408	1.1708	.76041	30						
31	967	458	.1702	022	29						
32	.64989	509	.1695	.76003	28						
33	.65011	559	.1688	.75984	27						
34	033	609	.1681	965	26						
35	.65055	.85660	1.1674	.75946	25						
36	077	710	.1667	927	24						
37	100	761	.1660	908	23						
38	122	811	.1653	889	22						
39	144	862	.1647	870	21						
40	.65166	.85912	1.1640	.75851	20						
41	188	.85963	.1633	832	19						
42	210	.86014	.1626	813	18						
43	232	064	.1619	794	17						
44	254	115	.1612	775	16						
45	.65276	.86166	1.1606	.75756	15						
46	298	216	.1599	738	14						
47	320	267	.1592	719	13						
48	342	318	.1585	700	12						
49	364	368	.1578	680	11						
50	.65386	.86419	1.1571	.75661	10						
51	408	470	.1565	642	9						
52	430	521	.1558	623	8						
53	452	572	.1551	604	7						
54	474	623	.1544	585	6						
55	.65496	.86674	1.1538	.75566	5						
56	518	725	.1531	547	4						
57	540	776	.1524	528	3						
58	562	827	.1517	509	2						
59	584	878	.1510	490	1						
60	.65606	.86929	1.1504	.75471	0						
	N. Cos.	N. Cot.	N. Tan.	N. Sin.							

41°						48°					
	N. Sin.	N. Tan.	N. Cot.	N. Cos.							
0	.65606	.86929	1.1504	.75471	60						
1	628	.86980	.1497	452	59						
2	650	.87031	.1490	433	58						
3	672	082	.1483	414	57						
4	694	133	.1477	395	56						
5	.65716	.87184	1.1470	.75375	55						
6	738	236	.1463	356	54						
7	759	287	.1456	337	53						
8	781	338	.1450	318	52						
9	803	389	.1443	299	51						
10	.65825	.87441	1.1436	.75280	50						
11	847	492	.1430	261	49						
12	869	543	.1423	241	48						
13	891	595	.1416	222	47						
14	913	646	.1410	203	46						
15	.65935	.87698	1.1403	.75184	45						
16	956	749	.1396	165	44						
17	.65978	801	.1389	146	43						
18	.66000	852	.1383	126	42						
19	022	904	.1376	107	41						
20	.66044	.87955	1.1369	.75088	40						
21	066	.88007	.1363	069	39						
22	088	059	.1356	050	38						
23	109	110	.1349	030	37						
24	131	162	.1343	.75011	36						
25	.66153	.88214	1.1336	.74992	35						
26	175	265	.1329	973	34						
27	197	317	.1323	953	33						
28	218	369	.1316	934	32						
29	240	421	.1310	915	31						
30	.66262	.88473	1.1303	.74896	30						
31	284	524	.1296	876	29						
32	306	576	.1290	857	28						
33	327	628	.1283	838	27						
34	349	680	.1276	818	26						
35	.66371	.88732	1.1270	.74799	25						
36	393	784	.1263	780	24						
37	414	836	.1257	760	23						
38	436	888	.1250	741	22						
39	458	940	.1243	722	21						
40	.66480	.88992	1.1237	.74703	20						
41	501	.89045	.1230	683	19						
42	523	097	.1224	664	18						
43	545	149	.1217	644	17						
44	566	201	.1211	625	16						
45	.66588	.89253	1.1204	.74606	15						
46	610	306	.1197	586	14						
47	632	358	.1191	567	13						
48	653	410	.1184	548	12						
49	675	463	.1178	528	11						
50	.66697	.89515	1.1171	.74509	10						
51	718	567	.1165	489	9						
52	740	620	.1158	470	8						
53	762	672	.1152	451	7						
54	783	725	.1145	431	6						
55	.66805	.89777	1.1139	.74412	5						
56	827	830	.1132	392	4						
57	848	883	.1126	373	3						
58	870	935	.1119	353	2						
59	891	.89988	.1113	334	1						
60	.66913	.90040	1.1106	.74314	0						
	N. Cos.	N. Cot.	N. Tan.	N. Sin.							

	N. Sin.	N. Tan.	N. Cot.	N. Cos.	
0	.66913	.90040	1.1106	.74314	60
1	935	093	.1100	295	59
2	956	146	.1093	276	58
3	978	199	.1087	256	57
4	.66999	251	.1080	237	56
5	.67021	.90304	1.1074	.74217	55
6	043	357	.1067	198	54
7	064	410	.1061	178	53
8	086	463	.1054	159	52
9	107	516	.1048	139	51
10	.67129	.90569	1.1041	.74120	50
11	151	621	.1035	100	49
12	172	674	.1028	080	48
13	194	727	.1022	061	47
14	215	781	.1016	041	46
15	.67237	.90834	1.1009	.74022	45
16	258	887	.1003	.74002	44
17	280	940	.0996	.73983	43
18	301	.90993	.0990	963	42
19	323	.91046	.0983	944	41
20	.67344	.91099	1.0977	.73924	40
21	366	153	.0971	904	39
22	387	206	.0964	885	38
23	409	259	.0958	865	37
24	430	313	.0951	846	36
25	.67452	.91366	1.0945	.73826	35
26	473	419	.0939	806	34
27	495	473	.0932	787	33
28	516	526	.0926	767	32
29	538	580	.0919	747	31
30	.67559	.91633	1.0913	.73728	30
31	580	687	.0907	708	29
32	602	740	.0900	688	28
33	623	794	.0894	669	27
34	645	847	.0888	649	26
35	.67666	.91901	1.0881	.73629	25
36	688	.91955	.0875	610	24
37	709	.92008	.0869	590	23
38	730	062	.0862	570	22
39	752	116	.0856	551	21
40	.67773	.92170	1.0850	.73531	20
41	795	224	.0843	511	19
42	816	277	.0837	491	18
43	837	331	.0831	472	17
44	859	385	.0824	452	16
45	.67880	.92439	1.0818	.73432	15
46	901	493	.0812	413	14
47	923	547	.0805	393	13
48	944	601	.0799	373	12
49	965	655	.0793	353	11
50	.67987	.92709	1.0786	.73333	10
51	.68008	763	.0780	314	9
52	029	817	.0774	294	8
53	051	872	.0768	274	7
54	072	926	.0761	254	6
55	.68093	.92980	1.0755	.73234	5
56	115	.93034	.0749	215	4
57	136	088	.0742	195	3
58	157	143	.0736	175	2
59	179	197	.0730	155	1
60	.68200	.93252	1.0724	.73135	0
	N. Cos.	N. Cot.	N. Tan.	N. Sin.	

	N. Sin.	N. Tan.	N. Cot.	N. Cos.	
0	.68200	.93252	1.0724	.73135	60
1	221	306	.0717	116	59
2	242	360	.0711	096	58
3	264	415	.0705	076	57
4	285	469	.0699	056	56
5	.68306	.93524	1.0692	.73036	55
6	327	578	.0686	.73016	54
7	349	633	.0680	.72996	53
8	370	688	.0674	976	52
9	391	742	.0668	957	51
10	.68412	.93797	1.0661	.72937	50
11	434	852	.0655	917	49
12	455	906	.0649	897	48
13	476	.93961	.0643	877	47
14	497	.94016	.0637	857	46
15	.68518	.94071	1.0630	.72837	45
16	539	125	.0624	817	44
17	561	180	.0618	797	43
18	582	235	.0612	777	42
19	603	290	.0606	757	41
20	.68624	.94345	1.0599	.72737	40
21	645	400	.0593	717	39
22	666	455	.0587	697	38
23	688	510	.0581	677	37
24	709	565	.0575	657	36
25	.68730	.94620	1.0569	.72637	35
26	751	676	.0562	617	34
27	772	731	.0556	597	33
28	793	786	.0550	577	32
29	814	841	.0544	557	31
30	.68835	.94896	1.0538	.72537	30
31	857	.94952	.0532	517	29
32	878	.95007	.0526	497	28
33	899	062	.0519	477	27
34	920	118	.0513	457	26
35	.68941	.95173	1.0507	.72437	25
36	962	229	.0501	417	24
37	.68983	284	.0495	397	23
38	.69004	340	.0489	377	22
39	025	395	.0483	357	21
40	.69046	.95451	1.0477	.72337	20
41	067	506	.0470	317	19
42	088	562	.0464	297	18
43	109	618	.0458	277	17
44	130	673	.0452	257	16
45	.69151	.95729	1.0446	.72236	15
46	172	785	.0440	216	14
47	193	841	.0434	196	13
48	214	897	.0428	176	12
49	235	.95952	.0422	156	11
50	.69256	.96008	1.0416	.72136	10
51	277	064	.0410	116	9
52	298	120	.0404	095	8
53	319	176	.0398	075	7
54	340	232	.0392	055	6
55	.69361	.96288	1.0385	.72035	5
56	382	344	.0379	.72015	4
57	403	400	.0373	.71995	3
58	424	457	.0367	974	2
59	445	513	.0361	954	1
60	.69466	.96569	1.0355	.71934	0
	N. Cos.	N. Cot.	N. Tan.	N. Sin.	

'	N. Sin.	N. Tan.	N. Cot.	N. Cos.	
0	.69466	.96569	1.0355	.71934	60
1	487	625	.0349	914	59
2	508	681	.0343	894	58
3	529	738	.0337	873	57
4	549	794	.0331	853	56
5	.69570	.96850	1.0325	.71833	55
6	591	907	.0319	813	54
7	612	.96963	.0313	792	53
8	633	.97020	.0307	772	52
9	654	076	.0301	752	51
10	.69675	.97133	1.0295	.71732	50
11	696	189	.0289	711	49
12	717	246	.0283	691	48
13	737	302	.0277	671	47
14	758	359	.0271	650	46
15	.69779	.97416	1.0265	.71630	45
16	800	472	.0259	610	44
17	821	529	.0253	590	43
18	842	586	.0247	569	42
19	862	643	.0241	549	41
20	.69883	.97700	1.0235	.71529	40
21	904	756	.0230	508	39
22	925	813	.0224	488	38
23	946	870	.0218	468	37
24	966	927	.0212	447	36
25	.69987	.97984	1.0206	.71427	35
26	.70008	.98041	.0200	407	34
27	029	098	.0194	386	33
28	049	155	.0188	366	32
29	070	213	.0182	345	31
30	.70091	.98270	1.0176	.71325	30
31	112	327	.0170	305	29
32	132	384	.0164	284	28
33	153	441	.0158	264	27
34	174	499	.0152	243	26
35	.70195	.98556	1.0147	.71223	25
36	215	613	.0141	203	24
37	236	671	.0135	182	23
38	257	728	.0129	162	22
39	277	786	.0123	141	21
40	.70298	.98843	1.0117	.71121	20
41	319	901	.0111	100	19
42	339	.98958	.0105	080	18
43	360	.99016	.0099	059	17
44	381	073	.0094	039	16
45	.70401	.99131	1.0088	.71019	15
46	422	189	.0082	.70998	14
47	443	247	.0076	978	13
48	463	304	.0070	957	12
49	484	362	.0064	937	11
50	.70505	.99420	1.0058	.70916	10
51	525	478	.0052	896	9
52	546	536	.0047	875	8
53	567	594	.0041	855	7
54	587	652	.0035	834	6
55	.70608	.99710	1.0029	.70813	5
56	628	768	.0023	793	4
57	649	826	.0017	772	3
58	670	884	.0012	752	2
59	690	.99942	.0006	731	1
60	.70711	1.0000	1.0000	.70711	0
	N. Cos.	N. Cot.	N. Tan.	N. Sin.	'

EXPLANATION OF TABLES.

1. Definition of logarithms. The logarithm of a number to the base k is defined as the power to which k must be raised in order to equal the number. That is, if the logarithm of A to the base k is a , then $k^a = A$. We may then write $\log_k A = a$. This equation is read "The logarithm of A to the base k equals a ."

Since $2^3 = 8$, we have $\log_2 8 = 3$. Similarly since $2^2 = 4$ and $3^2 = 9$, we have $\log_2 4 = 2$ and $\log_3 9 = 2$.

Example: Find the values of $\log_3 27$, $\log_{10} 1000$, $\log_2 32$, $\log_k k$, $\log_k 1$.

2. Logarithms of products, quotients, powers and roots.

Let $A = k^a$ and $B = k^b$ so that $\log_k A = a$ and $\log_k B = b$.

Then $AB = k^a k^b = k^{a+b}$.

Hence $\log_k (AB) = a + b$.

Therefore $\log_k (AB) = \log_k A + \log_k B$.

Also $A \div B = k^a \div k^b = k^{a-b}$.

Hence $\log_k (A \div B) = a - b$.

Therefore $\log_k (A \div B) = \log_k A - \log_k B$.

And $A^n = (k^a)^n = k^{an}$.

Hence $\log_k (A^n) = na$.

Therefore $\log_k (A^n) = n \log_k A$.

And $\sqrt[n]{A} = (k^a)^{\frac{1}{n}} = k^{\frac{1}{n}a}$.

Hence $\log_k \sqrt[n]{A} = \frac{1}{n}a$.

Therefore $\log_k \sqrt[n]{A} = \frac{1}{n} \log_k A$.

These four results may be stated as follows:

1. The logarithm of the product of two numbers equals the sum of the logarithms of the two numbers.

2. The logarithm of the quotient of two numbers equals the logarithm of the numerator minus the logarithm of the denominator.
3. The logarithm of the n th power of a number equals n times the logarithm of the number.
4. The logarithm of the n th root of a number equals $1/n$ times the logarithm of the number.

3. Common Logarithms. For numerical calculations the most convenient system of logarithms is that in which the base is 10. Logarithms to the base 10 are called common logarithms. In all that follows common logarithms will be used and $\log_{10} A$ will be denoted simply by $\log A$, etc.

The Characteristic. Since $10^1 = 10$ and $10^2 = 100$, we have $\log 10 = 1$ and $\log 100 = 2$, and hence the logarithm of any number between 10 and 100 equals a number between 1 and 2, and is therefore made up of 1 plus a decimal. The whole number 1 is called the *characteristic* of the logarithm and the decimal part is called the *mantissa*. Similarly the logarithm of a number between 100 and 1000 equals 2 plus a decimal, 2 being the characteristic and the decimal the mantissa of this logarithm. Further, since $10^0 = 1$ and $10^1 = 10$, the logarithm of a number between 1 and 10 is between 0 and 1 and the characteristic of such a logarithm is then 0.

Since $10^0 = 1$ and $10^{-1} = 0.1$, the logarithm of a number between 0.1 and 1 is between -1 and 0 and may therefore be written $-1 +$ a decimal. The characteristic of such a logarithm is then -1 . The mantissa is always considered positive whether the logarithm is a positive or negative number.

Suppose the characteristic of a logarithm is -2 and the mantissa .56253. This logarithm is then $-2 + .56253$ for which we use the notation $\bar{2}.56253$, the minus sign being written above the characteristic to show that it alone is negative. In almost all numerical calculations it is found to be more convenient to add and subtract 10, so that the logarithm takes the form $8.56253 - 10$. In practice the -10 is generally left off when there is no danger of confusion, but its existence must be remembered.

Example. What are the characteristics of the logarithms of

the following numbers: 315.72, 7.6523, 0.354, 0.000673, 27.0054?

The Mantissa. Consider two numbers made up of the same sequence of digits but in which the position of the decimal place is different, such as 723.51 and 7.2351. Now $723.51 = 10^2 \times 7.2351$. Hence, by art. 2, rule 1,

$$\log 723.51 = \log 10^2 + \log 7.2351.$$

But $\log 10^2 = 2$, and hence .

$$\log 723.51 = 2 + \log 7.2351.$$

That is, the logarithms of these two numbers differ only in the characteristics, and the mantissæ are the same. Obviously two numbers made up of the same sequence of digits, but differing in the position of the decimal point, are such that either is equal to the product of the other by an integral power of 10, and hence *the mantissæ of the logarithms of numbers differing only in the position of the decimal point are the same.* For this reason in tables of logarithms the mantissæ alone are given, the characteristics being determined by the two rules below,

From the foregoing we may formulate a rule for finding the characteristic of the logarithm of any number. A number with five digits to the left of the decimal place, such as 13256.7, is obviously between 10,000 and 100,000, that is, between 10^4 and 10^5 . Its logarithm is then between 4 and 5, and the characteristic is therefore 4.

Similarly the characteristic of the logarithm of a number having three digits to the left of the decimal place is 2, and finally the logarithm of a number having n digits to the left of the decimal place is $n - 1$

Hence, *the characteristic of the logarithm of a number is one less than the number of digits to the left of the decimal place.*

This rule obviously does not apply to the logarithms of numbers less than 1. Consider the logarithm of a number having no integral part and two zeros to the right of the decimal before the first significant digit appears, such as 0.00325. This number is greater than one one-thousandth and less than one one-hundredth and is therefore between 10^{-3} and 10^{-2} . Its

logarithm is then $-3 +$ a decimal, so that the characteristic is -3 . Similarly the characteristic of $\log 0.041$ is -2 .

Hence for the logarithms of numbers less than 1 *the characteristic is minus one more than the number of zeros to the right of the decimal place before the first significant digit.*

TABLE I.

4. To find the Logarithm of a Number.

(a) *When the number has four figures.* Find the first three figures of the number on pages 104–121 in the column under **N** and the fourth figure in the top row. The last three figures of the mantissa are found in the column and row so determined, and the first two figures are found in the column under **L**. The proper characteristic is then prefixed. Thus to find $\log 62.48$ we look for 624 under **N**. This is found on page 114, and the first two figures of the mantissa are 79. The last three figures of the mantissa are found opposite 624 and under 8. They are 574. Finally, the characteristic is 1, and therefore $\log 62.48 = 1.79574$.

Whenever the three last figures of the mantissa are preceded in the tables by an asterisk (*) it will be found that in this row under **L** two numbers are bracketed, as $\left. \begin{smallmatrix} 78 \\ 79 \end{smallmatrix} \right\}$ opposite 616 on page 114. In all such cases the upper of the bracketed numbers is to be used for the first two figures of the mantissa when there is not an asterisk, and the lower when there is one. Thus $\log 616.4 = 2.78986$ and $\log 616.7 = 2.79007$. Similarly we find $\log 0.7562 = \bar{1}.87864$ or $9.87864 - 10$; $\log 0.02543 = \bar{2}.40535$ or $8.40535 - 10$.

(b) *When the number has three or less figures.* Add zeros to the right of the number and then find the logarithm as above. Thus $\log 23 = \log 23.00 = 1.36173$ (page 106), etc.

(c) *When the number has five or more figures.* Let us be required to find $\log 277.53$. Since the mantissa does not depend upon the position of the decimal point, we may find it by finding the mantissa of $\log 27753$. The number is between 27750 and 27760, and hence its logarithm is between the logarithms of these numbers. It is therefore equal to $\log 27750 + x$, where

x is a *correction* to be added to $\log 27750$. We *assume* that the difference between the logarithms is proportional to the difference between the corresponding numbers. This assumption is only approximately true. Now,

$$\begin{array}{lcl} \text{the mantissa of } \log 27753 = .44326 + x & \left. \begin{array}{l} \\ \\ \end{array} \right\} & \begin{array}{l} \text{difference of numbers} \\ = 3, \text{ and of logs} = x, \\ \text{difference of numbers} \\ = 10, \text{ and of logs} = 16. \end{array} \\ \dots\dots\dots \log 27750 = .44326 & & \\ \dots\dots\dots \log 27760 = .44342 & & \end{array}$$

We then assume that $3 : 10 = x : 16$.

And hence $x = .3 \times 16 = 4.8 = 5 -$

Therefore $\log 277.53 = 2.44326 + (5^*$
 $= 2.44331$

The multiplication of .3 by 16 is most easily done by aid of the table of proportional parts found in the logarithm tables to the right of the logarithms. Thus under 16 and opposite 3 we find 4.8 which is the product of 16 by .3.

	16
1	1.6
2	3.2
3	4.8
4	6.4
5	8.0
6	9.6
7	11.2
8	12.8
9	14.4

From the foregoing we see that we may find the logarithm of a number of five figures as follows.

Find the mantissa of the logarithm of the first four figures, and add to this a correction found by multiplying the difference between this mantissa and the one next following in the tables (tabular difference) by the remaining figure of the given number considered as a decimal. Finally to the mantissa so obtained prefix the proper characteristic.

If the number whose logarithm is required has more than five figures we find the logarithm of the nearest number of five figures. Thus, for $\log 380567$ we take $\log 380570$. With logarithmic tables giving only five places of decimals there is little or no accuracy gained by making a correction for a sixth figure.

In the correction any decimal less than .5 is neglected, and one over .5 is increased to 1. Thus the correction for $\log 18.202$ is found to be (page 105) 4.8, and this is taken as 5 so that

$$\log 18.202 = 1.26007 + (5,$$

* The symbol (is written before the correction 5 to indicate that this correction is to be added to the last figure of the mantissa.

the 5, of course, being added to the last figure of the mantissa. Then $\log 18.202 = 1.26012$.

When a correction involves exactly .5 it is customary to either neglect this .5 or to increase it to 1 as may be necessary in order to make the last figure of the mantissa an even number. Thus $\log 19115 = 4.28126 + (11.5)$. We then take the correction as 12 so that $\log 19115 = 4.28138$. On the other hand, $\log 19105 = 4.28103 + (11.5)$, and the correction is here taken as 11, so that $\log 19105 = 4.28114$. This arbitrary rule is followed as such errors are more apt to neutralize one another than would be the case were every .5 to be either neglected or increased to 1.

5. *To find the number corresponding to a given logarithm.* Let us be required to find the number n when given that $\log n = 1.08337$. The given mantissa .08337 is not to be found in the tables, but it lies between .08314 and .08350. Hence n , apart from the position of the decimal point, lies between 1211 and 1212. Therefore $n = 1211 + x$ where x is a correction we may find by assuming that the difference between two numbers is proportional to the difference between their logarithms.

We have, then,

$$\begin{array}{l} \text{mantissa of } \log 1212 = .08350 \\ \dots\dots\dots \log 1211 = .08314 \\ \dots\dots\dots \log n = .08337 \end{array} \left. \begin{array}{l} \text{difference of logs} = 36, \text{ and of} \\ \text{numbers} = 1, \\ \text{difference of logs} = 23, \text{ and of} \\ \text{numbers} = x. \end{array} \right\}$$

Therefore $x : 1 = 23 : 36$.

or $x = 23 \div 36 = .6 +$

Hence the sequence of numbers making up n is 1211.6.

Finally, since the logarithm has the characteristic 1, we must point off two places, and hence $n = 12.116$.

In finding the correction x we take the nearest tenth only. When we find six significant figures of a number by means of its logarithm from tables giving the mantissae to five places only, the sixth figure is very probably incorrect, and any attempt to find a seventh figure would be of absolutely no value.

The finding of the correction x may be performed more easily by means of the tables of proportional parts, as follows:

As we have seen $x = 23 \div 36$. That is x is the quotient obtained by dividing the difference between the given mantissa and the next smaller one in the tables by the tabular difference (the difference between successive mantissæ in the Tables).

	36
1	3.6
2	7.2
3	10.8
4	14.4
5	18.0
6	21.6
7	25.2
8	28.8
9	32.4

In the table of proportional parts under the tabular difference (36 in this case) find the nearest number to 23. This is seen to be 21.6, opposite to which is found 6. We then have $x = 6$.

Examples.

1. Find the logarithms of 25.897, 0.057281, 2537.3, 526.88.
2. Find n when given that $\log n = .32380$, $\log n = 8.58720 - 10$.
3. Verify the equation $\log 23.6 + \log 7.0004 = \log (23.6 \times 7.0004)$.
4. By means of logarithms find to five figures $\sqrt{227.54}$.

10 over 90° , use 180° - angle

TABLE II.

6. By $\log \sin \theta$ we mean the logarithm of that number which is equal to $\sin \theta$, and similarly for the logarithms of the other functions of θ . In Table II will be found the logarithms of the sines, cosines, tangents and cotangents of angles from 0° to 90° , computed for each minute. For angles from 0° to and including 44° the number of degrees is printed at the top of the page and the number of minutes in the column under ' (to the left of the column **L. Sin.**). The logarithms of the sines are found *under* the heading **L. Sin.**, those of the tangents under **L. Tan.**, etc. For angles from 45° to 90° the number of degrees is printed at the bottom of the page, the number of minutes is found in the right hand column, and the logarithms of the sines in the column *over* **L. Sin.**, and similarly for the other functions. Thus on page 147, we find $\log \sin 23^\circ 52' = 9.60704$, $\log \tan 23^\circ 31' = 9.63865$, $\log \sin 66^\circ 18' = 9.96174$, etc.

Since the sine and cosine of any angle are less than 1, the logarithms of these functions have negative characteristics. Also since the tangent of an angle less than 45° is less than 1, the logarithm of the tangent of such an angle has a negative characteristic. All of the logarithms in the columns *under* **L. Sin.**, **L. Tan.**, and **L. Cos.**, are too great by 10. Thus,

$\log \sin 23^\circ = 9.59188 - 10$, etc. In practice it is customary to leave off the -10 but its existence must be remembered (art. 3). The logarithms in the column *under L. Cot.* have correct characteristics.

7. *To find the logarithm of the sine of an angle.* Let us be required to find $\log \sin 42^\circ 24' 35''$. The required logarithm will be between $\log \sin 42^\circ 24'$ and $\log \sin 42^\circ 25'$. On page 166 we find $\log \sin 42^\circ 24' = 9.82885$, and the difference between this logarithm and $\log \sin 42^\circ 25'$ is given in the column under *d* as 14. Then the required logarithm will be $9.82885 + x$, where x is a correction found by assuming that the difference between two angles is proportional to the difference between the logarithms of their sines. That is, $x : 14 = 35 : 60$, or $x = \frac{35 \times 14}{60} = 8.1 +$. Hence $\log \sin 42^\circ 24' 35'' = 9.82885 + 8 = 9.82893$.

The correction to be added to 9.82885 will be found in the table of proportional parts under the tabular difference (14) and opposite 35''. With a little practice this correction may be made mentally and the final logarithm alone written down.

8. *To find the logarithm of the tangent of an angle* the method employed is exactly similar to that of the last article. Thus $\log \tan 56^\circ 11' 45'' = 0.17401 + (21 = 0.17422)$.

9. *To find the logarithms of the cosine and cotangent of an angle.* Since the cosine and cotangent of angles less than 90° decrease in value as the angles increase the logarithms of these functions also decrease as the angles increase, and hence the correction for the given number of seconds must be *subtracted*. Thus to find $\log \cos 57^\circ 32' 25''$, we have, page 156,

$$\begin{aligned} \log \cos 57^\circ 32' &= 9.72982 && (\text{tabular difference} = 20) \\ \text{correction for } 25'' &= && 8.3 \\ \therefore \log \cos 57^\circ 32' 25'' &= 9.72974. \end{aligned}$$

Similarly,

$$\begin{aligned} \log \cot 33^\circ 25' &= 0.18059 && (\text{tabular difference} = 27) \\ \text{correction for } 20'' &= && 9 \\ \therefore \log \cot 33^\circ 25' 20'' &= 0.18050. \end{aligned}$$

10. To find the angle corresponding to the logarithm of one of its functions. Let us be required to find θ when given

$$\log \sin \theta = 9.78350.$$

On page 161 we find that the given logarithm is between $\log \sin 37^\circ 24'$ and $\log \sin 37^\circ 25'$ (tabular difference = 16). Hence $\theta = 37^\circ 24' + x''$. The difference between the given logarithm (9.78350) and $\log \sin 37^\circ 24'$ (= 9.78346) is 4.

Hence $x'' : 60'' = 4 : 16$. Then $x'' = \frac{4 \times 60''}{16} = 15''$. And

therefore $\theta = 37^\circ 24' 15''$. The tables of proportional parts may be used to find the number of seconds as follows: in the column under the tabular difference find the number *nearest* to the difference between the given logarithm and the logarithm corresponding to the next smaller angle in the tables. Opposite to this will be found the proper number of seconds.

Find θ when given $\log \cos \theta = 9.80541$. On page 163 we find that the given logarithm is between $\log \cos 50^\circ 17' = 9.80550$ and $\log \cos 50^\circ 18'$. The tabular difference is 16 and the difference between the given logarithm and that corresponding to the *smaller* of these two angles ($50^\circ 17'$) is 9. Hence the number of seconds is found by looking for the number nearest to 9 in the table of proportional parts under 16. This is 9.3 and corresponds to $35''$. Hence $\theta = 50^\circ 17' 35''$.

In finding the number of seconds we take the nearest number of seconds given in the tables of proportional parts because we are not sure of obtaining accurate results closer than five seconds with tables giving only five places of decimals.

Examples.

1. Find $\log \sin 63^\circ 13' 15''$, $\log \cos 37^\circ 42' 55''$, $\log \cot 62^\circ 22' 5''$, $\log \tan 21^\circ 14' 40''$, $\log \cot 14^\circ 12' 50''$.
2. Find θ when given $\log \sin \theta = 9.88645$, $\log \cos \theta = 9.32253$, $\log \tan \theta = 0.25436$, $\log \cot \theta = 9.34152$, $\log \tan \theta = 9.22335$.
3. Given $a = 24.762 \times \tan 36^\circ 42' 45''$, find a .
4. Given $a = 2.0034 \sqrt{\cos 32^\circ 24' 25''}$, find a .
5. Find the numerical values of $\cos^2 37^\circ 23' 5''$ and $\sin^2 37^\circ 23' 5''$, and show that their sum is unity. (Owing to the fact that

the logarithmic tables give only five places of decimals, there may be an error in the last decimal.)

11. *To find the logarithms of the sine and tangent of angles near 0° .* The method we have employed for correcting the logarithmic sines and tangents for seconds does not give reliable results for angles between 0° and 4° . To find such logarithms we make use of two quantities S and T defined by the equations

$$\begin{aligned} S &= \log \frac{\sin \theta}{\theta''} = \log \sin \theta - \log \theta'', \\ T &= \log \frac{\tan \theta}{\theta''} = \log \tan \theta - \log \theta'', \end{aligned} \quad (1)$$

where θ'' is the number of seconds in θ . So that

$$\log \sin \theta = S + \log \theta'',$$

and

$$\log \tan \theta = T + \log \theta''. \quad (2)$$

On pages 124 to 128 will be found the proper values of S and T , the first three figures being printed at the top of the column and the last three opposite the required angle. The characteristics of S and T in the tables should be decreased by 10. Also in the column headed " will be found the number of seconds in each angle given in the tables.

Let us be required to find $\log \sin 2^\circ 32' 25''$.

On page 126 we find that the number of seconds in $2^\circ 32' = 9120$, and hence $\theta = 2^\circ 32' 25'' = 9120'' + 25'' = 9145''$.

$$\begin{aligned} \text{Hence we have} \quad S &= 4.68543 - 10 \\ \log \theta'' &= \log 9145 = 3.96118 \text{ (page 120)} \\ \therefore \log \sin 2^\circ 32' 25'' &= 8.64661 - 10. \end{aligned}$$

Similarly we may find $\log \tan 0^\circ 26' 35''$ as follows.

$$26' 35'' = 1560'' + 35'' = 1595''.$$

$$\begin{aligned} \text{Hence we have} \quad T &= 4.68558 - 10 \\ \text{and} \quad \log 1595 &= 3.20276 \\ \therefore \log \tan 0^\circ 26' 35'' &= 7.88834 - 10. \end{aligned}$$

12. *To find the logarithm of the cotangent of an angle near 0° .*

Since $\cot \theta = \frac{1}{\tan \theta}$, we have $\log \cot \theta = -\log \tan \theta$. We may

then find $\log \tan \theta$ by the method of the last article and, upon changing its sign, obtain the required logarithm. Thus, in article 11 we have found that $\log \tan 0^\circ 26' 35'' = 7.88834 - 10$. Hence we have $\log \cot 0^\circ 26' 35'' = 10 - 7.88834 = 2.11166$.

13. To find an angle between 0° and 5° when given the logarithm of its sine, tangent or cotangent.

Let us be required to find θ when given $\log \sin \theta = 8.34106$. On page 125 we find that θ is between $1^\circ 15'$ and $1^\circ 16''$, and that the corresponding value of S is $4.68554 - 10$. Then, since $S = \log \sin \theta - \log \theta$, we have $\log \theta = \log \sin \theta - S$.

$$\begin{aligned} \text{But} \quad \log \sin \theta &= 8.34106 - 10 \\ S &= 4.68554 - 10 \\ \log \theta'' &= 3.65552 \end{aligned}$$

$$\text{and} \quad \theta'' = 4524'' = 1^\circ 15' 24''.$$

In a similar manner we may find θ when given $\log \tan \theta$. Let $\log \tan \theta = 8.44932 - 10$. Then θ is between $1^\circ 36'$ and $1^\circ 37'$ and the corresponding value of T is $4.68569 - 10$. Then,

$$\begin{aligned} \log \tan \theta &= 8.44932 - 10 \\ T &= 4.68569 - 10 \\ \text{therefore} \quad \log \theta'' &= 3.76363 \end{aligned}$$

$$\text{and} \quad \theta'' = 5803'' = 1^\circ 36' 43''.$$

As a third example let us find θ when $\log \cot \theta = 1.63442$. Then as in article 12, $\log \tan \theta = -\log \cot \theta = -1.63442 = 8.36558 - 10$. Hence θ is between $1^\circ 19'$ and $1^\circ 20'$, and the corresponding value of T is $4.68565 - 10$.

Then

$$\begin{aligned} \log \tan \theta &= 8.36558 - 10 \\ T &= 4.68565 - 10 \\ \text{therefore} \quad \log \theta'' &= 3.67993 \end{aligned}$$

$$\text{and} \quad \theta'' = 4786'' = 1^\circ 19' 46''.$$

13. The ordinary method for finding the correction for seconds also fails when we attempt to find the logarithms of the tangent, cotangent and cosine of angles near 90° . Now, $\tan \theta = \cot (90^\circ - \theta)$, and hence we may find $\log \tan \theta$ by finding $\log \cot (90^\circ - \theta)$. But when θ is near 90° , $90^\circ - \theta$ is near 0° ,

and therefore $\log \cot (90^\circ - \theta)$ may be found by the method of article 12. In like manner we may find $\log \cos \theta$ by finding $\log \sin (90^\circ - \theta)$, and $\log \cot \theta$ by $\log \tan (90^\circ - \theta)$.

Thus $\log \cot 89^\circ 44' 45'' = \log \tan 0^\circ 15' 15'' = \log \tan 915''$.

Then

$$T = 4.68558 - 10$$

$$\log 915 = 2.96142$$

$$\log \cot 89^\circ 44' 45'' = 7.64700 - 10.$$

As a second problem let us find θ when given $\log \cos \theta = 8.24104 - 10$. Then we have $\log \sin (90^\circ - \theta) = 8.24104 - 10$. Hence $90^\circ - \theta$ is between $0^\circ 59'$ and $1^\circ 0'$, and the corresponding value of S is $4.68555 - 10$.

Then

$$\log \sin (90^\circ - \theta) = 8.24104 - 10$$

$$S = 4.68555 - 10$$

therefore

$$\log (90^\circ - \theta)'' = 3.55549$$

and

$$(90^\circ - \theta)'' = 3593'' + = 59' 53''.$$

Hence $\theta = 90^\circ - 59' 53'' = 89^\circ 0' 7''$.

14. *To find the logarithms of functions of angles not in the first quadrant.* We may find $\log \sin 120^\circ$ by making use of the relation $\sin 120^\circ = \sin (180^\circ - 120^\circ)$, and looking up $\log \sin 60^\circ$. The case is, however, somewhat different if we need to find $\log \cos 120^\circ$, for $\cos 120^\circ$ is a negative number and a negative number has no real logarithm. We avoid this difficulty as follows: we have $\cos 120^\circ = -\cos 60^\circ$, and we use $\log \cos 60^\circ$. Since logarithms are in general only used in problems involving multiplication and division, we may consider any product or quotient as made up only of positive quantities, and after all the logarithmic work is completed, give the result the proper sign, positive when an even number of negative quantities are involved and negative when this number is odd. Even though N be a negative number it is customary to write $\log N$, this symbol being used instead of the strictly proper $\log (-N)$. Thus we write $\log \cos 120^\circ = \log \cos 60^\circ = 9.69897$.

Examples.

- Find $\log \cos 132^\circ 45' 35''$, $\log \sin 2^\circ 22' 15''$, $\log \tan 1^\circ 21' 55''$;
 $\log \cot 0^\circ 1' 5''$, $\log \cos 87^\circ 59' 30''$, $\log \cot 88^\circ 53' 50''$.

2. Find the numerical value of $a = 312.87 \times \sin 2^\circ 2' 20'' \times \cos 100^\circ 43' 45''$.
3. By means of logarithms verify $\sin 140^\circ 26' 40'' = 2 \times \sin 70^\circ 13' 20'' \times \cos 70^\circ 13' 20''$.
4. Verify the relation $\cos 2x = 2 \cos (45^\circ - x) \sin (45^\circ - x)$, when $x = 10^\circ 13' 15''$

TABLE III.

15. In this table are found the numerical values of the trigonometric functions of angles between 0° and 90° . The method of using this table is exactly similar to that made use of in finding the logarithms of the trigonometric functions in Table II. Tables of proportional parts are, however, not given, and so any necessary corrections must be performed by actual multiplication or division.

To find the value of $\sin 36^\circ 24' 25''$ we proceed as follows: From page 88 we find

$$\sin 36^\circ 24' = .59342$$

and

$$\sin 36^\circ 25' = .59365$$

The difference between these numbers is 23, and hence the correction to be added to $\sin 36^\circ 24'$ is $\frac{25 \times 23}{60} = 9.5 +$. The correction is then taken as 10 and we have $\sin 36^\circ 24' 25'' = .59342 + (10 = .59352$.

Examples.

1. Find the values of

$$\begin{aligned} \tan 65^\circ 51' 50'', \quad \cos 32^\circ 12' 15'', \\ \cot 14^\circ 16' 55'', \quad \cos 154^\circ 24' 25''. \end{aligned}$$

2. Find $\sin 55^\circ 24' 35''$. Then look up the logarithm of this number in table I, and show that the result so obtained is the same as $\log \sin 55^\circ 24' 35''$ as found from Table II.

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